Effects of thermogenesis parameters and Biot numbers on a nonlinear heat conduction model of the human head

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Abstract

This paper considers the effects of thermogenesis parameters and biot numbers on the temperature of the human head. A non heat conduction model was considered for this work and its solution was graphically analysed .The results show that when the thermogenesis heat parameter is zero, then the temperature of the human head is at set point. Also, an increase in the thermogenesis heat or metabolic parameters may lead to an increase in the temperature of the human head. Moreover, the temperature of the human head is higher at the core and decreases transversely with minimum value at the periphery. The results also show that the temperature of the human head increases or decreases as the biot number decreases or increases with increase or decrease in thermogenesis heat parameter.

Keywords: Thermogenesis parameters,Biot numbers, nonlinear, heat conduction, human head

1. INTRODUCTION

Temperature control or thermoregulation is part of homeostatic mechanism that keeps the organism at optimum temperature. Hence temperature cycle goes up and down through the day as controlled by someone's circadian rhythm.

The core body temperature is the operating temperature of an organism, especially in deep structures of the body such as the liver in comparison to the temperature of the peripheral tissue.

A healthy person maintains its core (i.e brain and internal organs of the trunk) temperature of about 37^0C which fluctuates about 0.5^0C throughout the day with lower temperatures in the late morning and evening as the season, body's need and activities change. The temperatures in the remaining part of the body (i.e the periphery) are much less.

Different parts of the body have different temperature, for example the oral temperature under the tongue is slightly lower than the temperature taken under the arm.

Body temperature is sensitive to many hormones; for example, production of estrogen in women decreases body temperature and progesterone increases body temperature. Also, hormonal contraceptives suppress circa mensal rhythm and raise body temperature.

Hence, a temperature set point is the level at which the body attempts to maintain its temperature. When the set point is increased, the result is the body fever. Fever increases the core temperature through the action of the brain. Also, temperature elevation (hyperthermia) increases core body temperature without the consent of the heat control centers and temperature depression (hypothermia) reduces the core body temperature by 1^0C to 2^0C below the normal temperature.

Moreover, mathematical models of temperature distribution in human head have a role to play both in the treatment and diagnosis. They can aid in predicting the time and giving information on the temperature where thermometry is lacking.

Colin et al (1966) presented experimental data on the distribution of temperature in the human head. These data have been summarized by Flesch (1975) who interpreted them successfully on a theoretical basis. His result show that the generation of heat was greater in the periphery of the human head than in the centre and it was sensitive to ambient temperature changes at the periphery.

Gray (1980) investigated a simplified linear model for heat sources distribution in the human head. His results showed that in response to a drop in ambient temperature, the peripheral heat generation increases more than the central heat generation.

Anderson and Arthur’s (1981) described the complementary variational principles and the associated approximate solutions to a non linear model of heat condition in the human head.

Duggan and Goodman (1986) applied the theory of maximum principle to a non linear heat conduction model of the human head to obtain accurate analytical upper and lower bounding curves.

Celik and Gokmen (2003) presented a finite difference numerical solution for a non linear model for the distribution of the temperature in the human head with variable thermal conductivity.

Janssen et al (2005) presented a computer model that describes heat transfer in the human head during scalp cooling.

Makinde (2010) provided a non-perturbative, semi conductive solution to a non linear singular boundary value analytical problem modeling the distribution of heat sources in the human head and their dependence on environmental temperature by using Adomian decomposition method. The results show that human temperature is higher at the core and decreases transversely with minimum value at the periphery.

In this paper, we consider the effects of thermogenesis slope or heat parameters and biot numbers on a non linear heat conduction model of the human head. Graphs are plotted to support our results.

2. Mathematical model

Considering a steady state one-dimensional spherically symmetrical heat condition equation modeling a simplified human head and following Anderson and Arthur’s (1981), Duggan and Goodman (1986) and Makinde (2010), the governing energy balance equations together with its corresponding boundary conditions are:

$$\frac{d^2T}{d\bar{r}^2}+\frac{2}{\bar{r}}\frac{dT}{d\bar{r}}+\frac{\delta}{k}e^{-\alpha T}=0 \qquad (2.1)$$

$$\frac{dT}{d\bar{r}}=0, \qquad on \quad \bar{r}=0 \qquad (2.2)$$

$$-K\frac{dT}{d\bar{r}}=\beta(T-T_a), \qquad on \quad \bar{r}=R \qquad (2.3)$$

Where k is the thermal conductivity inside the head, T is the absolute temperature, $0\le\bar{r}<R$ is the radial distance measure from the centre to the periphery of head, β is a heat exchange coefficient from the head to the surrounding medium, T_a is the ambient temperature, α and δ are the metabolic thermogenesis slope parameter and thermogenesis heat production parameter respectively.

