MHD STAGNATION POINT FLOW AND HEAT TRANSFER OF MICROPOLAR FLUID IN POROUS MEDIUM OVER A STRETCHING SURFACE WITH THERMAL RADIATION, HEAT GENERATION AND DISSIPATION

¹E.O. Fatunmbi & ²A.S Odesola ¹Department of Mathematics and Statistics, Federal Polytechnic, Ilaro, Ogun state. ²Department of Mathematics, University of Lagos, Akoka. ¹<u>olusojiephesus@yahoo.com</u>, ²<u>sotontop@yahoo.com</u>

ABSTRACT

The problem of a steady two dimensional MHD stagnation point flow and heat transfer of an incompressible micropolar fluid over a stretching surface in porous medium with radiation, heat generation/absorption and dissipation is investigated. The governing partial differential equations are transformed to ordinary differential equations using similarity transformation and the resulting ordinary differential equations are nonlinear. These nonlinear ordinary differential equations are then solved using Runge-Kutta method alongside the Shooting method. The effect of the included parameters on the fluid flow and heat transfer characteristics are presented and discussed.

Keywords: Micropolar fluid, stagnation flow, magnetohydrodynamics, similarity solution

Introduction

Micropolar fluids belongs to the class of fluids with non-symmetric stress tensor that are called polar fluids, these are fluids with microstructures and have the ability to shrink and expand, change their shape and may rotate independently of the rotation and movement of the fluids. Physically, micropolar fluids may represent fluids consisting of rigid, randomly oriented fluids (or spherical) particles suspended in a viscous medium, where the deformation of the particles is ignored. (Lukaszewick,1999).

The problem of flow and heat transfer over a stretching surface has gained wide application in many engineering processes such application include hot rolling paper, production, wire and fibercoating, foodstuff processing, glass blowing continuous casting of metal and spinning of fiber and so on.

Since introduction of the model of micropolar fluids by Eringen (1966) it has attracted the attention of many scientists, engineers, mathematicians and others. The attractiveness and

power of the model of micro-polar fluids come from the fact that it is both a significant and simple generalization of the classical Navier-Stokes, model which does not describe the physical properties of polymer fluids, colloidal solutions, suspension, liquid crystals and fluid containing small additives animal blood, paints etc.

A stagnation flow describes that the situations where the fluid motion near the stagnation region exist on a solid body and the fluid moves towards it. The stagnation region encounters the highest pressure, highest heat transfer and highest rate of mass deposition. The two dimensional flow of a fluid near stagnation point was first considered by Hiemenz (1911) who observed that the Navier-Stokes equation governing the flow can be reduced to a third order ordinary differential equations by similarity transformation. The problem was extended to the axisymmetric case by Homann (1936). Raptis (1998) investigated the flow of micropolar stationary fluid past a continuously moving plate in the presence of radiation.

The problem of stagnation flow of a micropolar fluid towards a vertical permeable surface in which the surface temperature and velocity are assumed to vary linearly was considered by Ishak et al (2008) with the distance from the stagnation point. Olanrewaju et al (2011) investigated a steady MHD flow towards a stagnation point on a vertical surface immersed in a micropolar fluid in the presence of thermal radiation. Mahmoud (2011) considered the two dimensional stagnation point towards a permeable stretching surface subject to a transverse magnetic filed in the presence of heat generation/absorption.

Yacob and Ishak (2011) considered the steady two-dimensional flow of a micropolar fluid over a shrinking sheet in its own plane where the shrinking velocity is assumed to vary linearly with the distance from a fixed point on the sheet. The problem was later extended to two dimensional stagnation point flow over a shrinking sheet immersed in an incompressible micropolar fluid (Ishak, Lok & Pop, 2012).

Kazeem et al (2011) studied stagnation point flow past a porous stretching sheet. Bachok, Ishak and Pop (2013) investigated the similarity solution of stagnation point flow toward a stretching/shrinking sheet with a convective boundary condition.

Hydromagnetic flow of a conducting micropolar fluid over a plane wall with heat transfer was considered by Attia and Ewis (2011). Hussain et al (2013) investigated the boundary layer flow towards a permeable stretching sheet. Aurangzaib et al (2013) studied unsteady MHD mixed convection flow with heat and mass transfer over a vertical plate in micropolar fluid saturated porous medium.

The purpose of this study is to investigate the effect of magnetohydrodynamic and heat transfer flow on two dimensional stagnation point embedded in a porous medium in the presence radiation, suction/blowing, viscous dissipation and heat generation/absorption. This is an extension of Sayed etal (2015). The numerical solution has been sought to examine the nature of fluid flow, microrotation and heat transfer. The effects of included parameters on temperature, velocity, microrotation of the fluid have been discussed.

