Smoothing non-stationary noise of the Nigerian Stock Exchange All-Share Index data using variable coefficient functions

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Abstract *We employed variable coefficient models or generalized additive models (GAMs) with scatterplot smoothers such as smoothing splines to establish non-linear relationships that exist between non-stationary Nigeria Stock Exchange All-Share Index and three major monetary policy macroeconomic indicators using data collected from Central Bank of Nigeria and Nigeria Statistical Bulletin between 2004 and 2014. Two flexible smoothing splines were postulated using backfitting algorithm with weights and stringent control on convergence precision and maximum number of iterations for convergence. This was to ensure convergence of the iterative estimation procedure and to provide unbiased estimate of the regression coefficients and their standard errors. Generalized Cross-validation (GCV) re-sampling technique was employed in determining the effective degrees of freedom EDF and smoothing parameters for the smoothing spline functions. We compared models accuracy using the residual deviance for the GAMs. Smoothing splines without interaction terms was selected. The analysis shows that volatile exchange rates, rising inflation rates and Treasury bill rates remain the major monetary policy macroeconomic variables causing instability in the growth of the country's major stock market index, which is in line with conclusions drawn by [1].* ***R*** *programming language packages were employed throughout the analysis.*
Keywords: Nigerian Stock Exchange All-Share Index, variable coefficient models, generalized additive models, generalized cross-validation, smoothing splines and ***R*** programming language.

1.0 Introduction

[1] employed a modified autoregressive distributed lag modelling procedure of bounds testing or a conditional restricted equilibrium correction model as proposed by [2] to establish non-linear relationships existing between Nigeria Stock exchange (NSE) All-Share Index and three major macroeconomic indicators such as treasury bills, nominal exchange rate and inflation rate. The paper's analysis produced a model with predicted values of NSE All-share index very close to the actual values (**Figure** A). One advantage of this conventional method was that it outperforms most of the other econometric techniques when the sample size is small. In addition to reduced bias, we were able to minimize the variance of the residual error term. Furthermore, since the data generating process underlying the macroeconomic variables are all of the same integrated order one (i.e. I(1)), fitting an ARDL model allowed us to establish both the long-run cointegrating relations and short-run dynamic effects amongst the endogenous variables using the traditional ordinary least squares method of estimation. However, the estimates of the model parameters are average effects that provided limited information about the true pattern inherent in the data.

Plot on NSE All-Share Index between 2004 and 2014

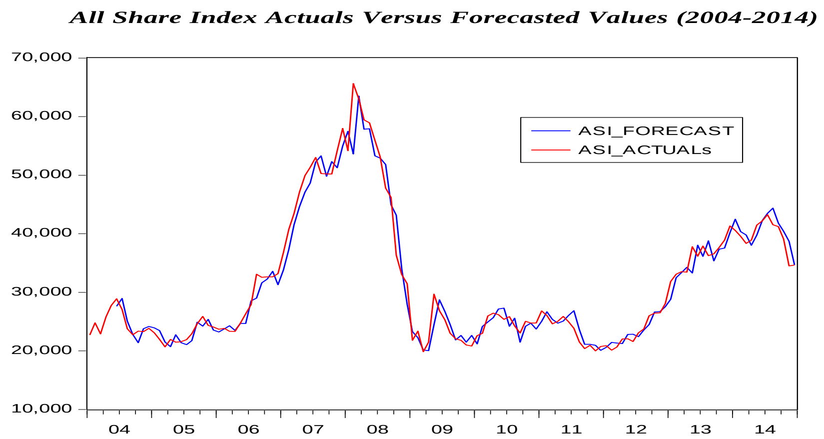


Figure A *Forecast and Actual values plot on NSE All-Share Index using the ARDL (4,3,2,2) model. This model was estimated using four, three, two and two lags of NSE All-Share Index, exchange rates, inflation rates and Treasury bill rates respectively. All variables are integrated of order one and the model's AIC = -2.6087.*
Source: *[1].*

In this current work, we introduce an alternative method of studying the non-linear relationships existing between the same set of macroeconomic variables in a non-parametric manner. This method unlike the ARDL is a semi-parametric and non-parametric regression learning method first introduced by [3]. It is known as the Variable Coefficient Model or Generalized Additive Model (GAM, hereon). GAM belongs to the family of Generalized Linear Models (GLM) or the Exponential family of distribution. Unlike most members of GLM, GAM replaces the linear predictor of a set of predictors with a sum of unspecified smooth functions whose parameters are estimated using a scatterplot smoother such as kernel estimate or the cubic spline amongst several other methods such as running mean and running median [3,4]. In the last decade, this statistical technique has gained popularity amongst statisticians and econometricians alike because it allows for nonparametric adjustments for nonlinear confounding effects of seasonality and trends in a time series data. Also, GAM's relevance in terms of flexibility, bias-variance trade-off optimization and model interpretability cannot be overlooked which has made it an important econometric analysis tool.

