COMPARISON OF DIFFERENTIAL TRANSFORM METHOD AND MATCHED ASYMPTOTIC EXPANSION FOR SOME BOUNDARY VALUE PROBLEMS

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Abstract

In this paper the properties of Differential Transform Method (DTM) and Matched asymptotic Expansion (MAE) Perturbation method are first presented. The properties of the methods presented are then applied to some selected linear homogeneous and non-homogeneous boundary value problems (bvp) which have perturbation parameter multiplying the highest derivative. The selected boundary value problems are solved with both methods and the results are then compared with each other. The solutions to the selected boundary value problems are graphically presented while the similarities and differences of the methods are also discussed in the paper. The results showed that both methods are effective and capable of solving boundary value problem which have a perturbation quantity multiplying the highest derivative in a given problem and also reveal the complementing usefulness of both methods for the homogeneous and non-homogeneous boundary value problems.

Keywords: differential transform method, asymptotic expansion, boundary value problems, homogeneous and non-homogeneous equation.

1.0 INTRODUCTION

Cha'o kuang chen and shung Huei Ho (1999) applied two dimensional Differential transform method to solve variable and constants partial differential equations problems in which the problem was transformed into algebraic equation and then inverting the algebraic equations to obtain a closed form series solution or an approximate solution. Ming – Jyi Jang, Chien – Li Chen, Yung-Chen and Yung – Chin Liy (2000) applied DTM to solve Initial value problem for linear and non linear ordinary differential equations. They compared the results obtained using differential transform method with that Runge-Kutta method and found out computation time is reduced using differential transform method.

Siray-ul Islam, Surajul Haq, Tarid Ali (2009) studied numerical solution of special 12th order boundary value problems using Differential transform method with two point boundary conditions and compared the result obtained with Adomian Decomposition method. They reported that differential transform method is better in terms of computational efforts. Fatma Ayaz (2004) investigated the application of two & three dimensional differential transform method on linear and non linear system of ordinary differential equations and compared the result with Adomian decomposition method and the result showed a good agreement linear and non linear. Differential transform method has also been found to be in good agreement with an analytic solution obtained from Laplace transformation and Pade approximant as investigated by Moustafa (2007). Ravi Kanth and Aruna (2008) showed that differential transform method can be applied to find approximate solutions for the linear and non-linear system of partial differential equations and the method is capable of reducing the size of computational work.

Differential transform method is successfully applied to singularly perturbed volterra integral equations and the result showed perfect agreement to the exact solution as shown by Nurettin Dogan, Vedat, Shalur Momani, Ahmet Yildrim (2010).

Many authors such as Aytac Arikoglu, Ibrahim Ozkol (2006) applied differential transform method to difference equations and fractional differential equations.

Perturbation methods have been very applicable in many branches of science and engineering.

Piotr Skrzypacz and lutz Tobiska (2005) investigated the applications of Matched asymptotic expansion and Finite element method for chemical reactor flow problems. Matched asymptotic expansion is not only useful the area of in science and technology, Sam Howison (2005) studied the application of matched asymptotic expansion in finance (option pricing) and showed that the method is effective in handling complex mathematical model and model that are characterized by mathematical technicalities and difficulties such as those that arise when levy processes are used to model asset prices and when non-linearities arise as in model of illiquid market etc. Wong and Sze (1998), showed the usefulness of analytic solutions by applying Matched asymptotic expansion to the free vibration of a Hemetic shell. Chang and Cheng (1983) studied the problem of steady free convection in a porous medium adjacent to a horizontal impermeable heated surface using Matched asymptotic expansion method. Abderrahmane, Abdelkader Makhlon and Sebasthieu Tordeuy (2007) applied Matched asymptotic expansion for the determination of the electromagnetic field near the edge of a patch antenna.

Han Hansen and Kember (2010) studied the application of Matched asymptotic expansion method on highly unsteady porous media flows. The method is used to analyse a space time entrance zone in highly unsteady flow through an elastic porous medium and also used to match flows at early times and near the entrance with complementary forms that are away from the entrance.

Moreso, solving initial value problem using Matched asymptotic expansion was considered by Yuri Skrynnikov (2012) the author applied Matched asymptotic expansion to solve initial value problems posed for an advection-diffusion equation modeling orientation of pulp fibres in a steady fully turbulent flow as a singular perturbation technique. Matched asymptotic expansion has also be compared with another singular perturbation technique called multiple scales by David Wollkind (1977) using a biological population dynamics as illustration.

MATCHED ASYMPTOTIC EXPANSION

A regular perturbation problem is ordinary differential equations involves an equation with a small parameter $\epsilon > 0$ that posses a solution $y(x;\epsilon)$ which has uniformly valid asymptotic expansion for $x \mathcal{E} [0,1]$ A similar problem possessing a solution which its asymptotic expansion is not uniformly valid in this interval is said to be a singular perturbation problem.