2.0 Analysis and Method

The governing equations under the above assumption are:

Continuity equation

Momentum equation

$$U\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = -\frac{1}{p}\frac{\partial p}{\partial x} + \left(\frac{u+k}{p}\right)\frac{\partial^2 u}{\partial x^2} + \frac{k\partial u}{p\partial y} - \frac{u}{kp}u - \frac{\sigma Bo^2}{p}u \dots \dots \dots (2)$$

Angular Momentum equation

$$U\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = \frac{r}{pj}\frac{\partial^2 u}{\partial y^2} - k\left(\partial\mu + \frac{\partial u}{\partial y}\right)\dots\dots\dots\dots(3)$$

Energy Equation

$$U\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{pAP}\frac{\partial^2 T}{\partial y^2} - \frac{1}{pAP}\frac{\partial^2 T}{\partial y^2}\frac{\sigma Bo^2}{pAP}(u - ue)^2 + \frac{\mu}{pAP}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q(T - T_{\infty})}{pAP}(4)$$

But at free stream

 $U(x) = U_e(x)$

equation (2) implies

$$U_e(x)\frac{du_e(x)}{dx} + v\frac{\partial u_e(x)}{\partial v} = -\frac{1}{p}\frac{\partial p}{\partial x} + \left(\frac{u+k}{p}\right)\frac{\partial^2 u}{\partial y^2} + \frac{k\partial u}{p\partial y} - \frac{v}{kp}u_e(x) - \frac{\sigma Bo^2}{p}u_e(x)$$
$$= -\frac{1}{p}\frac{\partial p}{\partial x} = u_e(x)\frac{\partial u_e}{\partial x} + \frac{v}{kp}u_e(x) - \frac{\sigma Bo^2 u_e(x)}{p}$$

hence equation (2) becomes

$$U\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e \frac{\partial u_e}{\partial x} + \left(\frac{u+k}{p}\right)\frac{\partial^2 u}{\partial y^2} + \frac{k\partial u}{p\partial y} - \frac{v}{kp}(u-ue) - \frac{\sigma Bo^2}{p}(u-ue)$$

the governing equations with the boundary conditions are:

subject to:

$$u = ax$$
, $v = v_w$, $\mu = -m\frac{du}{dy}$, $T = Tw at y = 0$

$$U \to U_e(x) = bx, \quad \mu \to o. \quad T \to Tw \text{ as } y \to \infty$$

where x and y are Cartesians coordinates u and v are velocity components in the direction of x and y respectively

T: the temperature of the fluid in the boundary layer

 μ = the micro rotation vector (angular velocity)

p = dynamic viscosity

- J: micro inert density
- r: spin gradient viscosity
- k: vortex viscosity
- σ : electric conductivity

 β_0 : Magnetic field strength

- q: radiative heat flux
- A_p: specific heat at constant pressure
- K: thermal conductivity

m is a constant and $0 \le m \le 1$. The case when m is 0 is called strong concentration (Yacob & Ishale, 2012) indicates $\mu = 0$ near the wall, represents concentration particles flows in which the micro elements close to the wall surface are unable to rotate the care $\mu = 1/2$ indicates the vanishing of antisymmetric part of the stress tension and indicates weak concentrations (Ahmadi, 1976). the case, m – 1 is used for he modelling of turbulent boundary later flows (Peddieson, 1972)

As used by many authors, we assume that $r = \left(\mu + \frac{x}{2}\right)f = \mu\left(1 + \frac{x}{2\mu}\right)f = \mu\left(1 + \frac{x}{2\mu}\right)f$ where $\left(k = \frac{x}{\mu}\right)j$ is he material parameter micropolar parameter), $j = \frac{u}{a}$ is the reference length where $v = \frac{\mu}{\rho}$ (kinematic viscosity).

the assumption is involved to allow the field of equation predicts the correct behaviour in the limiting case when the microstructure effect become negligible and the total spin μ reduces to the angular velocity (Ahmadi, 1976).

