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On the diffusion reaction of fourth-grade hydromagnetic fluid flow and thermal criticality in a plane Couette medium

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ABSTRACT

This study investigates hydromagnetic fourth-grade reactive diffusion, exothermic temperature distribution of the fluid flow and ignition criticality in a channel. This is important to improve the optimal efficiency of engineering devices and industrial machines. The fully exothermic combustible reaction occurs in a Couette medium with molecular diffusion. Ignoring material reacting consumption, the heat exchange in the system is greater than the ambient heat exchange, and the viscoelastic liquid is propelled by pre-exponential reaction rate and upper wall motion of the device. The resulted highly nonlinear dimensionless time variation, boundary value equations are solved by finite semi-discretization difference method. The obtained solutions for various parameters dependent flow characteristic are plotted and discussed. It is seen that activation energy increases the system thermal criticality while chemical kinetics decreases thermal ignition. Also, it is noticed from the study that the terms that raises the heat generation must be managed in order to prevent solution explosion and enhances optimal performance of engineering devices. The results from this study will assist in understanding the safe and unsafe regime of a system which can help in the chemical synthesis industries and thermal engineering operations.

1. Introduction

The flow of viscoelastic hydromagnetic liquid in a device in which one surface is fixed while the other moves in the absence of pressure gradient is important in lubricants. This has significant applications in nature, and in technology as well as in industrial design [1–3]. In the presence of heat transfer, non-Newtonian hydromagnetic fluid flow in a channel is useful in thrust bearing design, drag reduction, cooling transpiration, oil thermal recovery, polymer production and so on [4–7]. The mechanism of such applications which is characterized by immensely nonlinear derivative equations has led to numerous theoretical and experimental studies. Due to its industrial and environmental usefulness as well as its high viscous properties and strong elastic nature, the study of fourth-grade liquid has fascinated several scholars among are [8–12]. The fourth-grade fluid model is known to capture the interesting non-Newtonian flow properties such as shear thinning and shear thickening that many other non-Newtonian models do not exhibit. The liquid obeys order four fluid model, and the shear thickening and thinning are well described by fourth-grade liquid.

Non-Newtonian fluids are generally categorized as integral, rate and differential type, Neeraja et al. [13]. Fourth-grade fluid is an essential subclass of differential type that involves high number of complex terms that described the shear effects. Hayat et al. [8] investigated the steady and unsteady state of fourth-grade fluid in a porous Couette device with magnetic field influence but without heat transfer. Numerical solution to the highly nonlinear formulated fluid equation was constructed, and it was reported that a rise in the fluid material terms boosted the flow velocity. Moakher et al. [9] considered in the presence of slip condition, the magnetic field impact on the fourth-grade fully developed fluid flow in a device. The flow momentum equation along with the mixed Robin conditions were analytically solved, and it was revealed that the slip term decreases the velocity components. Sajid et al. [10] investigated the fourth-grade fluid solution analytically by explicit analysis of homotopy method. Obtained from the study without heat distribution, the second-grade viscoelastic properties strongly differs in behaviour when compared with fourth-grade fluid. However, the heat transfer impact on the fluid molecules can not be over emphasized. Hayat et al. [14,15] examined the heat transfer influence on a viscoelastic fourth-grade fluid. It is seen that the fourth-grade term decreases both the momentum and temperature components. Sahoo and Poncet [11] analyzed the effect of

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non-Newtonian, magnetic field and slip terms on the fourth-grade temperature distribution in a permeable infinite plate. The Broyden's technique was utilized for the computational analysis, and the results show that the slip and non-Newtonian terms have different impacts on the thickness of the heat boundary layer. All these studies were carried out without considering the fourth-grade exothermic diffusion process. However, this is essential in understanding the many engineering material properties used in technological innovation.

No matter the amount of heat that begins a reaction process, for an exothermic reactive diffusion system without heat loss, the fourth-grade reactive combustion will reach a thermal ignition critical level [16,17]. Thermal criticality arises when there is an approximated kinetic diffusion at high temperature with time history, this is appreciably essential from the practical point of view. For instance, it is applicable in chemical synthesis, explosion safety, ignition propulsion and others; Zeldovich et al. [18]. Steady reactants heat for a combustion process is needed to correctly monitor chemical synthesis and engineering device's critical state, Hassan et al. [19]. Cullis and Foster [20] employed the theory of thermal ignition to study a spontaneous chemical mixture of heat ignition limits. Okoya [21] examined the chemistry of thermal explosion branched-chain ignition time with heat loss. It was noticed that criticality is difficult for some terms as a result of reaction rate suppression. Salawu et al. [22] discussed the criticality and entropy minimization of a non-Newtonian hydromagnetic liquid with variable properties. It was reported that critical regimes need to be estimated in order to prevent blowup of reactive solution. Okoya [23] considered crtiticality transition of third-grade liquid with variable viscosity in a device. It was found that non-Newtonian term enhances the criticality disappearance in an exothermic reaction.

This study was stimulated by the reports from the works of [9,11,15] which were an extension of the studies done by Ref. [8,10]. The significance of the researchers reports on the industrial and engineering usefulness of fourth-grade working fluid has motivated further study on the reaction diffusion of the fluid and thermal criticality of thermal engineering system. Apart from the engineering applications of fourth-grade fluid in drag reduction, radial diffusers, thrust bearing and so on, the positive results achieved in the previous studies and suggested future extension has encouraged further investigations on the combustible diffusion reaction of the fluid. Of several studies carried out on fourth-grade liquids, little or no study has been done on the exothermic reactive diffusion and thermal criticality of fourth-grade fluids. Meanwhile, it is important to examine the critical regimes in order to determine safe and unsafe conditions of industrial chemical synthesis and machines. Also, it will help improving lubricant efficiency for optimal performance of industrial engines. However, this study explicitly considered theoretical mathematical analysis of the diffusion reaction of unsteady fourth-grade hydromagnetic fluid flow and thermal criticality in a Couette channel using finite difference techniques for the solution procedures.