2.0

1.0 DIFFERENTIAL TRANSFORM METHOD

Differential Transform Method (DTM) is similar to the Taylors expansion method. The Taylors expansion for a function $y(x)$ about the point $x = x_0$ is defined as

$$y(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!} \frac{d^k}{dx^k} y(x) \bigg|_{x=x_0}.$$

We move on to define the differential transform of $y(x)$ as

$$D\{y(x)\} = Y[k] = \frac{1}{k!}\frac{d^k}{dx^k}y(x)\bigg|_{x=x_0}$$

and the inverse differential transform defined as

$$D^{-1}\{Y[k]\} = y(x) = \sum_{k=0}^{\infty} Y[k]x^k.$$

The differential transform converts the differential equations to algebraic equation and therefore makes it easy to solve. In what follows, we shall present some theorems here. In the following theorems, we shall suppose that

$$D\{y(x)\} = Y[k]$$

and define the Dirac delta function as

$$\delta_{n,m} = \begin{cases} 0 & n \neq m \\ 1 & n = m \end{cases}$$

Theorem 3.1

If $y(x) = c$ then

$$Y[k] = c\delta_{0,k}$$

Theorem 3.2

If $y(x) = ax^n$, then

$$Y[k] = a\delta_{n,k}$$

Theorem 3.3

If $y(x) = e^{ax}$, then

$$Y[k] = \frac{a^k}{k!}$$

Theorem 3.4

a) $D\left\{\frac{dy}{dx}\right\} = (k+1)Y[k+1]$

b) $D\left\{\frac{d^2y}{dx^2}\right\} = (k+2)(k+1)Y[k+2]$

c) $D\left\{\frac{d^ny}{dx^n}\right\} = (k+1)(k+2)\cdots(k+n)Y[k+n]$

Theorem 3.5

If $D\{u(x)\} = U[k], D\{v(x)\} = V[k]$ and then $y(x) = u(x)\cdot v(x)$ implies that

$$D\{y(x)\} = Y[k] = \sum_{r=0}^{k} U[r]\cdot V[k-r]$$

2.0 NUMERICAL EXAMPLE OF LINEAR NONHOMOGENEOUS EQUATION

Consider the second order boundary value problem

$$\epsilon y'' + 2y' + 2y = 0, 0 < x < 1 \qquad (4.0.1)$$

with the boundary condition

$$y(0) = 0 \qquad (4.0.2a)$$

$$y(1) = 1. \qquad (4.0.2b)$$

Observe that the second order homogeneous differential can be solved for the exact solution using eigenvalue method to get

$$y(x) = \begin{cases} \dfrac{e^{\frac{1-x}{\epsilon}} \sinh\left(\frac{x}{\epsilon}\sqrt{1-2\epsilon}\right)}{\sinh\left(\frac{1}{\epsilon}\sqrt{1-2\epsilon}\right)} & \epsilon < 0.5 \\ \dfrac{e^{\frac{1-x}{\epsilon}} \sin\left(\frac{x}{\epsilon}\sqrt{1-2\epsilon}\right)}{\sin\left(\frac{1}{\epsilon}\sqrt{1-2\epsilon}\right)} & \epsilon > 0.5 \end{cases} \qquad (4.0.3)$$

and for $\epsilon = 0.5$, we have

$$y(x) = xe^{2-2x}. \qquad (4.0.4)$$

2.1 Solution using MAE

We get the outer solution by substituting a regular solution of the form

$$y = y_0 + \epsilon y_1 + O(\epsilon^2), \qquad y_0(0) = 0, y_0(1) = 1, y_i(0) = y_i(1) = 0 \ \ \forall\, i \geq 1,$$

into equation (4.0.1) and comparing coefficients, we have

$$\varepsilon^0 : 2y_0^{'} + 2y_0 = 0 \Rightarrow y_0^{'} + y_0 = 0 \Rightarrow y_0 = A^* e^{-x} \qquad (4.1.1)$$

$$\epsilon^1 : \ \ y_0^{''} + 2y_1^{'} + 2y_1 = 0 \qquad (4.1.2)$$

The presence of only one arbitrary constant in (4.1.1) indicates that that there is a boundary layer. Using the condition (4.0.2a) (i.e. $y_0(0) = 0$) will make produce $A^* = 0$ and consequently, $y_0(x) = 0$ but using the condition (4.0.2b) (i.e. $y_0(1) = 1$)we get $A^* = e$ and so