2.0 Generalized Additive Model (GAM) on NSE All-Share Index

GAM is an extension of the generalized linear models with nonlinear function $\eta=\Sigma_j s_j(x_j)$, where each $s_j(x_j)$ is unspecified nonparametric function estimated using smoothing splines or local regression (LOESS) smoothers. We postulated a generalized additive model on NSE All-Share Index (***as***) and exchange rate (***er***), inflation rate (***ir***) and treasury bill rate (***tb***) using monthly data extracted from the database of Central Bank of Nigeria and Nigeria Statistical Bulletin between 2004 and 2014 as expressed in equation 1.1. The GAM is of the form

$$\log(as_t) = \beta_0 + s_1(er_t,\Omega_1) + s_2(ir_t,\Omega_2) + s_3(tb_t,\Omega_3) + \xi_t \qquad 1.1$$

Where each unspecified univariate smooth function s_j is estimated using a combination of very fast additive backfitting algorithm with weights (ARBAW). This approach uses a scatterplot smoother to generalize the Fisher scoring procedure for calculating MLE [3,4]. The s_j's are standardized smooth functions such that $\mathbf{E}[s_j(x_j)]=0$. Each smooth function is only estimable to within an additive constant. The *backfitting* algorithm is appropriate for fitting several additive models, and in GAM it is used within the local scoring iteration when several smooth functions are included in the model [5]. The error term is assumed to be gaussian and for simplicity that each covariate is standardized within the [0,1] interval. Scatterplot smoother is a method of estimating the smooth non-parametric functions $s_j(.)$ i.e. splines. According to [6], the response, $\boldsymbol{y}_i$ is usually measured with noise and it is generally more useful to smooth ($\boldsymbol{x}_{ij}$, $\boldsymbol{y}_i$) data rather than interpolating them by setting $g(.)$ in the penalized residual sum of squares (*PRESS*) equation as T free parameters of the cubic spline. The non-parametric form of the functions in equation 1.1 allows for relatively flexible pattern of the dependence of ***as*** on the three macroeconomic variables, but by specifying the model only in terms of smooth functions s_j. This flexibility and convenience comes at the cost of two conjectural problems. It is essential to define the smooth functions involving basis dimension and location of region boundary and to determine the degree of smoothness controlled by a tuning parameter Ω_j. According to [3], GAMs allow for implicit non-linear relationships between the response and the covariates without suffering from "*curse of dimensionality*". We estimated each of the functions in equation 1.1 separately as an iterative approximation in order to find the optimal estimates using the scatterplot smoothers. The convergence of the *backfitting* algorithm for the GAM in equation 1.1 was controlled by stringent parameters for the control of convergence precision and the maximum number of iterations permissible (ε and M for the control of convergence precision and maximum number of iterations respectively and user-defined parameters ε_{bf} for the control of convergence precision and M_{bf} for the control of the maximum number of iterations to employ in the *backfitting*). In line with the work of [5], the extra stringent control was to ensure the convergence of the iterative estimation procedure and produce unbiased estimates of the regression coefficients and standard errors. We are able to fit non-linear s_j to each predictor allowing non-linear relationships missed by a linear model.

2.1 *The Smoothing Splines Model (SSM) on NSE All-share index in Nigeria*

We establish the natural cubic spline scatterplot smoother which involves fitting the model in equation 1.1 by using additive regression backfitting algorithm with weights (ARBAW) in which each nonparametric smooth function s_1, s_2, s_3 are cubic smoothing splines. GAM estimation technique of backfitting has the advantage that the component functions of an additive model to be represented using almost any smoothing or modelling technique. However, it has the disadvantage that estimation of the degree of smoothness of a model is difficult to incorporate into this approach [6]. Smoothing splines result from minimizing a penalized residual sum of squares criterion subject to a smoothness penalty. They are natural cubic splines with knots at every unique observation of x_i. The details are not covered in this research work. Additive regression backfitting algorithm with weights (ARBAW) fits a multiple predictors model by repeatedly updating the fit for each predictor in turn holding the others fixed. This approach uses the scatterplot smoothers to generalize the Fisher scoring procedure for calculating [3].

Scatterplots of likely association between NSE All-Share Index and its predictors in Nigeria from 2004 to 2014