Using Rosseland approximation the radiation heat flux is (Mohamed & Abo-Dahab, 2009)

$$qr = \frac{4\sigma * \partial T^4}{3a * \partial y}$$

where $\sigma *$ is the Stefan BoltzMann constant and a * is the mean absorption co-efficient. Assuming that the temperature within the flow are sum all such that T² can be expanded in Taylor series about T_∞

$$T^{4} = 4T_{\infty}^{4} - 3T_{\infty} + \dots \dots \text{ (Neglecting higher order term)}$$
$$\frac{\partial qr}{\partial y} = -\frac{160^{*}}{3a *} T_{\infty}^{3} \frac{\partial^{2} u}{\partial y^{2}}$$

Introduction stream function defined as $u = \frac{\partial u}{\partial y}$, $v = \frac{\partial u}{\partial x}$

Continuity equation (1) is automatically satisfied

I order to solve equation (1) - (5), we use the following similarity transformation (Olanrewaju et al., 2011) to transform the governing partial differential equations to ordinary differential equation

$$n = \left(\frac{a}{v}\right)_{y}^{1/2}, \qquad \varphi = (av)^{1/2}, \qquad xf(n), \quad \mu = ax\left(\frac{a}{v}\right)^{1/2}, \quad g(n)$$

$$\theta = \frac{T - T_{w}}{T_{w} - T_{\infty}} => T = (T_{w} - T_{\infty})\theta(n) + T_{\infty}$$

$$U = \frac{\partial \varphi}{\partial x} = -(av)^{1/2} xf(n)$$

$$\frac{\partial u}{\partial y} = \left(\frac{a}{v}\right)^{1/2} axf^{11}(n), \frac{\partial u}{\partial x} af^{11}(n)$$

$$\frac{\partial^{2}u}{\partial y^{2}} = \frac{a^{2}x}{v} f^{111}(n)$$

$$\frac{\partial n}{\partial y} = \frac{a^{2}x}{v} g^{1}(n)$$

$$\frac{\partial u}{\partial x} = a \left(\frac{a}{v}\right)^{1/2} g(n)$$

$$\frac{\partial T}{\partial y} = \left(\frac{a}{v}\right)^{1/2} (T_{w} - T_{w})\theta^{1}(n)$$

$$\frac{\partial T}{\partial y^{2}} = \frac{a}{v} (T_{w} - T_{w})\theta^{11}(n)$$

6) using (7) in (2) - (5) we obtain

$$(1+K)f^{111} + ff^{11} + A^2 - f^1 + Kg^1 + m(A - f^1) + D_a(A - f^1) = 0 \dots \dots (8)$$
$$\left(1 + \frac{k}{2}\right)g^{11} + fg^1 - f^1g - K(2g + f^{11}) \dots \dots \dots (9)$$
$$\left(1 + \frac{4}{3}R_n\right)\theta^{11} + P_rf\theta^1 + mB_r(C - f^1)^2 + B_rf^{11^2} + BP_r\theta = 0 \dots \dots (10)$$

subject to

where $M = \frac{\sigma B o^2}{a p}$ is the magnetic parameter,

 $B = \frac{Q}{apAp}$ is dimensionless heat generation / absorption parameter. $B_r = \frac{\mu u^2}{K(T_w - T_w)}$ is the Brikman number $P_r = \frac{\mu A_p}{K}$ is the prandtl number
$$C_f R_{ex}^{1/2} = \left(1 + \frac{K}{2}\right) f^{11}(0)$$

Where R_{ex} is the local Raynolds number defined as R_{ex} = $\frac{xu}{v}$

3.0 Results

Table 1:	The variation of <i>f</i> and <i>f</i> ' and θ at <i>A</i> = 0.5,1.0, and 1.5 while other parameters
	are constant

		A = 0.5		A = 1.0			A = 1.5		
η	f	f'	θ	f	f'	θ	f	f'	θ
0	0	1.00	1.000	0	1.0000	1.000	0	1.000	1.000
1	0.7366	0.5819	0.6588	1.0000	1.000	0.5835	1.3039	1.4592	0.5226
2	1.2715	0.5098	0.6588	2.0000	1.0000	0.2726	2.27916	1.4988	0.1964
3	1.7752	0.5008	0.1939	3.0000	1.0000	0.09943	4.2913	1.4999	0.0509
4	2.2755	0.5000	0.0820	4.0000	1.0000	0.0275	5.7913	1.4999	0.0088
5	2.7755	0.5000	0.0250	5.0000	1.0000	0.0052	7.2913	1.4999	0.0009
6	3.7755	0.5000	0.	6.000	1.0000	0	8.7913	1.5000	0