$$y_0 = e^{1-x}. \qquad (4.1.3)$$

Hence, we know the boundary layer is at $x = 0$. Substituting (4.1.3) into (4.1.2) and solving for y_1, we have

$$y_1 = \frac{1}{2} x e^{1-x} + Be^{-x} \qquad (4.1.4)$$

and with the condition that $y_1(0) = 0$,we have that $B = \frac{1}{2}e$ and so

$$y_1 = \frac{1}{2}(1-x)e^{1-x}. \qquad (4.1.5)$$

The outer solution therefore is

$$y = e^{1-x} + \epsilon\left(\frac{1}{2}(1-x)e^{1-x}\right) = \frac{1}{2}\big(2 + \epsilon(1-x)\big)e^{1-x} \qquad (4.1.6)$$

To make the solution valid for the inner region, a transformation is needed to move the perturbation parameter away from the term of the highest derivative. We choose the boundary layer coordinateξ as

$$\xi = \frac{x}{\epsilon} \ \Rightarrow \quad x = \epsilon\xi$$

Transforming the outer solution (4.1.6) to the boundary layer coordinate and ignoring $O(\epsilon^2)$, we have

$$y^i = e + \frac{1}{2}e(1 - 2\xi)\epsilon \tag{4.1.7}$$

Finally, we transform back to x and we have the outer solution as

$$y = e + \frac{1}{2}e\left(1 - \frac{2x}{\epsilon}\right)\varepsilon = e(1 - x) + \frac{1}{2}e\epsilon. \tag{4.1.8}$$

Now, we turn to find the inner solution by introducing the boundary layer coordinate into the original problem (4.0.1). The transformed problem becomes

$$Y'' + 2Y' + 2\epsilon Y = 0, \tag{4.1.9}$$

Introducing the regular perturbation

$$Y = Y_0(\xi) + \epsilon Y_1(\xi) \tag{4.1.10}$$

into equation (4.1.9), comparing coefficients and using $Y_0(0) = Y_1(0) = 0$ we have

$$Y_0 = B\left(1 - e^{-2\xi}\right) \tag{4.1.11}$$

$$Y_1 = -B\xi - \frac{1}{2}(2B\xi + C_2 + B)e^{-2\xi} + \frac{1}{2}(B + C_2) \tag{4.1.12}$$

and thus, putting (4.1.11) and (4.1.12) in (4.1.9), we have the inner solution as

$$Y = B\left(1 - e^{-2\xi}\right) + \left(-B\xi - \frac{1}{2}(2B\xi + C_2 + B)e^{-2\xi} + \frac{1}{2}(B + C_2)\right)\epsilon \tag{4.1.13}$$

Transform inner solution back to a function of x then,

$$Y = B\left(1 - e^{-\frac{2x}{\epsilon}}\right) + \left(-B\frac{x}{\epsilon} - \frac{1}{2}\left(2B\frac{x}{\epsilon} + C_2 + B\right)e^{-2\frac{x}{\epsilon}} + \frac{1}{2}(B + C_2)\right)\epsilon \tag{4.1.14}$$

as $\epsilon \to 0$ then

$$Y = B(1 - x) + \frac{1}{2}(B + C_2)\epsilon. \tag{4.1.15}$$

We expect the two solutions to be equal at the overlapping region, so we equate (4.1.8) and (4.1.15) and then compare coefficients as follows

	$e(1-x)+\frac{1}{2}e\epsilon = B(1-x)+\frac{1}{2}(B+C_2)\epsilon$	
	$B=e,\quad \text{and}\quad B+C_2=e \Rightarrow C_2=0$	(4.1.16)

By substituting the results in (4.1.16) into (4.1.14), the inner solution is

	$Y=e\left(1-e^{-\frac{2x}{\epsilon}}\right)+\left(\frac{1}{2}e-\frac{ex}{\epsilon}-\frac{1}{2}\left(\frac{2ex}{\epsilon}+e\right)e^{-2\frac{x}{\epsilon}}\right)\epsilon$	(4.1.17)

and the solution at the overlapping region is

$$e(1-x)+\frac{1}{2}e\epsilon. \qquad (4.1.18)$$

The solution to the problem (4.0.1) is then obtained using the relationship

	$y=y+Y-e(1-x)+\frac{1}{2}e\epsilon.$	

So that the solution is given as

	$y=e^{1-x}-(1+x)e^{1-\frac{2x}{\epsilon}}+\frac{1}{2}\epsilon\left((1-x)e^{1-x}-e^{1-2\frac{x}{\epsilon}}\right)$	(4.1.19)