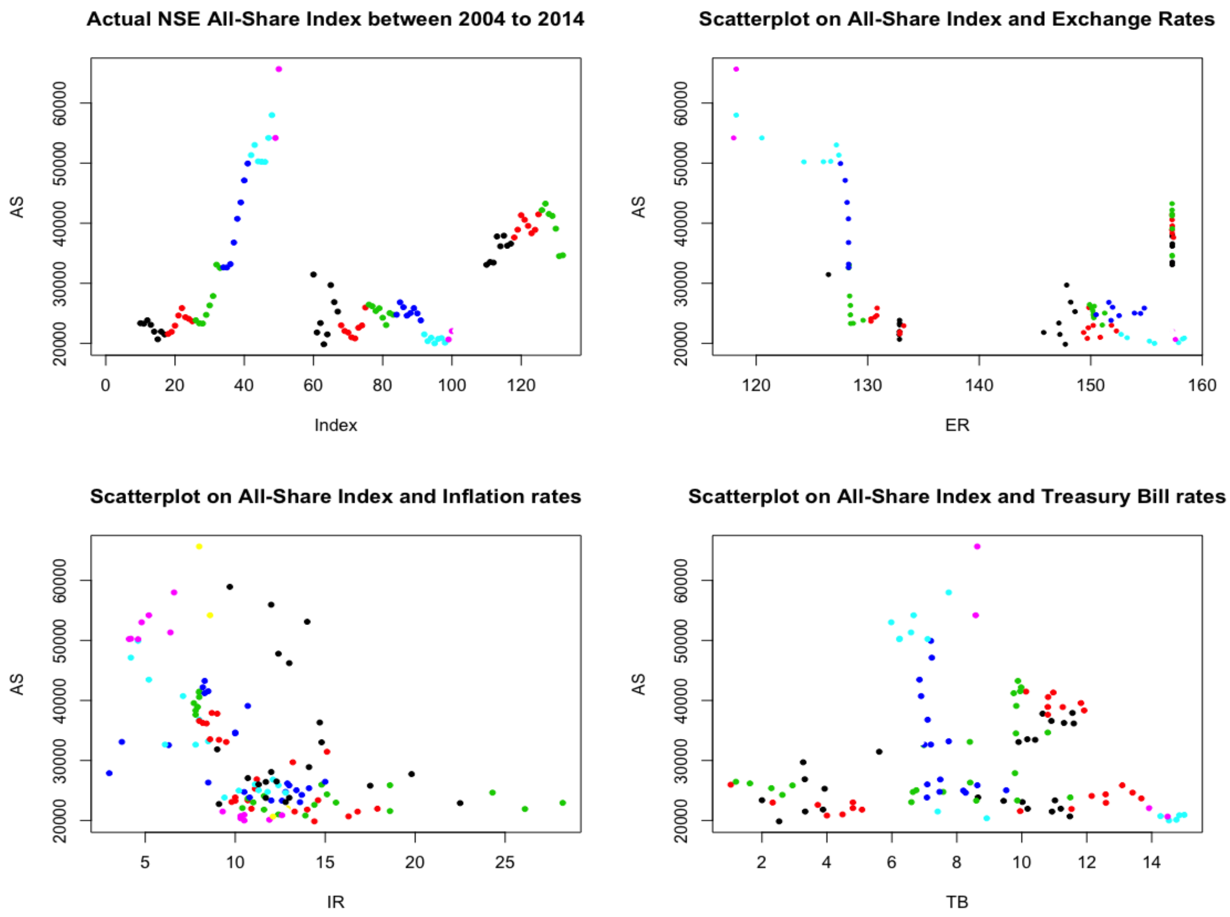


Figure 1: *Scatterplots on NSE All-Share Index (**as**) and the three macroeconomic variables. These plots exhibit patterns of non-linearity. The upper left panel is a time plot showing the non-stationary noise exhibited by NSE All-Share Index between 2004 and 2014.*
***Source**: Personal computation using **R** Studio ggplot2 package [7].*

The backfitting algorithm [3,4,8,9] generates a smoothing spline that minimizes the penalized residual sum of squares (PRESS) in equation 1.2

$$PRESS = \sum_{t=1}^{132}(y_t - \beta_0 - \sum_{j=1}^{3} s_j(x_{jt}))^2 + \sum_{j=1}^{3}\Omega_j \int g_j^{||}(t_j)^2 dt_j \qquad 1.2$$

$$g(x_{jt}) = \beta_0 + \sum_{j=1}^{3} s_j(x_{jt}) \qquad 1.3$$

ARBAW for the model in equation 1.1 is as given below;

- Set $\hat{\beta} = \hat{s}_j = 0$ for j=1,2,3 , m=0.
- Iterate: m=m+1< M (outer loop)
  1. Form the adjusted dependent variable:

$$z = \eta^{m-1} + (y - \mu^{m-1})\partial\eta / \partial\mu^{m-1}$$

where

$$\eta^{m-1} = \beta^{m-1} + \sum_{j=1}^{3} s_j^{\,m-1} = \log(\mu^{m-1})$$

and

$$z = (z_1, \ldots\ldots\ldots\ldots z_T), \eta = (\eta_1 \ldots\ldots\ldots\ldots, \eta_T), \mu = (\mu_1, \ldots\ldots\ldots, \mu_T), y = (y_1, \ldots\ldots y_T)$$

2. Form the weights $w = (\partial\mu / \partial\eta^{m-1})^2 V^{-1}$, where V is the variance of y at μ^{m-1}
3. Fit an additive model to ***z*** using the *backfitting* algorithm with weights ***w*** and estimates $\beta^m, s_1^{\,m}, s_2^{\,m}, s_3^{\,m}$ and η^m as follows:
   3.1 Cycle j = 1,2,3 (inner loop)
   3.2 Compute residuals by removing the estimated functions or covariate effects of all the other variables: t=1,2,…….,T

$$r_{t1} = y_t - s_2^{\,m-1} - s_3^{\,m-1}$$

$$r_{t2} = y_t - \beta^{m-1} - s_1^{\,m-1} - s_3^{\,m-1}$$

$$r_{t3} = y_t - \beta^{m-1} - s_1^{\,m-1} - s_2^{\,m-1}$$

   3.3 Estimate the $s_j^{\,m}$ by smoothing the residuals with respect to the next covariate:

$$s_1^m = s \quad (r_1 / x_{1t})$$

$$s_2^m = s \quad (r_2 / x_{2t})$$

The term $smooth(r_j / x_{jt})$ denotes a smoothing of the data (r_j , x_j) at the point x_{jt}. The parameter estimate β^m is obtained by fitting weighted least squares on the data (r_j, x_j).