2. The flow mathematical setup

The flow of unsteady incompressible hydromagnetic, exothermically combustible fourth-grade fluid is considered. The diffusion reaction and variable heat reliant pre-exponential factor are taken into account. A viscous, special homogenous Rivlin-Ericksen fluid type, namely fourth-grade fluid is examined in a plane Couette channel. The stress Cauchy compatible tensor $\mathbf{T}$ that is thermodynamically related to the fourth-grade fluid is written as Hayat et al. [8].

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{H}_1 + \delta_1\mathbf{H}_2 + \delta_2\mathbf{H}_1^2 + \mathbf{B}_1 + \mathbf{B}_2, \tag{1}$$

In which $\mathbf{B}_1$ and $\mathbf{B}_2$ are defined as follows:

$$\mathbf{B}_1 = \varphi_1\mathbf{H}_3 + \varphi_2(\mathbf{H}_2\mathbf{H}_1 + \mathbf{H}_1\mathbf{H}_2) + \varphi_3\left(tr\mathbf{H}_1^2\right)\mathbf{H}_1, \tag{2}$$

$$\mathbf{B}_2 = \Lambda_1\mathbf{H}_4 + \Lambda_2(\mathbf{H}_3\mathbf{H}_1 + \mathbf{H}_1\mathbf{H}_3) + \Lambda_3\mathbf{H}_2^2 + \Lambda_4\left(\mathbf{H}_2\mathbf{H}_1^2 + \mathbf{H}_1^2\mathbf{H}_2\right) + \Lambda_5(tr\mathbf{H}_2)\mathbf{H}_2 +$$

$$\Lambda_6(tr\mathbf{H}_2)\mathbf{H}_1^2 + (\Lambda_7 tr\mathbf{H}_3 + \Lambda_8 tr(\mathbf{H}_2\mathbf{H}_1))\mathbf{H}_1,$$

where μ denotes viscosity, $\mathbf{I}$ connotes tensor identity, p represents pressure and $\delta_i(i=1,2)$, $\varphi_i(i=1,2,3)$ and $\Lambda_i(i=1-8)$ are the material constants. Note that for Navier-Stokes liquid, $\delta_i = \varphi_i = \Lambda = 0$. When $\delta_i \neq 0$ but $\varphi_i = \Lambda = 0$, second-grade model is obtained. Also, when $\delta_i \neq 0$ and $\varphi_i \neq 0$ but $\Lambda = 0$, then third-grade model is obtained. The Rivlin-Ericksen kinematical tensors $\mathbf{H}_1$ to $\mathbf{H}_4$ are defined as

$$\mathbf{H}_1 = \mathbf{A} + \mathbf{A}^T, \tag{3}$$

$$\mathbf{H}_n = \frac{d\mathbf{H}_{n-1}}{dt} + \mathbf{H}_{n-1}\mathbf{A} + \mathbf{A}^T\mathbf{H}_{n-1}(n>1), \tag{4}$$

$$\mathbf{A} = \nabla\mathbf{W}, \tag{5}$$

the terms ∇ is an operator, $\mathbf{W}$ represents the velocity field and d/dt is the time derivative of material taken the form

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{W}\cdot\nabla). \tag{6}$$

The incompressible fluid flow equations for the mass conservation and momentum balance are given as

$$div\mathbf{W} = 0, \tag{7}$$

$$\frac{d\mathbf{W}}{dt} = \frac{1}{\rho}\mathbf{J}\times\mathbf{B} + \frac{1}{\rho}div\mathbf{T}, \tag{8}$$

where $\mathbf{B}$ denotes the total magnetic field expressed as $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$ in which $\mathbf{b}$ connotes magnetic field induced, while $\mathbf{J}$ stands for current density. According to Rossow [24] assumptions with small Reynolds number, $\frac{1}{\rho}\mathbf{J}\times\mathbf{B} = -\frac{\sigma}{\rho}B_0^2\mathbf{W}$. The equation for the exothermic heat reaction balance is

$$\rho Cp\frac{d\psi}{dt} = k\nabla^2\psi + \mathbf{T}\cdot\mathbf{A} + CDQ, \tag{9}$$

where k is heat conductivity, Cp connotes the heat capacity, ψ is the internal energy, C denotes reactant concentration species and Q represents reaction heat. The reaction generalized branch-chain rate D, follow from Ref. [23,25] takes the form

$$D = D_0\left(\frac{KT}{\hbar\nu_0}\right)^n exp\left(-\frac{E}{RT}\right), \tag{10}$$

for which D_0 is the branching chain constant order, K denotes Boltzmann's constant, R is constant universal gas, ν_0 is frequency of vibration, E stands for activation energy, $\hbar$ represents planck's number, n connotes pre-exponential numerical index factor, T is temperature. Here, the reaction mechanism is of *mth*-order satisfying

$$F + G \rightarrow P + Hr. \tag{11}$$

where F represents the fluid, G is oxidizer, P denotes product and Hr is the reaction heat. The thermal ignition criticality conditions are fulfilled when there is small temperature reference or large activation energy $(i.e.RT/E \ll 1)$ see, Boddington et al. [26].

In the study, the flow is propelled by the upper motion of the plate along the x direction in the channel center with y direction perpendicular to it under the influence of uniform magnetic field. With the above stated assumptions, the flow geometrical illustration for a Couette device is seen in (Fig. 1) and the dimensional reactive exothermic fourth-grade fluid flow equations according to Ref. [9,15,27] are given as:with conditions

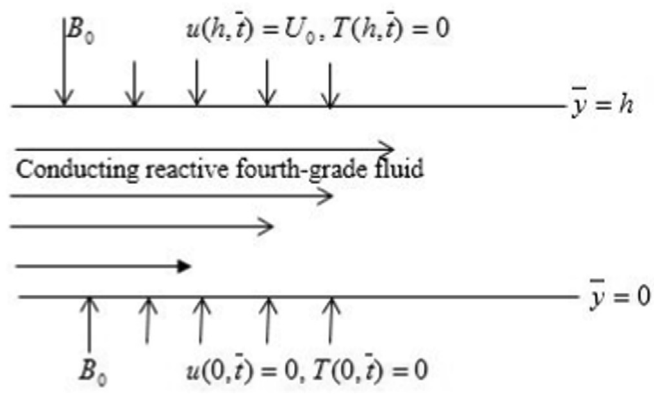

Fig. 1. Flow schematic diagram.