Table 2: The variation of θ at k = 1,3 and 9 where other parameters are constant

		K = 1		K = 2			<i>K</i> = 3		
η	f	f'	θ	f	f'	θ	f	f'	θ
0	0	1.0000	1.0000	0	1.0000	1.0000	0	1	1
1	2.9631	3.9221	0.33629	2.82388	3.8316	0.34494	271313	3.7349	0.35218
					4				0
2	6.96665	4.0069	0.045568	6.809034	4.0152	0.048616	6.665000	4.017785	0.05140
					2	6		9	9
3	10.96834	4.00005	0.002156	10.81431	4.0005	0.002399	10.67350	4.00170	0.00263
			3	5					6
4	14.96835	4.00000	0.000003	14.81442	4.0000	0.00003	14.67795	4.00002	0.00004
	4	0		9	0				
5	18.96835	4.00000	0.00000	18.81442	4.0000	0.00000	1867395	4.00000	0.00000
	4	0		9	0		7		
6	22.96835	1.71886	0.0000	22.17442	4.0000	0.0000	22.67395	4.00000	0.00000
				9	0		7		

Ishak and Mazar (2010) Sayed Α et al(Present 2015) Result 0.1 - 0969381 -0.969436 -0.9643623 0.2 -0918102 -0.918108-0.91811319 0.5 -0.667265 -0.667256 -0.667263721 0.00000 0.00000 0.0000000 2 2.017531 2.017481 2.0175028 3 4.72923 4.7292823 4.729283

Table 3: Comparison of skin friction of $C_f R_{e_x}^{1/2}$ the material parameter K = 0 for various values of the stretching parameter A

Fig.13 shows the variation of magnetic parameter (M) on the flow, it shows that increasing the values of M decreases the velocity profile f'

Fig.12 presents the temperature profile of θ for various values of Pr corresponding decrease the temperature profile.

Fig.10 shows the effect of Darcy number (Da) on the velocity profile f' and reveals that increasing the values of Darcy number decreases the velocity profile and reduces the thickness of the velocity boundary layer.

Fig.9 shows the effect of Darcy number (Da) on the profile which reveals that the thermal boundary layer thickness decreases with increasing in the values of Darcy number (Da).

Fig.8 presents the velocity profile f for various values of Darcy number. It shows that as the Darcy number increases beyond 1 the thickness of the velocity boundary layer increases.

Fig.7 illustrates the effect of suction (f_w) parameter on the velocity profile. It is noticed that the velocity boundary layer thickness decreases with increase in the suction (f_w) parameter up to when 1 is 2 after which there is a uniform distribution of the velocity profile.

Fig 5a shows the effect of Darcy number on the profile it shows that increasing the value of Darcy number decreases the velocity profile (f).

Fig.5b and 7 present the velocity profile of f and f' respectively for various values of f_w

Fig.6 illustrates the effect of suction (f_w) parameter on the temperature (θ) it is noticed that as the suction parameter (f_w) increases there is an increase in the temperature distribution on the profile.

This indicates that wall transpiration (suction or injection) provides an effective means of controlling the flow and heat transfer characteristics.

Fig.5c reveals the effect of the suction parameter on the velocity profile (f). It is noticed that increasing the suction parameter (f_w) , the velocity profile (f) is decreased.

From Fig. 14. It can be seen that an increase in M leads to an increase in the temperature profile. This is due to the fact that application of a magnetic field to an electrically conducting fluid produced a drag-like force known as Lorentz force. This force brings decrease in the fluid velocity and the microrotation but increase in the fluid temperature.

Fig. 15 and 16 depict the effect of the material parameter K on the dimensionless microrotation (g). It is noticed that near the plate the microrotaton (g) decreases with increase in K, while the reverse is true far away from the surface. Moreover, the velocity increases as the materials parameter (K) increase.





















Fig.5 Effect5 of fw on the profile at M=1,A=4,Pr=0.7,K=1

4.0 Conclusion

We have considered MHD stagnation point flow and heat transfer of a micropolar fluid over a stretching surface embedded in a porous medium with radiation, viscous dissipation and heat generation. Similarity transformation was used to transform the governing partial differential equations of the problem to ordinary differential equations. The resulting equations with the boundary conditions are solved by fourth order Runge-Kutta method alongside with Shooting method .The effects of material parameter, magnetic parameter, Prandtl number, radiation parameter, Darcy number, heat generation parameter and suction or injection on the temperature and velocity profiles are considered.

Some of the discoveries are as follow:

• Increase in the material parameter K shows that there is decrease in velocity profiles *and f'*.

- Increase in the material parameter K shows that there is decrease in the temperature profiles θ
- Temperature profile θ decreases when the stretching parameter (A) increases
- Velocity profiles f and f' increase with increasing values of the stretching parameter
- When radiation parameter increases there is a decrease in temperature profile.
- Temperature profile increases with increase in Darcy number.

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