Then using the following dimensionless variables on equations (2.1) to (2.3);

$$\theta=\frac{T}{T_a},\ r=\frac{\bar{r}}{R},\ \lambda=\frac{\delta R^2}{KT_a},\ m=\alpha T_a,\ \beta_i=\frac{\beta R}{K} \qquad (2.4)$$

We obtain the dimensionless governing equations together with their corresponding boundary conditions as:

$$\frac{d^2\theta}{dr^2}+\frac{2}{r}\frac{d\theta}{dr}+\lambda e^{-m\theta}=0 \qquad (2.5)$$

$$\frac{d\theta}{dr}=0 \ \text{ on } \ r=0 \qquad (2.6)$$

$$\frac{d\theta}{dr}=B_i(1-\theta) \ \text{ on } \ r=1 \qquad (2.7)$$

where B_i, m and λ represent the Biot number, metabolic thermogenesis slope parameters and the thermogenesis heat production parameter respectively.

The analytical solution of the boundary value problem (2.5) to (2.7) is

$$e^{m\theta}(1-\theta) = -\frac{\lambda}{3B_i}$$

3. Computational Method

In the computation of results, there are three cases of solutions

Case 1: When $\lambda = 0$, then $\theta(r) = 1$ …………..(3.1)

The graph of θ against r is

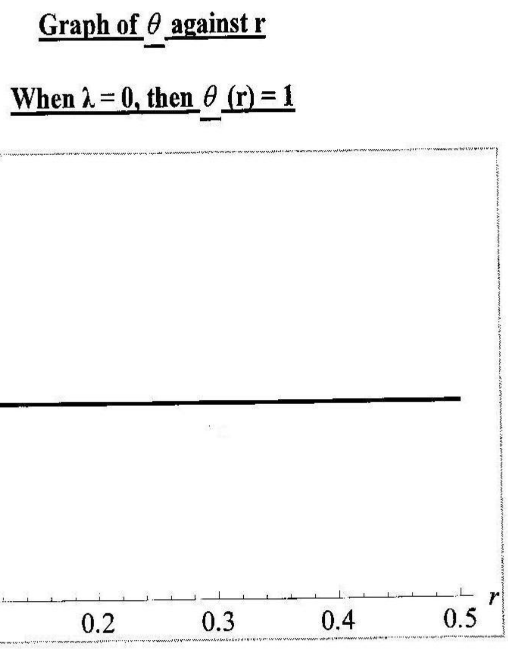

Figure 3.1.1

Case II: When $m = 0$

$$\theta = 1 + \frac{\lambda}{3B_i} \quad \text{……… (3.2)}$$

Then, the graph of θ against λ for different B_i is

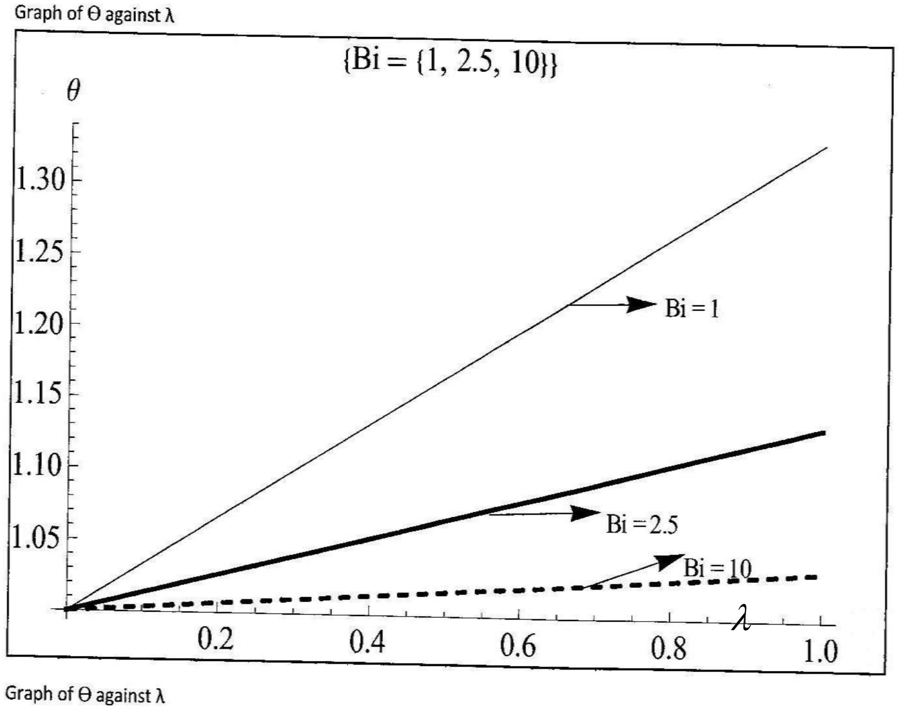

Figure 3.2.1

Also,the graph of θ against B_i , for different λ is

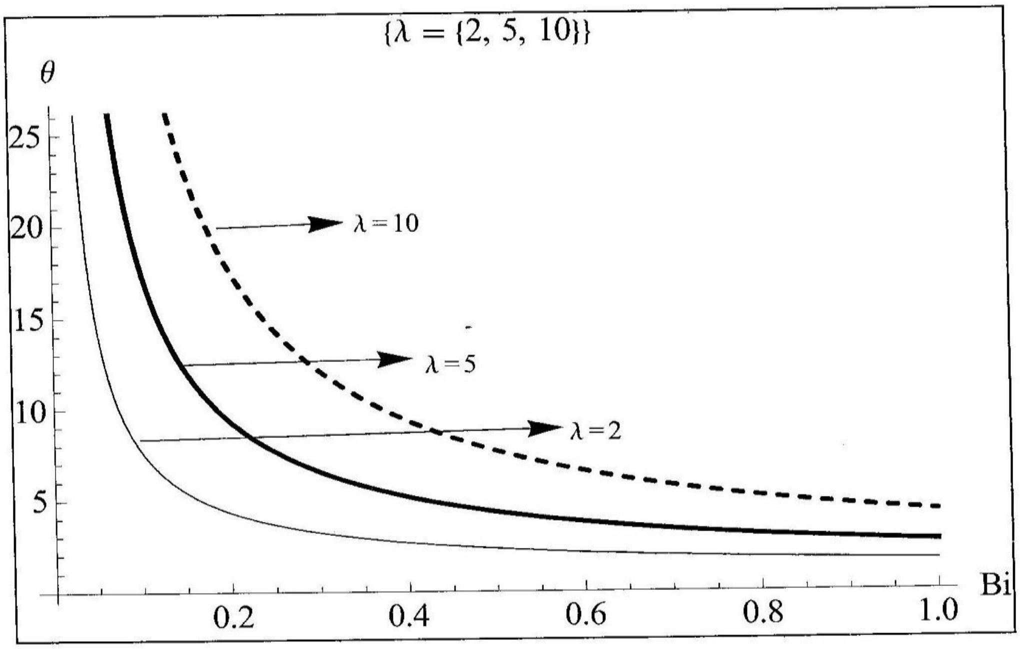

Figure 3.2.2

Case III: When $m \neq 0$ and $\lambda \neq 0$

Since $e^{m\theta} = 1 + m\theta + \frac{(m\theta)^2}{2!} + \frac{(m\theta)^3}{3!} + \ldots\ldots.$

Then, $e^{m\theta}(1-\theta) = \frac{-\lambda}{3B_i}$ becomes

$(1+m\theta)(1-\theta) = \frac{-\lambda}{3B_i}$ by truncating $e^{m\theta}$ at second term, then

$$\theta = \frac{(m-1) \pm \sqrt{(m-1)^2 + 4m\left(1 + \frac{\lambda}{3B_i}\right)}}{2m} \qquad \ldots\ldots\ldots\ldots(9)$$

We then obtain the following graphs from (9)