$$\frac{\partial u}{\partial \bar{t}} = \nu\frac{\partial^2 u}{\partial \bar{y}^2} + \frac{\delta_1\nu}{\rho}\frac{\partial^3 u}{\partial \bar{y}^2\partial \bar{t}} + \frac{\varphi_1\nu^2}{\rho}\frac{\partial^4 u}{\partial \bar{y}^2\partial \bar{t}^2} + \frac{6(\varphi_2+\varphi_3)}{\rho}\left(\frac{\partial u}{\partial \bar{y}}\right)^2\frac{\partial^2 u}{\partial \bar{y}^2} + \frac{\Lambda_1\nu^3}{\rho}\frac{\partial^5 u}{\partial \bar{y}^2\partial \bar{t}^3} + \tag{12}$$

$$\frac{2\nu(3\Lambda_2+\Lambda_3+\Lambda_4+\Lambda_5+3\Lambda_7+\Lambda_8)}{\rho}\left(\frac{\partial u}{\partial \bar{y}}\frac{\partial^2 u}{\partial \bar{y}^2}\frac{\partial^2 u}{\partial \bar{y}\partial \bar{t}} + \left(\frac{\partial u}{\partial \bar{y}}\right)^2\frac{\partial^3 u}{\partial \bar{y}^2\partial \bar{t}}\right) - \frac{\sigma B_0^2}{\rho}u,$$

$$\frac{\partial T}{\partial \bar{t}} = \frac{k}{\rho Cp}\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{\nu}{Cp}\left(\frac{\partial u}{\partial \bar{y}}\right)^2 + \frac{\delta_1\nu}{\rho Cp}\frac{\partial^2 u}{\partial \bar{y}\partial \bar{t}}\frac{\partial u}{\partial \bar{y}} + \frac{\varphi_1\nu^2}{\rho Cp}\frac{\partial^3 u}{\partial \bar{y}\partial \bar{t}^2}\frac{\partial u}{\partial \bar{y}} + \frac{2(\varphi_2+\varphi_3)}{\rho Cp}\left(\frac{\partial u}{\partial \bar{y}}\right)^4 + \tag{13}$$

$$\frac{\Lambda_1\nu^3}{\rho Cp}\frac{\partial^4 u}{\partial \bar{y}^2\partial \bar{t}^2}\frac{\partial u}{\partial \bar{y}} + \frac{2\nu(3\Lambda_2+\Lambda_3+\Lambda_4+\Lambda_5+3\Lambda_7+\Lambda_8)}{\rho Cp}\frac{\partial^2 u}{\partial \bar{y}\partial \bar{t}}\left(\frac{\partial u}{\partial \bar{y}}\right)^3 + \frac{\sigma B_0^2}{\rho Cp}u^2 + CDQ$$

$$\frac{\partial u}{\partial \bar{y}}(0,\bar{t}) = 0, \frac{\partial T}{\partial \bar{y}}(0,\bar{t}) = 0, u(\bar{y},0) = 0, T(\bar{y},0) = 0, u(h,\bar{t}) = U_0, T(h,\bar{t}) = 0. \tag{14}$$

Applying the subsequent variables on equations (12)–(14) to transform it to dimensionless form

$$t = \frac{\bar{t}U_0^2}{\nu}, w = \frac{u}{U_0}, \theta = \frac{E(T-T_0)}{RT_0^2}, t = \frac{\bar{t}U_0^2}{\nu}, \delta = \frac{\delta_1 U_0^2}{\rho\nu^2}, \varphi_a = \frac{\varphi_1 U_0^4}{\rho\nu}, \varphi = \frac{2(\varphi_2+\varphi_3)U_0^4}{\rho\nu^3}, \tag{15}$$

$$M = \frac{\sigma\nu B_0^2}{\rho U_0^2}, \Lambda_a = \frac{\Lambda_1 U_0^6}{\rho\nu}, \varepsilon = \frac{RT_0}{E}, Pr = \frac{\rho Cp\nu}{k}, \lambda = \frac{QCA\nu E}{U_0^2 RT_0^2}\left(\frac{KT_0}{\nu_0\hbar}\right)^n exp\left(-\frac{1}{\varepsilon}\right),$$

$$\Lambda = \frac{2(3\Lambda_2+\Lambda_3+\Lambda_4+\Lambda_5+3\Lambda_7+\Lambda_8)}{\rho\nu^4}, Br = \frac{\nu E}{U_0 RT_0^2}.$$

The dimensionless form of the main equations is gotten as

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial y^2} + \Lambda_a\frac{\partial^5 w}{\partial y^2\partial t^3} + \Lambda\left(\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial y\partial t} + \left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^3 w}{\partial y^2\partial t}\right) + \delta\frac{\partial^3 w}{\partial y^2\partial t} + \varphi_a\frac{\partial^4 w}{\partial y^2\partial t^2} + \tag{16}$$

$$3\varphi\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial y^2} - Mw,$$

$$Pr\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} + Br\left(\frac{\partial w}{\partial y}\right)^2 + BrMw^2 + \lambda(1+\varepsilon\theta)^n exp\left(\frac{\theta}{1+\varepsilon\theta}\right) + \tag{17}$$

$$Br\left(\delta\frac{\partial^2 w}{\partial y\partial t}\frac{\partial w}{\partial y} + \varphi_a\frac{\partial^3 w}{\partial y\partial t^2} + \varphi\left(\frac{\partial w}{\partial y}\right)^4 + \Lambda_a\frac{\partial^4 w}{\partial y^2\partial t^2}\frac{\partial w}{\partial y} + \Lambda\frac{\partial^2 w}{\partial y\partial t}\left(\frac{\partial w}{\partial y}\right)^3\right),$$

along with the conditions

$$\frac{\partial w}{\partial y}(0,t) = 0, \frac{\partial\theta}{\partial y}(0,t) = 0, w(y,0) = 0, \theta(y,0) = 0, w(1,t) = 1, \theta(1,t) = 0, \tag{18}$$

where w and θ are the dimensionless velocity and temperature. The terms M, Br, ε, λ, Pr and n are respectively the magnetic field, Brinkman number, activation energy, Frank-Kamenetskii, Prandtl number and chemical kinetics. δ is second-grade term, φ_a and φ are the third-grade terms, and Λ_a and Λ are the fourth-grade term.