2.2 Solution using DTM

The problem (4.0.1)

$$\epsilon y'' + 2y' + 2y = 0, 0 < x < 1$$

is transformed using the differential transform and we have

$$\epsilon(h+1)(h+2)Y[h+2]+2(h+1)Y[h+1]+2Y[h]=0, 0<x<1. \qquad (4.2.1)$$

Rearranging (4.2.1), we have

$$Y[h+2]=-\frac{2}{\epsilon}\left(\frac{(h+1)Y[h+1]+Y[h]}{(h+1)(h+2)}\right) \qquad (4.2.2)$$

and the boundary condition (4.0.2a) is transformed into

$$Y[0]=0. \qquad (4.2.3)$$

Considering the recursive equation (4.2.2), we observe that one more condition is needed to start the iteration. We therefore set

$$Y[1]=\lambda. \qquad (4.2.4)$$

On solving (4.2.2) with initial conditions (4.2.3) and (4.2.4), we have the following results

$$Y[2]=-\frac{\lambda}{\epsilon}$$

$$Y[3] = -\frac{\lambda}{3\epsilon} + \frac{2\lambda}{3\epsilon^2}$$
$$Y[4] = \frac{\lambda}{3\epsilon^2} - \frac{\lambda}{3\epsilon^3}$$
$$Y[5] = \frac{2\lambda}{15\epsilon^4} - \frac{\lambda}{5\epsilon^3} + \frac{\lambda}{30\epsilon^2}$$

etc. So that the solution is

$$y(x) = \sum_{h=0}^{\infty} Y[h]x^h.$$

For the Nth order approximation, we write as

$$y(x) = \sum_{h=0}^{N} Y[h]x^h.$$

As an example, we write the 5th order approximation

	$y(x) = Y[0] + xY[1] + x^2Y[2] + x^3Y[3] + x^4Y[4] + x^5Y[5]$	
	$= \lambda\left\{x - \frac{1}{\epsilon}x^2 + \left(-\frac{1}{3\epsilon} + \frac{2}{3\epsilon^2}\right)x^3 + \left(\frac{1}{3\epsilon^2} - \frac{1}{3\epsilon^3}\right)x^4 + \left(\frac{2}{15\epsilon^4} - \frac{1}{5\epsilon^3} + \frac{1}{30\epsilon^2}\right)x^5\right\}.$	(4.2.5)

Now, we can estimate the value of λ by using the second boundary condition (4.0.2b) (i.e. $y(1) = 1$). Set □ $= 1$ in (4.2.5), we have

	$□(1) = 1 = □\left\{1 - \frac{1}{□} - \frac{1}{3□} + \frac{2}{3□^2} + \frac{1}{3□^2} - \frac{1}{3□^3} + \frac{2}{15□^4} - \frac{1}{5□^3} + \frac{1}{30□^2}\right\}.$	
	$\lambda = -\frac{30□^4}{10□^5 - 30□^4 + 30□^3 - 31□^2 + 16□ - 4}$	(4.2.6)

With this value of □, we now write the 5th order approximation as

	$= -\left(\frac{30□^4}{10□^5 - 30□^4 + 30□^3 - 31□^2 + 16□ - 4}\right)\left\{□ - \frac{1}{□}□^2 + \left(-\frac{1}{3□} + \frac{2}{3□^2}\right)□^3 + \left(\frac{1}{3□^2} - \frac{1}{3□^3}\right)□^4 + \left(\frac{2}{15□^4} - \frac{1}{5□^3} + \frac{1}{30□^2}\right)□^5\right\}.$	(4.2.7)

The beauty of this method is that □[□] can be easily computed using the symbolic programming packages. The value of □ is calculated as *29.0990* at the 100th order approximation.

3.0 NUMERICAL EXAMPLE OF LINEAR NONHOMOGENEOUS EQUATION

Consider the second order boundary value problem

$$□^{2}□'' + □□□' - □ = -□^{□}, 0 < □ < 1 \tag{5.0.1}$$

with the boundary condition

$$□(0) = 2 \tag{5.0.2a}$$

$$□(1) = 1. \tag{5.0.2b}$$

3.1 Solution using MAE

We get the outer solution by substituting a regular solution of the form

$$□ = □_{0} + □□_{1} + □(□^{2}), \qquad □_{0}(0) = 2, □_{0}(1) = 1, □_{□}(0) = □_{□}(1) = 0 \ \forall □ \geq 1,$$

into equation (5.0.1) and comparing coefficients, we have

$$□^{0}: -□_{0} = -□^{□} \tag{5.1.1}$$

The absence of arbitrary constants in (5.1.1) indicates that that there are two boundary layers and these layers are at both ends. The outer solution therefore is

$$□ = □^{□} \tag{5.1.6}$$

To make the solution valid for the inner region, two transformations will be needed to move the perturbation parameter away from the term of the highest derivative. We choose the two boundary layer coordinates □ and □ as

	$□ = \frac{□}{□} \Rightarrow \quad □ = □□$	(5.1.7)
and	$□ = \frac{□ - 1}{□} \Rightarrow □ = 1 + □□,$	(5.1.8)

where □ and □ represent the boundary layer coordinates near the $□ = 0$ and $□ = 1$ ends respectively.