(1). The graph of θ against λ; $B_i = 1$ for different m

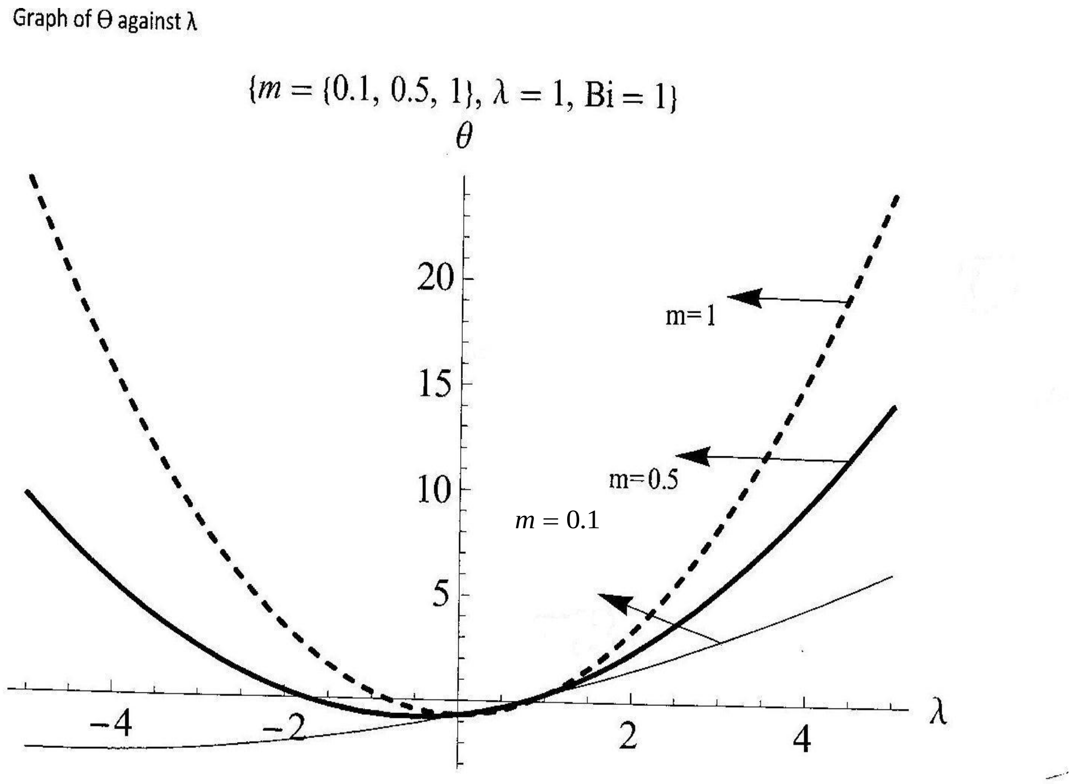

Figure 3.3.1

(2). The graph of θ against B_i ; $\lambda = 1$ for different m

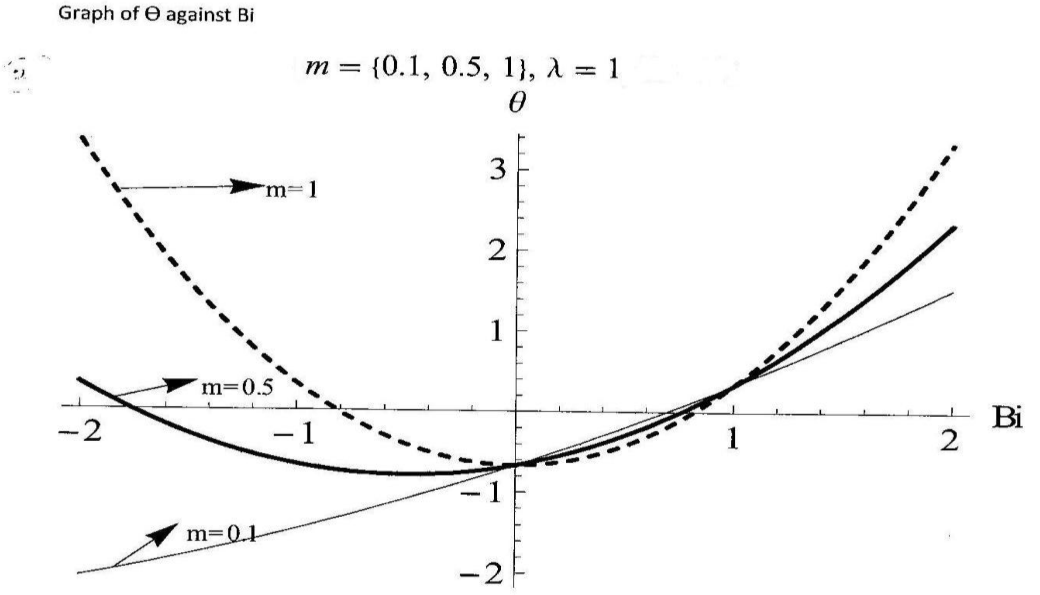

Figure 3.3.2

(3). The graph of θ against m, $B_i = 1$ for different λ

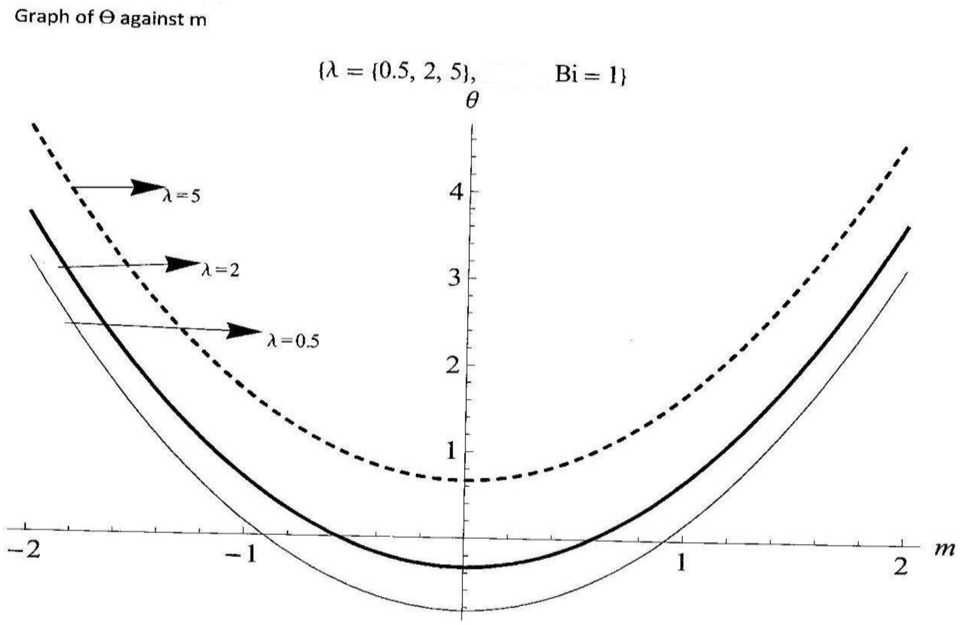

Figure 3.3.3

(4). The graph of θ against m, $\lambda = 1$ for different B_i

Graph of θ against m $B_i = \{0.5,\ 2.\ 5\},\ \lambda = 1$

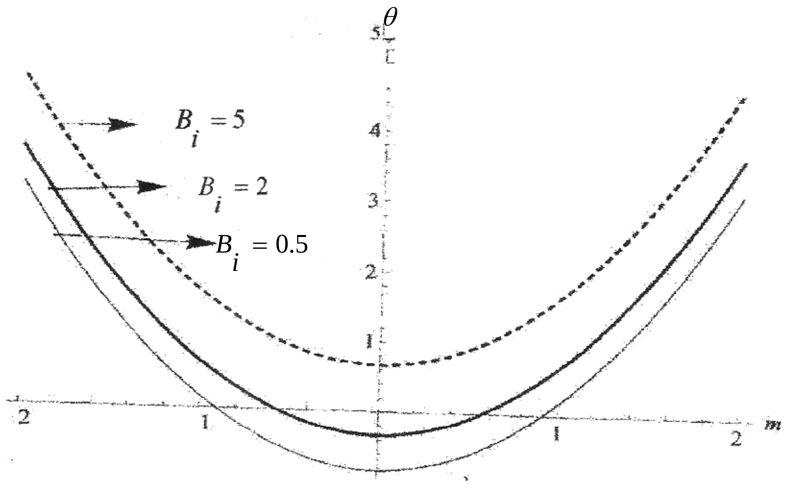

Figure 3.3.4

4. NUMERICAL RESULTS AND DISCUSSIONS

Considering the graphs in cases I, II, and III above, figure (3.1.1) shows that when the thermogenesis heat production parameter $\lambda \equiv 0$, then the temperature of the human head is constant and at a set point. Also, figures (3.2.1) and (3.2.2) show that the temperature of the human head increases or decreases as the boit number decreases or increases with increase or decrease in thermogenesis heat parameter. This is in agreement with Celik and Gokman (2005) together with Makinde (2010).

Moreover, figures (3.3.1), (3.3.2) and (3.3.3) shows that an increase in the thermogenesis heat parameter or increase in metabolic thermogenesis slope parameters may lead to increase in the temperature of the human head. This is in agreement with Makinde (2010).

In addition to this, figures (3.3.3) and (3.3.4) show that the temperature of the human head is higher at the core and decreases transversely with minimum value at the periphery. The minimum values increase with increase in theremogenesis heat or slope parameters and increase in biot numbers.

CONCLUSION

This paper explained graphically the solution of the stated problem, which analyzed the human head profile and confirmed the earlier results reported in the literature.

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