2.1. Limiting cases of comparison

Non-Newtonian liquid of fourth order model has been discussed in the literature, among are the works of [8,10] that coincided with the present study in the absence of energy equation. For steady state without exothermic reaction with $\lambda = 0$, this study is equivalent to Ref. [14,15]. With constant viscosity in a plane Couette when $\Lambda = \Lambda_a = \varphi_a = 0$ and $M = 0$ or $M \neq 0$, the present problem reduces to Ref. [28,33].

The wall physical property for the fluid velocity and energy that are of engineering interest are called the skin friction Cf and Nusselt number Nu. The quantities are individually defined as

$$Cf = -\frac{\partial w}{\partial y}\Big|_{y=1}, Nu = \frac{d\theta}{dy}\Big|_{y=1} \tag{19}$$

3. Numerical solution

Here, the numerical solution scheme for the reactive boundary value, heat fluid flow equations is based on a Cartesian regular grid and mesh discretization arising from finite difference techniques [28–30]. First, the dimensionless interval [0,1] is partition. The interval is divided into regular L parts with the grid size taken to be $\Delta y = 1/L$ and $y_i = (i-1)\Delta y$ defines the grid point for which $1 \le i \le L+1$. This is done in time space intermediate $(L+\tau)$ with $\tau = 1$. The central second order difference is used to approximate the spatial derivatives for the first and second order, [31,32]. The boundary conditions are integrated by modifying the contained last and first equations grid points. The numerical scheme is tested for stability and consistence. The dimensionless derivative equations (16) and (17) are approximated, and modification of the boundary conditions (18) is asymptotically augmented and incorporated into the core equations. The discretization of the momentum component is expressed as:

$$\frac{\partial}{\partial t}\left(w - \delta\frac{\partial^2 w}{\partial y^2}\right) = \left(1 + 3\varphi\left(\frac{\partial w}{\partial y}\right)^2\right)\frac{\partial^2}{\partial y^2}w^{(L+\tau)} + \left[\frac{\partial}{\partial y^2}\left(\Lambda_a\frac{\partial^3 w}{\partial t^3} + \varphi_a\frac{\partial^2 w}{\partial t^2}\right)\right]^{(L)} - \tag{20}$$

$$Mw^{(L+\tau)} + \Lambda\left(\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial y\partial t} + \left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^3 w}{\partial y^2\partial t}\right)^{(L)}.$$

Note that $\frac{\partial\xi}{\partial t} = (\xi^{(L+1)} - \xi^{(L)})/\Delta t$and $w^{(L+1)}$ is expanded to have:

$$q_1 w_{j-1}^{(L+1)} + (1+2q_1)w_j^{(L+1)} - q_1 w_{j+1}^{(L+1)} = \text{explicit terms}, \tag{21}$$

where $q_1 = \Delta t/(\Delta y^2)$. A matrices tri-diagonal inversion is obtained from the solution process which is then solved to have the velocity solution. The discretization of the combustible heat component is obtained as:

$$Pr\frac{\theta^{(L+1)} - \theta^{(L)}}{\Delta t} = \frac{\partial^2}{\partial y^2}\theta^{(L+\tau)}$$

$$+ \left[Br\left(\left(\frac{\partial w}{\partial y}\right)^2 + Mw^2\right) + \lambda(1+\varepsilon\theta)^n exp\left(\frac{\theta}{1+\varepsilon\theta}\right)\right]^{(L)} \tag{22}$$

$$+Br\left[\left(\delta\frac{\partial^2 w}{\partial y\partial t}\frac{\partial w}{\partial y} + \varphi_a\frac{\partial^3 w}{\partial y\partial t^2} + \varphi\left(\frac{\partial w}{\partial y}\right)^4 + \Lambda_a\frac{\partial^4 w}{\partial y^2\partial t^2}\frac{\partial w}{\partial y} + \Lambda\frac{\partial^2 w}{\partial y\partial t}\left(\frac{\partial w}{\partial y}\right)^3\right)\right]^{(L)}.$$

The term $\theta^{(L+1)}$ is expanded to becomes.

$$q_2\theta_{j-1}^{(L+1)} + (1+2q_2)\theta_j^{(L+1)} - q_2\theta_{j+1}^{(L+1)} = \text{explicit terms}, \quad (23)$$

where $q_2 = \tau\Delta t/\Delta y^2$. The solution translates to matrices tri-diagonal inverse and it is solved to get temperature solution. Maple code algorithm is written for the solution techniques and tested for temporal and spatial convergence. The calculation is done for 200 steps time at $\Delta t = 1$ but found to be the same with $\Delta t = 5$ for 40 steps time. The code runs faster for various values of the parameters with infinitesimal computational times.

4. Results and discussion

The theoretical investigation of reactive diffusion of hydromagnetic fourth-grade fluid flow in a plane Couette device is studied using stable, convergent semi-discretization of finite difference techniques. The initial and boundary conditions are asymptotically augmented. The adopted default values used is taken after [8,15] as $\delta = 0.2$, $\varphi_a = 0.001$, $\varphi = 0.05$, $\Lambda_a = 0.001$, $\Lambda = 0.03$, $Pr = 3.0$, $\lambda = 0.02$, $Br = 0.03$, $n = 0.5$, $\varepsilon = 0.1$ and $M = 1.0$ are based on related previous studies. Table 1 represents the computed results for the wall friction and heat gradient. As seen, some parameters increases while some terms decreases separately the shear rate or wall heat transfer. Figs. 2 and 3 demonstrate the effect of time variation on the reactive combustible fluid flow and heat field in a three dimensional system. Early pronounced rise in the flow velocity and temperature distributions is noticed close to the lower channel surface as the time varies. The profiles reduce gradually towards the upper moving surface of the device as the exothermic reaction reached the peak.