The inner solution near the boundary layer $□ = 0$ is obtained by transforming the original problem (5.0.1) using the transformation (5.1.7) and the problem becomes

	$□'' + □□□' - □ = -□^{□□}$ where $□(□) = □(□)$	(5.1.9)

By substituting

	$□ = □_{0} + □(□)$	(5.1.10)

we obtain

	$□_{0}'' - □_{0} = -1 \Rightarrow □_{0} = □□^{□} + □□^{-□} + 1$	(5.1.11)

and using $□_{0}(0) = 0$, we have that $□ = 1 - □$ and then,

$$\square_0 = 1 + \square\square^{-\square} + (1 - \square)\square^{\square} \tag{5.1.12}$$

Transform inner solution back to a function of $\square \left(\square = \frac{\square}{\square}\right)$we have

$$\square_0 = 1 + \square\square^{-\frac{\square}{\square}} + (1 - \square)\square^{\frac{\square}{\square}} \tag{5.1.13}$$

and setting $\square \to 0$, then

$$\square_0 = 2 - \square \quad \text{5.1.14)}$$

and the outer solution becomes

$$\square = 1 \tag{5.1.15}$$

We expect the two solutions to be equal at the overlapping region, so we equate (5.1.14) and (5.1.15) so that we get

$$1 = 2 - \square \Rightarrow \square = 1 \tag{5.1.16}$$

By substituting the results in $\square$ into (5.1.13), the inner solution is

$$\square = 1 - \square^{-\frac{\square}{\square}} \tag{5.1.17}$$

and the solution at the overlapping region is 1.

The inner solution near the boundary layer $\square = 1$ is obtained by transforming the original problem (5.0.1) using the transformation (5.1.8) and the problem becomes

$$\square'' + (1 + \square\square)\square' - \square = -\square^{1+\square\square} \quad \text{where} \quad \square(\square) = \square(\square) \tag{5.1.18}$$

By substituting

$$\square = \square_0 + \square(\square) \tag{5.1.19}$$

we obtain

$$\square_0'' + \square_0' - \square_0 = -\square \Rightarrow \square_0 = \square\square^{\square_1\square} + \square\square^{\square_2\square} + \square \tag{5.1.20}$$

where $\square_{1,2} = \left(-1 \pm \sqrt{5}\right)/2$and using $\square_0(0) = 1$, we have that $\square = 1 - \square - \square$and then,

$$\square_0 = \square\square^{\square_1\square} + (1 - \square - \square)\square^{\square_2\square} + \square \tag{5.1.21}$$

Transform inner solution back to a function of $\square$ $\left(\square = \frac{\square - 1}{\square}\right)$ and setting $\square \to -\infty$, we have

$$\square_0 = 1 - \square \qquad (5.1.22)$$

and setting $\square \to 1$, the outer solution becomes

$$\square = \square \qquad (5.1.23)$$

We expect the two solutions to be equal at the overlapping region, so we equate (5.1.22) and (5.1.23) so that we get

$$\square = 1 - \square \Rightarrow \square = 1 - \square \qquad (5.1.24)$$

By substituting $\square$ into (5.1.21), the inner solution is

$$\square = (1 - \square)\square^{\square_1 \square} + \square = (1 - \square)\square^{\square_1\left(\frac{\square - 1}{\square}\right)} + \square \qquad (5.1.25)$$

and the solution at the overlapping region is $\square$.

Now, the total solution at the overlapping region is

$$1 + \square$$

And we compute the final solution by the formula

$$\square = \square + \square + \square - (1 + \square)$$

So that the final solution becomes

$$\square = \square^{\square} + 1 - \square^{-\frac{\square}{\square}} + (1 - \square)\square^{\square_1\left(\frac{\square - 1}{\square}\right)} + \square - (1 + \square)$$

$$\square = \square^{\square} - \square^{-\frac{\square}{\square}} + (1 - \square)\square^{\square_1\left(\frac{\square - 1}{\square}\right)}$$

3.2 Solution using DTM

The problem (5.0.1)

$$\square^2\square'' + \square\square\square' - \square = -\square^{\square}, 0 < \square < 1$$

is transformed using the differential transform and we have

$$\square^2(h + 1)(h + 2)\square[h + 2] + \square\square[h] - \square[h] = -\frac{1}{h!}. \qquad (5.2.1)$$