   3.4 Compute the backfitting convergence criterion:

$$RSS^m = \frac{1}{132} \sum_{t=1}^{132} \left[y_t - \beta^m - \sum_{j=1}^{3} s_j^{\,m}(x_{jt}) \right]^2$$

   3.5 Stop when $\left| RSS^m - RSS^{m-1} \right| < \varepsilon_{bf}$

We attempt to find the function *g* or smoothing spline, which minimizes the equation 1.2 using the *backfitting* algorithm above. This function is the shrunken natural cubic spline with region boundaries at unique values of $\boldsymbol{x}_{ij}$*'s* with continuous first and second derivatives at each region boundary or *knot*. The level of shrinkage is controlled by the tuning parameter Ω in each smooth function in equation 1.1 determined by solving equation 1.6a through 1.6d. Equation 1.2 is a function of residual sum of squares plus a smoothness penalty associated with each s_j. In order to avoid the excessive flexibility associated with smoothing splines when Ω is too low and the over-smoothness when Ω is too high, we employed the cross-validation re-sampling method to determine the tuning parameter and the corresponding effective degrees of freedom so that the estimated model is as close as possible to the true model. Specifically, the generalized cross-validation (GCV) approach (by setting the "*cv*" argument in the "*smooth.splines*" function in ***R*** programming language to FALSE), which is based on orthogonal rotation of the residual matrix such that the diagonal elements of the influence or hat matrix **A** is as even as possible [6]. A hat (influence) matrix provides the estimates of vector of **E**(*Y*) when post-multiplied by the data vector ***y***. It has been proven that ordinary cross-validation approach suffers from two major problems. Firstly, there is a problem of cost associated with the number of smoothing parameters involved. Secondly, there exist a problem of lack of invariance from comparing the ordinary cross-validation scores. Generalized cross-validation (GCV) addressed these problems by rotating an influence matrix **A** such that the effective degrees of freedom are the sum of the diagonal elements in the matrix A_Ω [6]. That is

$$\mathrm{EDF}\,(\Omega) = \mathrm{tr}(\mathbf{A}) \qquad 1.6a$$

The equation 1.6a is derived from the decomposition of

$$A_\Omega = QAQ^I \qquad 1.6b$$

such that if $\mathbf{A}=\mathbf{VV}^I$, where V is any matrix square root, then

$$A_\Omega = QVV^IQ^I \qquad 1.6c$$

since the orthogonal matrix **Q** is such that each row of **QV** has the same mathematical length, the principal elements in the leading diagonal of the influence matrix A_Ω have the same value [10] i.e.

$$tr(A_\Omega) = tr(QAQ^I) = tr(AQ^IQ) = tr(A) \qquad 1.6d$$

[4] have shown that the GCV score for a GAM expressed in equation 1.6e derived from the rotation of the ordinary cross-validation score has predicted error unaffected by the rotation.

$$\upsilon_{gcv} = \frac{TD(\hat{\beta})}{[T - tr(A)]^2} \qquad 1.6e$$

where $D(\hat{\beta})$ is the estimate of the unpenalized model deviance. The integral part of equation 1.2 measures the overall change in the slope $g'(t)$ over its entire range measuring the roughness of the slope. As the smoothing parameter Ω approaches ∞, the EDF decreases from T to 2, where T is the sample size (here, T=132). If EDF reduce to 2, then all observations are utilized and the resulting curve tallies with the least squares regression line. Of course this will result in a model with low residual variance but high bias. Hence, the EDF are determined in some ways using the cross validation method of re-sampling. This controls the trade-off between the adequacy of the model and model smoothness. ARBAW allows us to update a function using the partial residual. The partial residuals relating to the j^{th} smooth term are the residuals resulting from subtracting all the current model term estimates from the response variable, except for the estimate of j^{th} smooth. We exploit ***R*** programming language packages "*splines*" and "*gam*" to compute the smoothing parameter, the EDF and diagnostic checks for the GAM.

The model is estimated as

$$\log(\hat{as}_t) = 11.46 - 0.01\hat{er}_t - 0.02\hat{ir}_t - 0.03\hat{tb}_t \qquad 1.7$$

Table 1 show the analysis of non-parametric effects of the covariate in the smoothing splines model of equation 1.7. Analysis of the effects of the individual smooth splines models indicate that the non-parametric effects of the predictors are all statistically significant at 1 per cent level. These effects were tested through the hypothesis of linearity against the alternative hypothesis of nonlinearity.

Table 1 Analysis of non-parametric effects in smoothing splines 1 model

Effect	**Non-parametric df**	**Non-parametric F-value**	***p*-value**
s(er, edf=33.79)	32.88	16.89	2.20×10^{-16}*
s(ir, edf=8.65)	7.70	12.26	3.44×10^{-11}*
s(tb, edf=5.02)	4.00	10.44	6.33×10^{-07}*

*Note: * statistically significant at 1 per cent level. Reject the null hypothesis of linearity. Null Deviance=13.09 on 131 degrees of freedom, Residual Deviance= 0.71 on 83.54 degrees of freedom, AIC= -215.68. **Source**: Personal computation using **R** Studio "gam" package [11]*

From equation 1.6d and deploying the "*smooth.splines*" function in the "*splines*" package of the **R** Studio programming language, we calculated the optimal tuning parameters as Ω_1=0.67, Ω_2=0.77 and Ω_3=0.99 for each smoothing functions s_j, j=1,2,3 respectively which control the trade-off between model adequacy and smoothness of the model in equation 1.7. The generalized cross-validated (GCV) scores (equation 1.6e) for the smooth functions in equation 1.7 are 0.02, 0.05 and 0.08 respectively.