4.1. Velocity dependent parameters solutions

The plot of velocity dependent parameters solutions for $w(y)$ against y is illustrated in Figs. 4–7 for different parameter values. Fig. 4 depicts the reaction of viscoelastic non-Newtonian fluid to rising in the values of the magnetic field term (M). The electrically conducting reactive fluid flow decreases due to induction of Lorentz force by magnetic field that opposes the flow rate. This resulted in the overall damping of the velocity field. Fig. 5 shows the response of the non-Newtonian fluid to an increase in the second-grade term (δ). The fluid viscosity is enhanced as the second-grade term is encouraged in the Couette device. A rise in the fluid viscosity decreases the reactive fluid particles collision, thereby reduces the magnitude of the flow velocity. In Fig. 6, the impact of third-grade material term (φ) on the reactive combustible viscoelastic fluid is demonstrated. An increase in the flow rate is observed, which is opposite to what is seen in other viscoelastic material terms (δ) and (Λ). A steady rise is noticed near the fixed wall as the exothermic reactive diffusion occurs which breaks the fluid bonding force and causes the liquid particles to move freely in the plane Couette device. As it moves towards the motioning surface with non-isothermal wall temperature, the viscoelastic

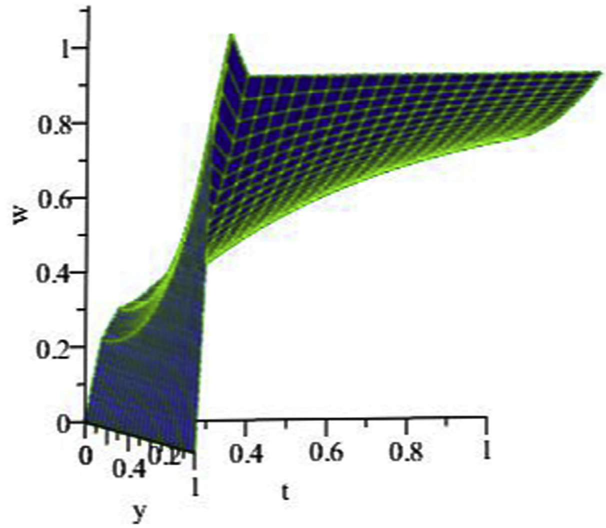

Fig. 2. Time variation for velocity field.

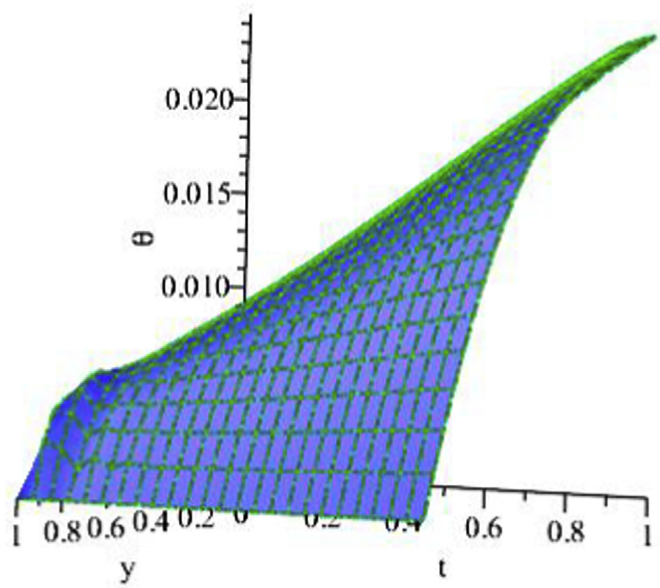

Fig. 3. Time variation for Heat profile.

started building up which leads to contraction and dragging in the fluid thereby gradually diminishes the flow rate. The fourth-grade material term (Λ) discourages the flow velocity by enhancing the fluid viscosity,

Table 1
Computed results for the skin friction (Cf) and Nusselt number.(Nu)

M	Λ	ε	φ	λ	Br	n	Cf	Nu
0.3	0.03	0.1	0.05	0.02	0.03	0.5	0.3321351974	0.0313964679
0.5							0.5924261257	0.0422272309
1.0							1.3746795318	0.0901775991
	0.1	0.2					0.3321351974	0.0314068167
	0.5	0.3					0.3321351974	0.0314171673
		0.1	0.03				0.3329120094	0.0313999996
		0.1	0.07				0.3313689016	0.0313929732
				0.3			0.3321351976	0.3513165476
				0.5			0.3321352584	0.6429898475
					0.1		0.3321351974	0.3809486990
					0.5		0.3321352584	0.8722616579
						−2.0	0.3321351974	0.3399412297
						0.0	0.3321352361	0.6333089578

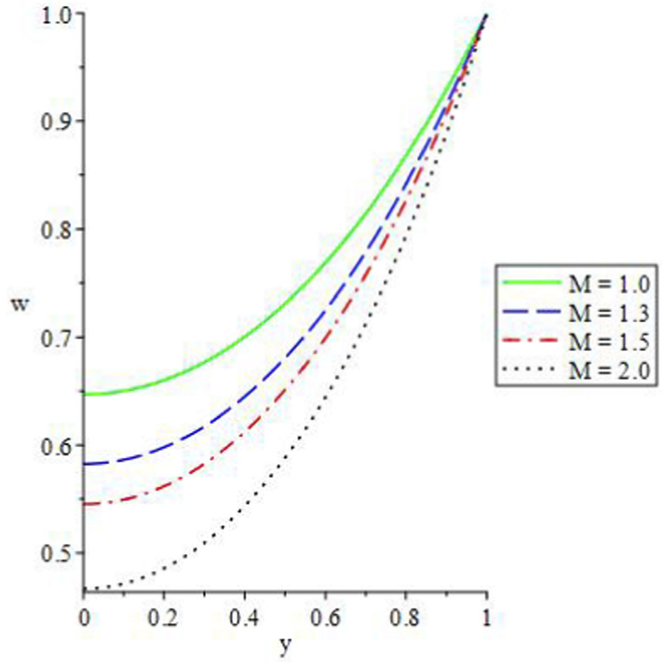

Fig. 4. Impact of rising M on velocity field.