Rearranging (5.2.1), we have

$$\square[h + 2] = \frac{\square[h] - \square\square[h] - \frac{1}{h!}}{\square^2(h + 1)(h + 2)} \qquad (5.2.2)$$

and the boundary condition (5.0.2a) is transformed into

$$\square[0] = 2. \tag{5.2.3}$$

Considering the recursive equation (5.2.2), we observe that one more condition is needed to start the iteration. We therefore set

$$\square[1] = \square. \tag{5.2.4}$$

On solving (5.2.2) with initial conditions (5.2.3) and (5.2.4), we have the following results

$$\square[2] = \frac{1}{2\square^2} - \frac{1}{\square}$$

$$\square[3] = \left(-\frac{1}{6\square} + \frac{1}{6\square^2}\right)\square - \frac{1}{6\square^2}$$

$$\square[4] = \frac{1}{24\square^2} + \frac{1}{24\square^4} - \frac{1}{8\square^3}$$

$$\square[5] = \left(\frac{1}{120\square^4} - \frac{1}{60\square^3} + \frac{1}{120\square^2}\right)\square - \left(\frac{1}{120\square^4} - \frac{1}{120\square^3} + \frac{1}{120\square^2}\right)$$

etc. So that the solution is

$$\square(\square) = \sum_{h=0}^{\infty} \square[h]\square^h.$$

For the $\square$th order approximation, we write as

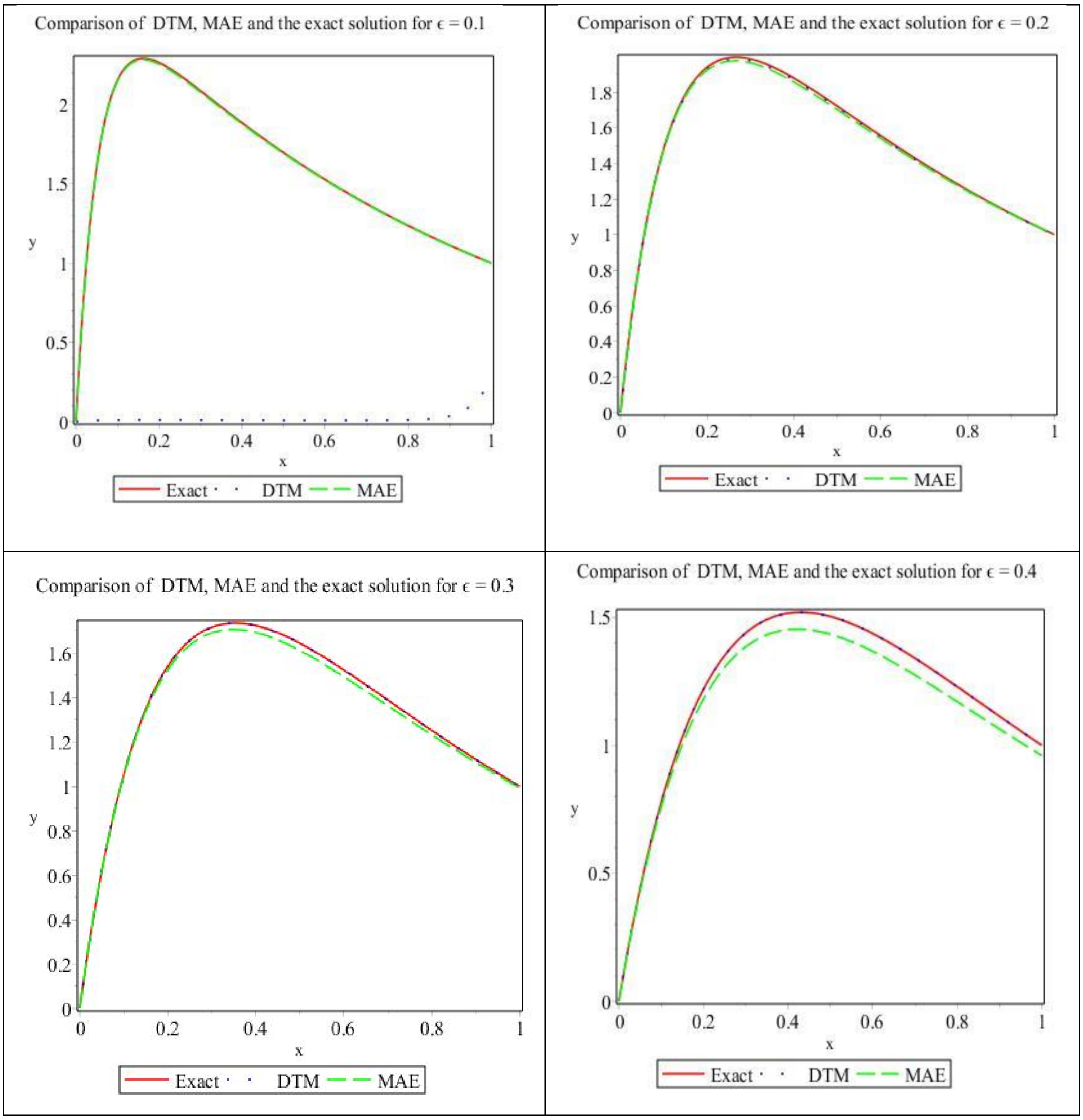
$$\square(\square) = \sum_{h=0}^{\square} \square[h]\square^h.$$