The null model which estimates one parameter for the data set has a deviance of 13.09. The deviance was reduced to 0.71 with about 83.54 degrees of freedom, one for the intercept and the remaining for the three predictors. The *p*-values under the non-parametric correspond to the null hypothesis of linearity versus the alternative of non-linear relationship. Extremely low *p*-values for all the smoothing functions indicate non-linear functions are sufficient for all the terms in the model in equation 1.7. Also, we tested for model adequacy using the residual deviance and the degree of freedom by conducting the deviance test. This test is based on the fact that the difference between the null model deviance and the alternative model deviance is distributed with chi-square with D_1-D_0 degrees of freedom, where D_1 and D_0 are the deviances for the null and alternative models respectively. Since test statistic=12.38 is greater than the table or critical value at all levels of significance, we conclude that the there no evidence of lack of fit and a model with three parameters is adequate for our data.

Smoothing Splines plots showing relationships between NSE All-Share index and its predictors

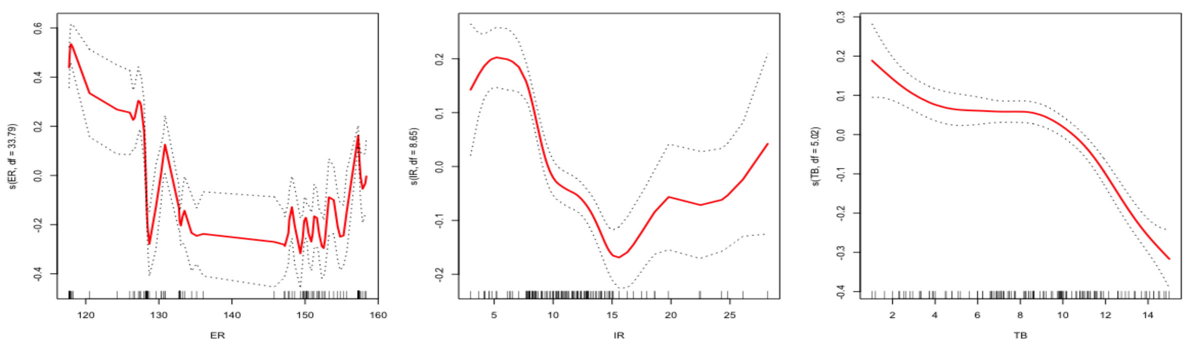


Figure 2: *Fitting a 3-term Generalized Additive Model using backfitting algorithm with stringent convergence parameters $\varepsilon=10^{-15}$, M=1,000, $\varepsilon_{bf}=10^{-15}$ and M_{bf}=1,000. The local scoring iteration converged at 3 iterations with 0.51 deviance. These are smooth spline plots of the relationships between each predictor and the NSE All-Share Index (**as**) in the fitted model (equation 1.7). By smoothing the partial residuals, an estimate of the corresponding smooth function (evaluated at the covariate values) is produced: these estimates are shown as red thick curves. For example, the third column, shows the partial residuals for s_3 on Treasury Bill rates (**tb**) at the final iteration, plotted against x_3 (i.e. **tb**): the thick curve is therefore the final iteration estimate of s_3, and results from smoothing the partial residuals with respect to **tb**. Also, each plot shows the fitted function and 2 times standard errors. The dotted curve is the estimated 95 per cent confidence interval for the smoothing splines. All functions are smoothing splines with 33.79, 8.65, 5.02 EDF respectively. **Source**: Personal computation using **R** Studio "splines" package [12].*

In **Figure** 2, the response variable ***as*** is expressed in terms of mean deviation (partial residuals), thus each smooth function s_j is centred and represents how ***as*** changes relative to its mean with unit changes in the predictors. Hence, the zero value on the response-axis is the mean of ***as***. The first panel shows that holding all other predictors fixed, ***as*** is extremely sensitive to increasing values of exchange rates (***er***). This is paprticuarly evident for exchange rates above the ₦150/$US. The middle panel showing the effect of inflation rates on ***as*** indicate that ***as*** is high for low and high values of inflation rates but lowest for intermediate values of inflation rates around the 15 per cent mark. The extreme right panel of **Figure** 2 shows the effects of treasury bill rates (***tb***) on ***as***. This curve shows a rather unsurprising consistent drop in ***as*** for every 1 per cent increase in the ***tb*** rate. Lawal *et al* mentioned that investors generally react to the increase in interest rate by balancing their portfolio in money market instruments such as the Treasury bills, which are risk-free and possess higher return.

2.2 *The Smoothing Splines Model (SSM) on NSE All-share index in Nigeria with interaction effects amongst predictors*

One of the snag associated with the GAM in equation 1.1 is that it is additive in the predictors, hence important interactions between variables was missed since p>2. Furthermore, theoretically, significant interactions exist between these major monetary macroeconomic indicators (see Lawal *et al* for details) leading to the inclusion of two-way and three-way interaction smooth functions in equation 1.8. In order to avoid this setback, we added interaction terms to the model in equation 1.1 by including additional predictors of the form ir_t*er_t, tb_t*er_t, tb_t*ir_t and $ir_t*er_t*tb_t$. Low-dimension interaction functions of the forms $s_4(ir_t,er_t)$, $s_5(tb_t,er_t)$, $s_6(tb_t,ir_t)$ and three-way interaction term $s_7(ir_t*er_t*tb_t)$ which were then estimated using the low-dimensional smoothing splines within the *backfitting* algorithm. This was done effortlessly using the "*gam*" function in the **R** programming language. The model with interaction terms is expressed in equation 1.8 below:

$$\log(as_t)=\beta_0+s_1(er_t,\Omega_1)+s_2(ir_t,\Omega_2)+s_3(tb_t,\Omega_3)+s_4(ir_t*er_t,\Omega_4)+s_5(tb_t*er_t,\Omega_5)+s_6(tb_t*ir_t,\Omega_6)+s_7(ir_t*er_t*tb_t,\Omega_7)+\xi_t \quad 1.8$$