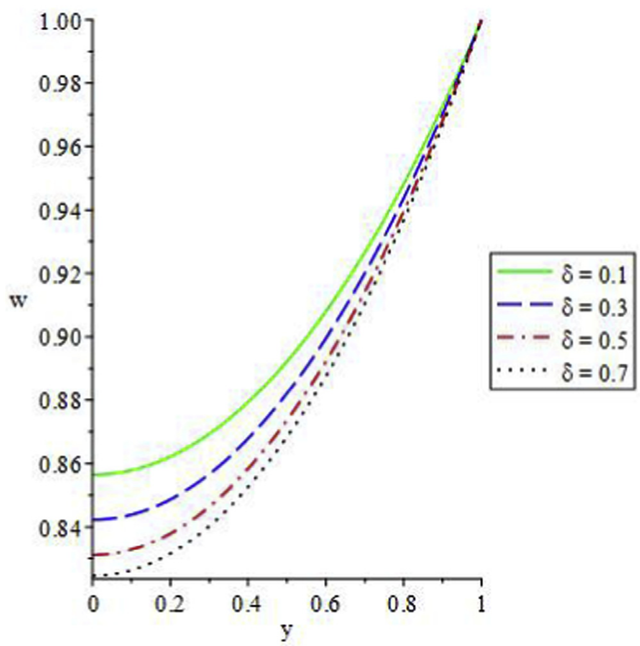

Fig. 5. Effect of δ on the flow rate.

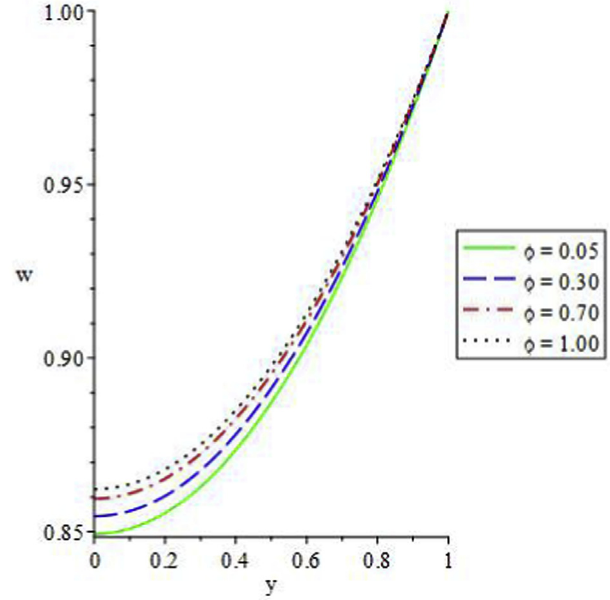

Fig. 6. Velocity profile for different φ

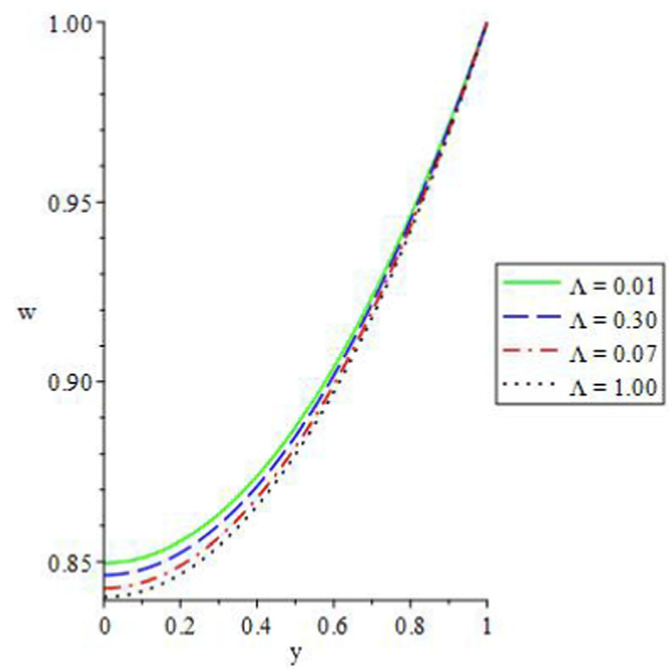

Fig. 7. Flow rate for various values of.Λ

this in turn reduces the overall magnitude of the flow as seen in Fig. 7. A significant impact is observed close to the fixed wall as the thermal boundary layer rises due to decrease in the exothermic chemical reaction. Therefore, to enhance the fluid viscosity for optimum performance of hydromagnetic lubricants for engine efficiency, the terms that encouraged fluid particles bonding force must be enhanced.