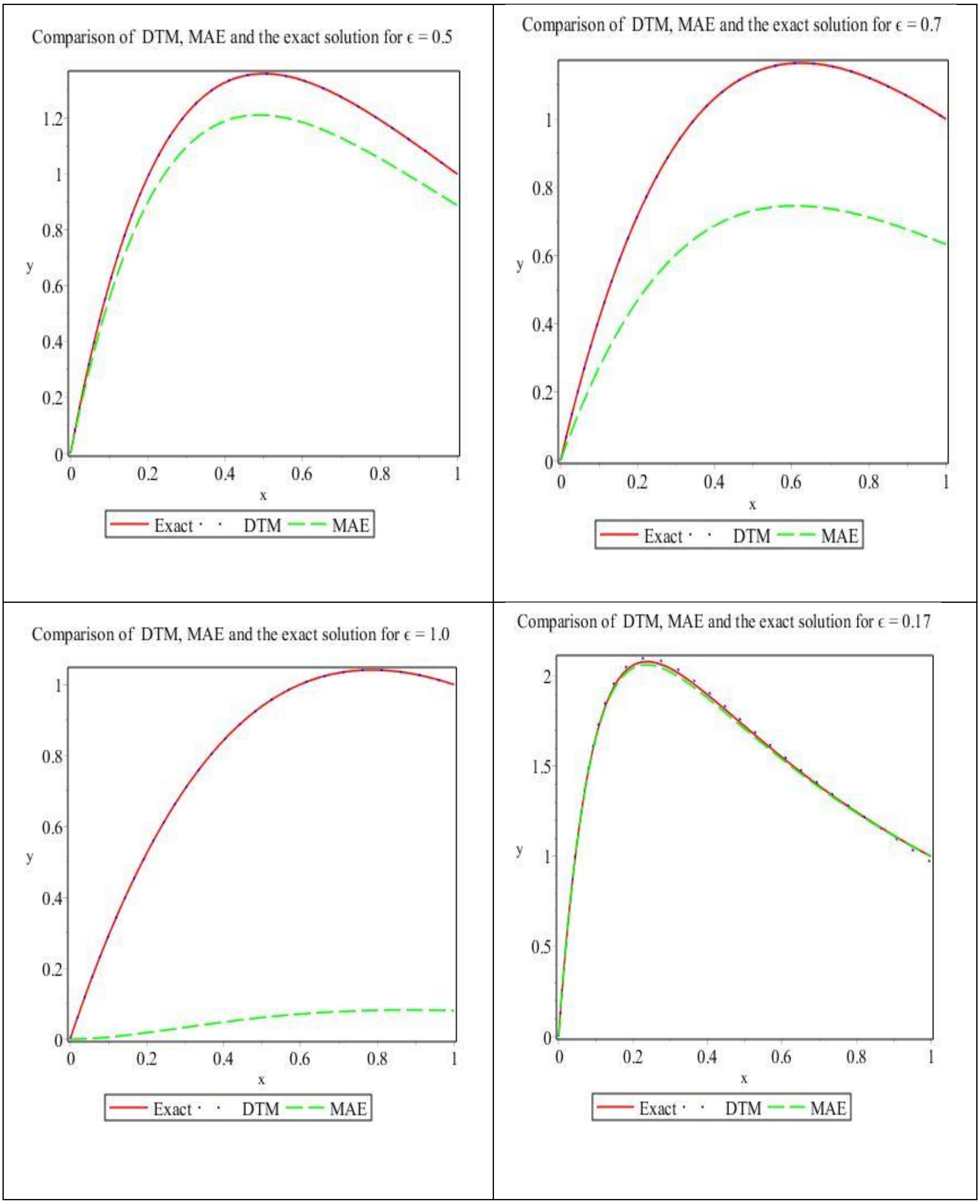
As an example, we write the 5th order approximation