The additional tuning parameters for the interaction terms was computed in similar manner using the "*smooth.splines*" function in the "spline" package of ***R*** programming language. The PRESS in equation 1.2 was modified to account for the extra interaction effects as expressed in equation 1.9

$$PRESS=\sum_{t=1}^{132}(y_t-\beta_0-\sum_{j=1}^{6}s_j(.))^2+\sum_{j=1}^{6}\Omega_j\int g_j^{||}(t_j)^2\,dt_j \quad 1.9$$

$$g(.)=\beta_0+\sum_{j=1}^{6}s_j(.) \quad 2.0$$

Where *g(.)* is the smoothing spline that minimizes the PRESS in equation 1.9. The *backfitting* algorithm under the GAM without interaction effects with stringent control on convergence parameters was easily modified to accommodate three interaction terms at the initialization stage and step 3 (the full detail is not covered in this paper). Whilst the control on the convergence precision ε and ε_{bf} were set at 10^{-20}, the maximum number of iterations required for convergence M and M_{bf} remained unchanged at 1,000. The *backfitting* convergence criterion was computed as

$$RSS^m=\frac{1}{132}\sum_{t=1}^{132}\left[y_t-\beta^m-\sum_{j=1}^{6}s_j^{\,m}(.)\right]^2 \quad 2.1$$

which terminates when $\left|RSS^m-RSS^{m-1}\right|<\varepsilon_{bf}$. Based on GCV, the EDF for each interaction smooth function in equation 1.8 are 6.55, 11.29, 4.29 and 4.15 for $s_4(er_t*ir_t)$, $s_5(er_t*tb_t)$, $s_6(ir_t*tb_t)$ and

s_7(***ir**$_t$****er**$_t$****tb**$_t$*)* respectively. The optimal tuning parameters are 0.99, 0.79, 1.15 and 1.14 respectively. Hence, the estimated model for equation 1.8 is

$$\log(\hat{as}_t) = 18.73 - 0.060\hat{er}_t - 0.707\hat{ir}_t - 0.893\hat{tb}_t + 0.005(\hat{ir} * \hat{er})_t + 0.007(\hat{tb} * \hat{er})_t + 0.082(\hat{tb} * \hat{ir})_t - 0.001(\hat{ir} * \hat{er} * \hat{tb}) \quad 2.2$$

Table 2 show the analysis of non-parametric effects of the covariates in the smoothing splines model of equation 1.9. The inclusion of additional interaction term between the three predictors in the analysis of the effects of the individual smooth function indicates that the non-parametric effect of the three-way interaction term and interaction effect between inflation rate and Treasury bill rates are also linear. This model revealed relationships between NSE All-Share index and its predictors, which are supported by theory.

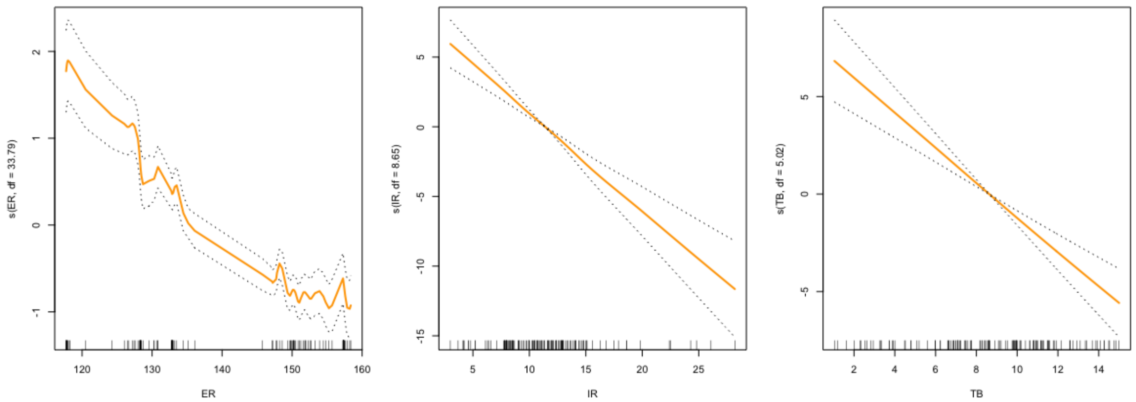
Table 2 Analysis of non-parametric effects in smoothing splines 2 model

Effect	Non-parametric df	Non-parametric F-value	*p*-value
s(er, edf=33.79)	32.88	15.97	$2.20\text{x}10^{-16}$*
s(ir, edf=8.65)	7.70	6.14	$1.36\text{x}10^{-05}$*
s(tb, edf=5.02)	4.00	1.68	0.17
s(er*ir, edf=6.55)	5.5	2.30	0.05***
s(er*tb, edf=11.29)	10.3	5.25	$1.66\text{x}10^{-05}$*
s(ir*tb, edf=4.29)	3.3	0.82	0.50
s(ir*er*tb, edf=4.15)	3.1	1.46	0.23

*Note: * , *** statistically significant at 1 per cent and 10 per cent levels. Reject the null hypothesis of linearity. Null Deviance=13.09 on 131 degrees of freedom, Residual Deviance= 0.43 on 57.26 degrees of freedom, AIC= -238.39.* ***Source****: Personal computation using* ***R*** *Studio "gam" package [11].*

The null model has a deviance of 13.09. The deviance was reduced to 0.43 with about 57.26 degrees of freedom, one for the intercept and the remaining for the three predictors. Extremely low *p*-values for all the smooth functions except the smooth function on treasury bill rates and interaction terms inflation rates and treasury bill rates, and the three-way interaction term amongst the predictors indicate non-linear functions are sufficient for all the terms in the model in equation 1.9. The smooth function on ***tb*** is a linear function as depicted in **Figure** 3. Model adequacy test also revealed that the model is adequate since the test statistic D_1-D_0 = (13.09-0.43)=12.66 is greater than the chi-sqaure with 7 degrees of freedom at all levels of significance (critical value).