4.2. Temperature dependent parameters solutions

Figs. 8–11 illustrate the effect of heat dependent terms on the exothermic combustible temperature distribution in a Couette medium. The graph is plotted in $(y, \theta(y))$ plane for variation in the parameter values. The terms (M), (Br) and (λ) as seen in Figs. 8–10 has a momentous influence on the temperature distribution due to decreases in the fluid friction and viscosity. The parameters are strong heat generation terms in a combustible reaction system that leads to thinner in the temperature boundary layer of the viscoelastic fluid, this therefore encourages heat diffusion in the system. Hence, these parameters must be consciously monitored to prevent engines or chemical species blowing up in a thermal system, this is because an increase in the heat production within a system will affect lubricants that may result into technology and industrial machine or device under performance. Thus, heat distribution in the reactive exothermic fourth-grade fluid is enhanced. Nevertheless, the Prandtl number (Pr) diminishes the energy distribution as depicted in Fig. 11. A rise in the values of (Pr) correspondingly affects the heat

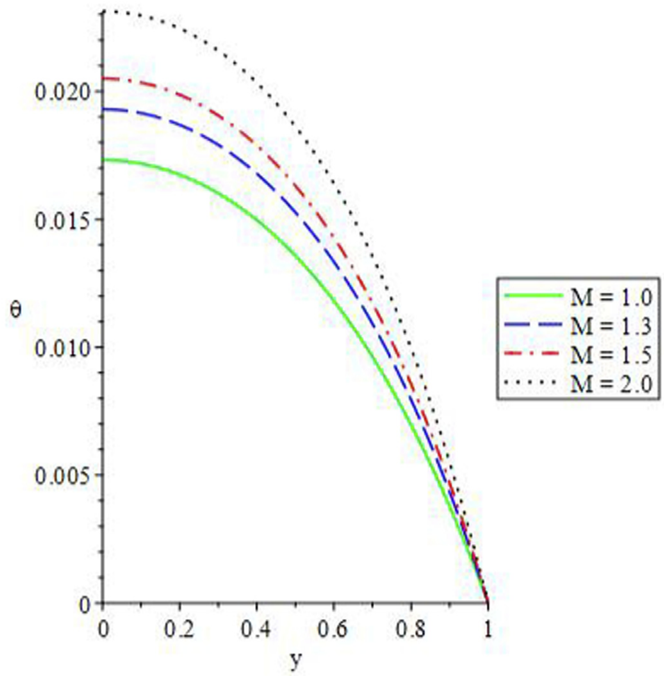

Fig. 8. Temperature field for various *M*.

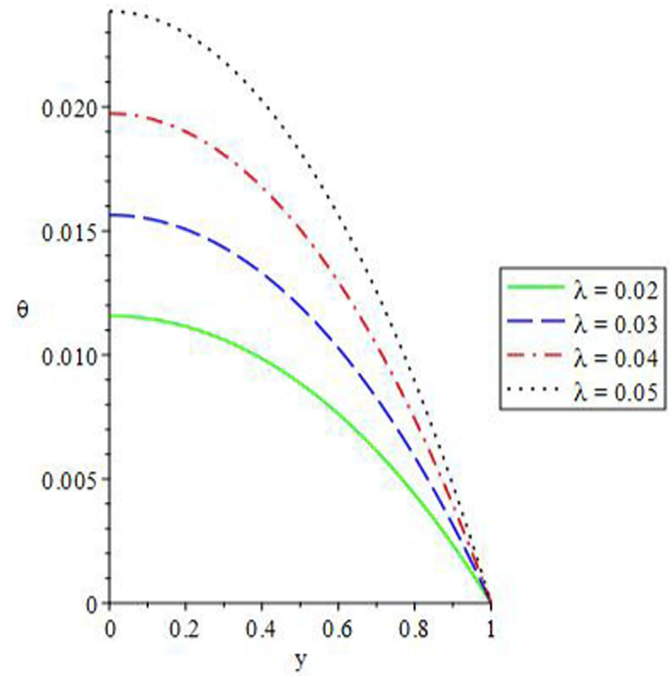

Fig. 10. Heat profile for different λ

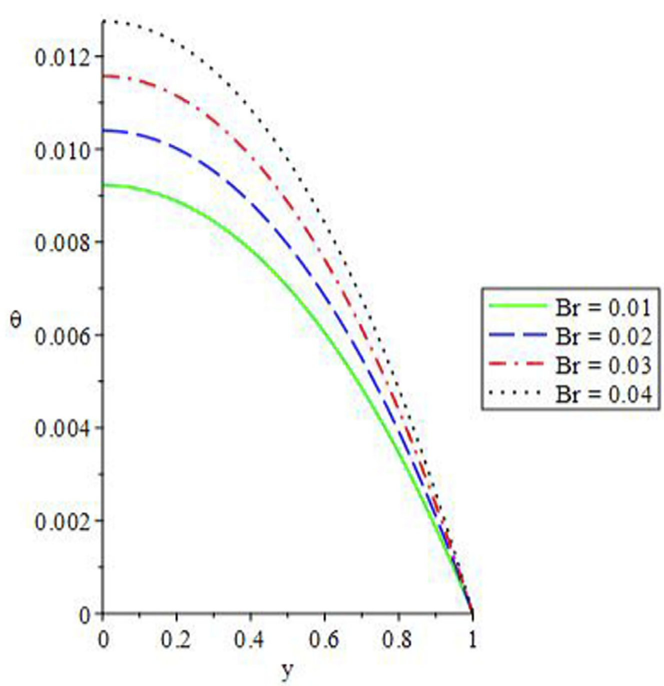

Fig. 9. Impact of *Br* on heat profile.

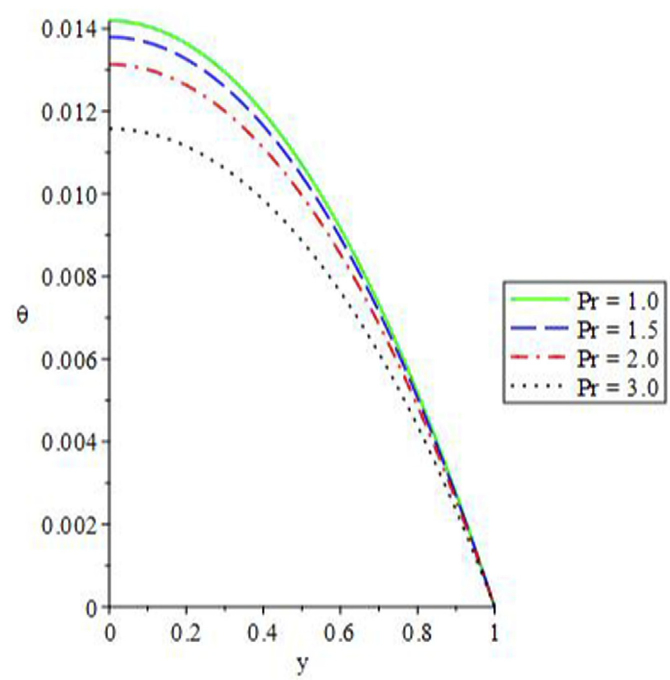

Fig. 11. Effect of *Pr* on temperature field.

boundary layer of the Couette channel, this leads to decrease in the magnitude of heat generation in the system that consequentially affect the temperature distribution as presented in the plot.