	$\square(\square) = \square[0] + \square\square[1] + \square^2\square[2] + \square^3\square[3] + \square^4\square[4] + \square^5\square[5]$	
	$= \left\{2 + \left(\frac{1}{2\square^2} - \frac{1}{\square}\right)\square^2 - \frac{1}{6\square^2}\square^3 + \left(\frac{1}{24\square^2} + \frac{1}{24\square^4} - \frac{1}{8\square^3}\right)\square^4 - \left(\frac{1}{120\square^4} - \frac{1}{120\square^3} + \frac{1}{120\square^2}\right)\square^5\right\} + \square\left\{\square + \left(-\frac{1}{6\square} + \frac{1}{6\square^2}\right)\square^3 + \left(\frac{1}{120\square^4} - \frac{1}{60\square^3} + \frac{1}{120\square^2}\right)\square^5\right\}$	

The value of $\square$ can be calculated by using the second boundary condition and is obtained as -0.0318354 at the 100th order approximation.

4.0 DISCUSSIONS


Comparison of DTM, MAE and the exact solution for ϵ = 0.1
Comparison of DTM, MAE and the exact solution for ϵ = 0.2
Comparison of DTM, MAE and the exact solution for ϵ = 0.3
Comparison of DTM, MAE and the exact solution for ϵ = 0.4
x
y
Exact
DTM
MAE


Comparison of DTM, MAE and the exact solution for ϵ = 0.5
Comparison of DTM, MAE and the exact solution for ϵ = 0.7
Comparison of DTM, MAE and the exact solution for ϵ = 1.0
Comparison of DTM, MAE and the exact solution for ϵ = 0.17
y
x
Exact
DTM
MAE

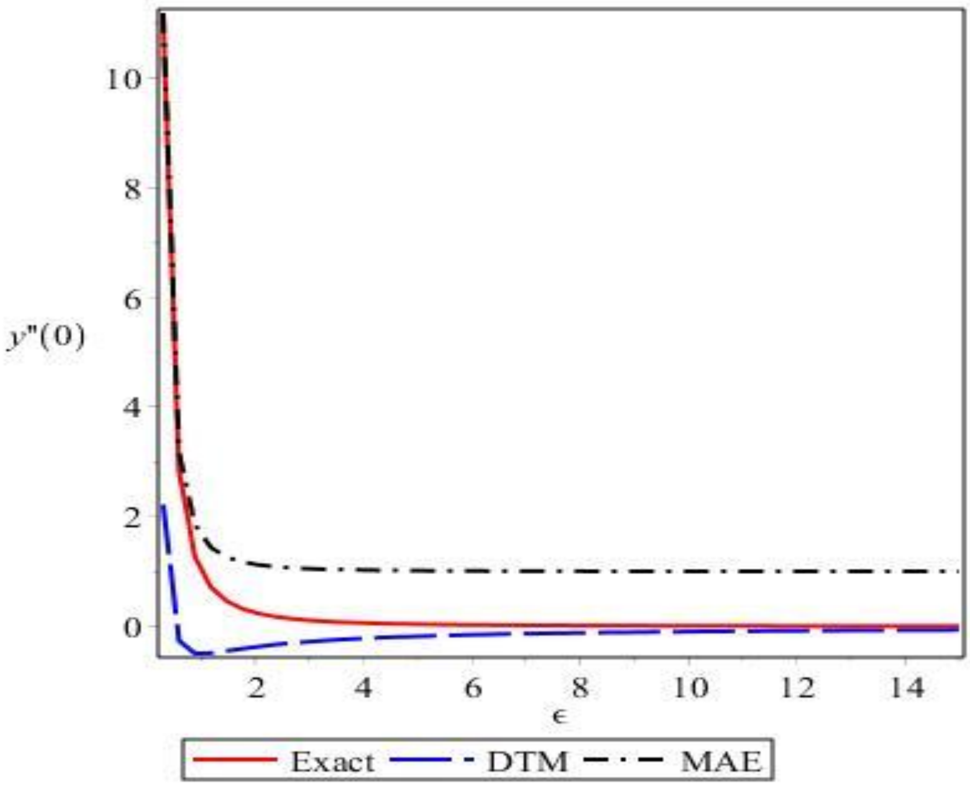


Figure 6.7: Comparison of DTM, MAE and Exact solution for values of $0 < \epsilon < 15$

5.0 CONCLUSION:

We have presented and compared two different techniques that can be used to approximate solution to a boundary value problem. The method of matched asymptotic expansion (MAE) which is useful when one wishes to investigate the inner and outer expansions separately in their own right. The differential transforms method (DTM) which is capable of reducing computational work and at same time effective. The knowledge of the method of matched asymptotic expansion and differential transforms method described herein can be a substantial aid to researchers in many diverse fields.