Smoothing Splines plots showing relationships between NSE All-Share index and its predictors with interaction terms



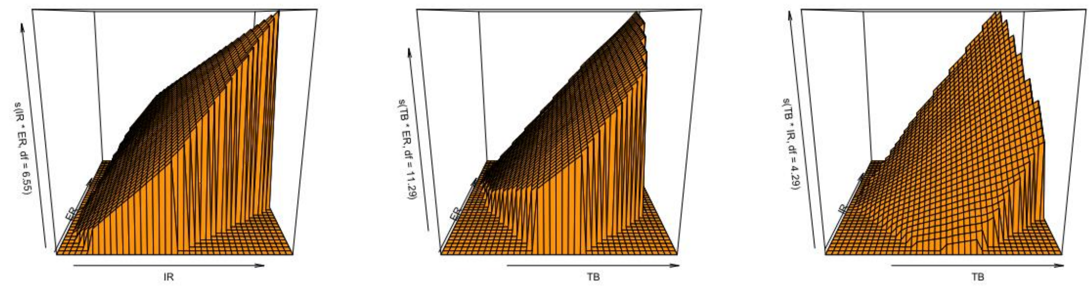


Figure 3: *Fitting a 7-term Generalized Additive Model using backfitting algorithm with stringent convergence parameters $\varepsilon=10^{-20}$, $M=1{,}000$, $\varepsilon_{bf}=10^{-20}$ and $M_{bf}=1{,}000$. The local scoring iteration terminated at 398 iterations with 0.43 deviance. The inclusion of the three-way interaction term in equation 1.9 has changed the nature of the relationships between NSE All-Share index and its predictors. These relationships are supported by theory. For example, increasing inflation rates and Treasury bill rates result in steadily declining index.* ***Source****: Personal computation using* ***R*** *Studio "splines" and "akima" packages [12,13].*

2.3 *Models comparison*

We test a null hypothesis that the GAM in equation 1.7 with fewer parameters is acceptable, against the alternative that the GAM in equation 1.9 with interaction effects is acceptable in a nested analysis of deviance. We use the difference between the alternative and null residual deviance as the test statistic. Since the sampling distribution of Ω under the null hypothesis is approximated assuming the smooth parameters are known by

$$\Omega \sim \chi^2_{(EDF_1 - EDF_0)}$$

where EDF_1 and EDF_0 are the total effective degrees of freedom under the alternative and null hypothesis respectively. EDF_1=73.74 and EDF_0 =47.46. Hence, we reject the null hypothesis that the simpler GAM is acceptable if 0.29 is greater than $\chi^2_{(1-(0.05)/2,26.28)}$. Since the test statistic is less than critical value of 14.57, we conclude that the model with no interaction effects is sufficient for the data on NSE All-Share Index. Hence, the smoothing splines with no interaction effects provide better estimates for the actual NSE All-Share Index values during the period under review as shown in the **Figure** 4.

Actual NSE All- Share Index versus Predicted values from Smoothing Spline 1 and Smoothing Spline 2 and residual plots