4.3. Thermal criticality bifurcation solutions

Figs. 12 and 13 demonstrate the bifurcation branched chain for the combustible fourth-grade fluid maximum heat (θ_{max}) to the Frank-Kamenetskii term (λ) with variation in the activation energy (ε) and chemical kinetics (n). For various values of (ε) and (n), a turning point with sensitive critical value (λ_c) at (θ_{max}) is obtained. When $d\theta_{max}/d\lambda = 0$ such that $0 \leq \lambda < \lambda_c$ mathematically described thermal ignition (or criticality). For $\varepsilon = (0.15, 0.20, 0.25, 0.30)$, the respectively approximate values for $(\lambda_c,\ \theta_{max,c})$=(1.06457, 1.68809), (1.10299, 1.95601), (1.14801, 2.23551), (1.20191, 2.53150) are obtained and for $n = (-2.0, 0.0, 0.5)$, the approximate values for $(\lambda_c, \theta_{max,c})$=(1.44914, 2.37903), (1.09387, 1.67477), (1.03212, 1.53419) are respectively obtained. As the parameters values rises, the critical values is raised for (ε) but reverse occurred for (n) with decrease in the critical values due to amount of internal heat generation in the system. Hence, from engineering

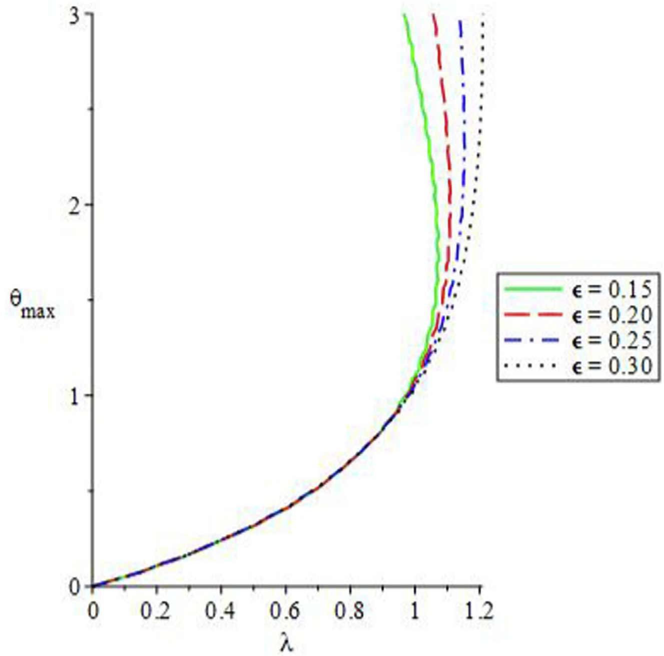

Fig. 12. Criticality slice for various ε

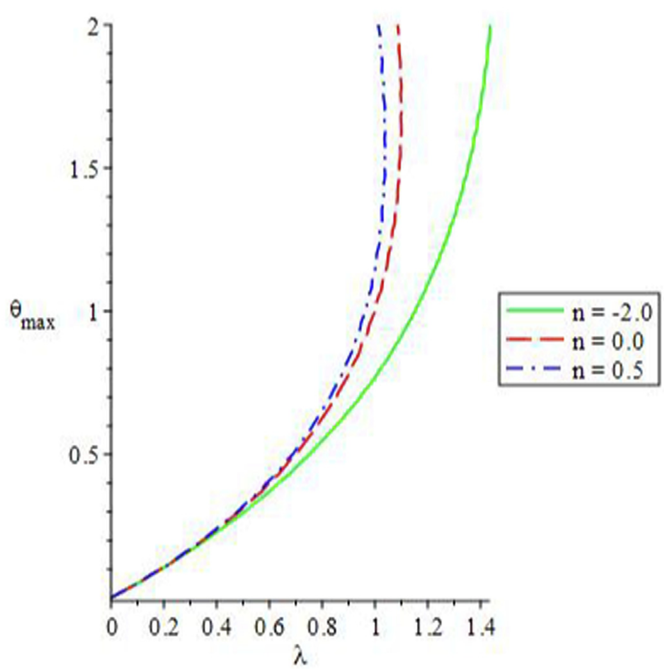

Fig. 13. Branch-chain for different *n*.

perspective, the critical state (or unsafe state) of exothermic combustible reaction must be avoided for maximal performance of their system. Also, reactive fourth-grade fluid can be used as lubricants to enhance industrial engines and machine efficiency due to its highly viscoelastic nature.

5. Conclusion

In the study, the heat distribution and thermal criticality diffusion reaction for a fourth-grade fluid flow in a plane Couette device is reported with fully exothermic reaction and heat dependent chemical kinetics. Using a finite semi-discretization difference method, the highly nonlinear model is solved. Taken from the results, the magnetic field and non-Newtonian material terms discourages flow velocity component by enhancing the viscoleastic effect. Thus, this will help in improving and strengthening hydromagnetic lubricants viscosity. It is revealed also that most of the terms in the energy equation enhance internal heat production and heat distribution in the device. Therefore, terms that raise the heat generation must be controlled in order to discourage solution explosion. The results from this study can assist in understanding the safe and unsafe regime of a system which can be of help to the thermal science and chemical synthesis industries.

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Credit author statement

This work was carried out in collaboration between both authors. S.O. Salawu: Conceived, designed the analysis, and reviewed the literatures. E.O. Fatunmbi: Contributed analysis tools or data, interpreted the data and discussed the findings. A.M. Ayanshola: Reviewed literature and discussed the findings. Both authors read and approved the final manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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