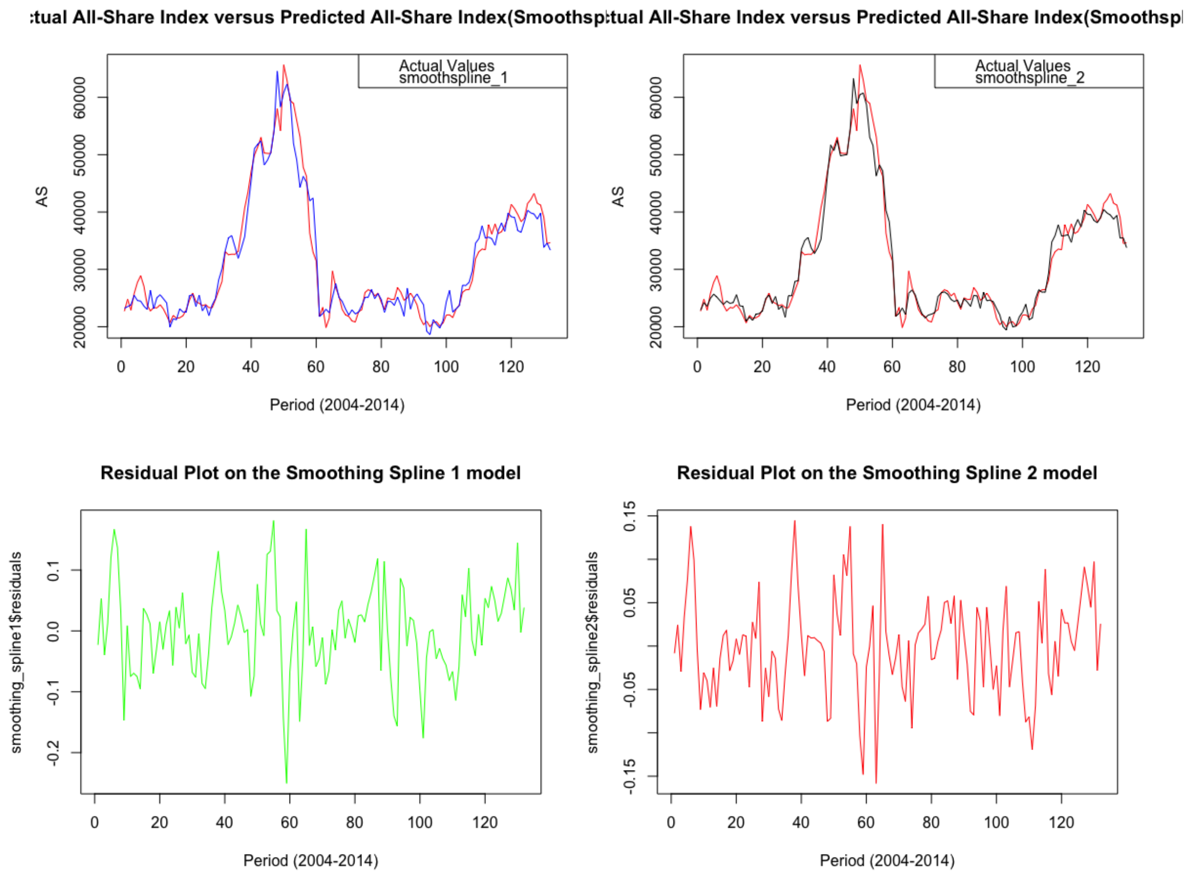


Figure 4 *Plots on the actual versus estimated NSE All-Share Index from the two postulated models. These plots do not reveal any obvious difference in the estimates of the two models. Hence, we need inference to reach a conclusion. Lower panels are the residual plots for the two smoothing spline models [12].*

Smoothing splines superimposed on scatterplots showing nonlinear long-term trends between All-Share Index and its predictors

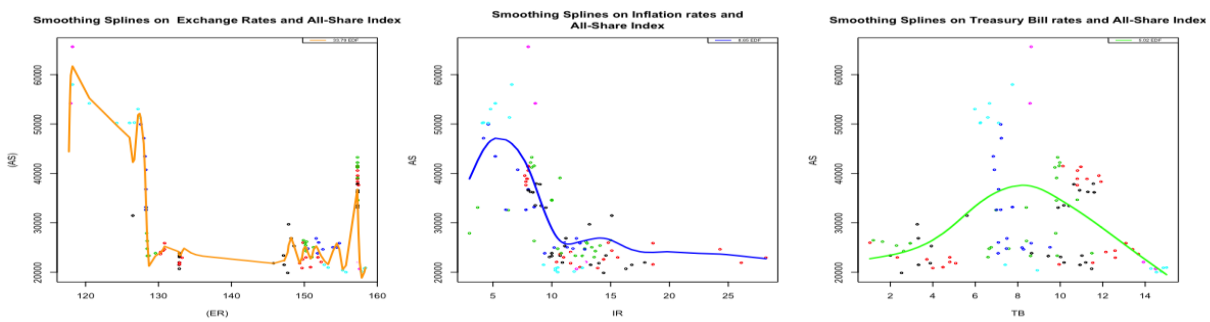


Figure 5: *Scatterplots with superimposed smoothing splines from model on smoothing splines. These show the long-term relationships existing between each predictor and the NSE All-Share Index (**as**) in the fitted model. Each plot shows various flexibilities and existence of non-linear long-term trend between **as** and each predictor. The left panel shows the relationship between NSE All-Share Index and Exchange rate. Rising exchange rate especially between ₦148/$US and ₦160/$US cause instability in stock market. Furthermore, a sharp drop was recorded in the index when exchange rate rose to about ₦128/$US. The middle panel is the plot on secular trend between the stock market index and inflation rate. This plot shows a sharp drop in NSE All-Share index when inflation rate rose to about 7 per cent. However it remained stable for values of inflation between 17 per cent and above. The right panel depicts the relationship between NSE All-Share Index and the Treasury bill rate. This* *plot revealed lower values in the index for low and high values of Treasury bill rate. Additionally, high values were recorded for intermediate values of the Treasury bill rate.* ***Source****: Personal computation using* ***R*** *Studio "splines" package [12].*

3.0 Conclusion

The purpose of this paper is to determine the nature of the relationships that exist between NSE All-Share Index and three major monetary macroeconomic indicators such as exchange rates, inflation rates and Treasury bill rates in Nigeria and make predictions that are comparable to conventional methods using variable coefficient models or generalized additive models. In the process, two very flexible models in form of smoothing splines with and without interaction terms were postulated. The analysis shows that smoothing spline without interaction terms provide better estimates of the NSE All-Share index during the period under review. Furthermore, the average effects of these monetary macroeconomic variables on the NSE All-share index indicate the existence of negative relationships. However, by varying the model coefficients, the index rose with increase in inflation rates above the 15 per cent mark based on the model without interaction effects. Also, varying coefficients of the model revealed that volatile exchange rates, rising inflation rates and Treasury bill rates remain the major macroeconomic variables causing instability in the growth of the country's major stock market index. According to [1] monetary policy stability is crucial to price level control because inflation is a monetary phenomenon in Nigeria. Therefore, the efficient use of Treasury bills as apparatus of monetary policy (inflation-targeting) and major source of government financing is essential to the growth of the Nigerian stock market.

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