Thermophoretic Mixed Convection Flow of MHD Micropolar Fluid Along an Inclined Surface with Soret-Dufour Effects and Variable Properties

E. O. Fatunmbi[1], S. O Bello[2]

[1-2]Department of Mathematics and Statistics, Federal Polytechnic, Ilaro, Nigeria.
E-mail: [1] olusojiephesus@yahoo.com, [2]sikiru.bello@federalpolyilaro.edu.ng
corresponding e-mail: *olusojiephesus@yahoo.com

Abstract
This study investigates the flow, heat and mass transfer of an electrically conducting micropolar fluid over an inclined permeable surface. In the analysis, the effects of thermophoresis, Soret and Dufour, non-uniform heat source/sink as well as chemical reaction are examined in the presence of variable dynamic viscosity and thermal conductivity. The set of governing equations modelling the problem is transformed from partial to ordinary differential equations with the use of appropriate similarity transformation variables. Subsequently, Runge-Kutta-Fehlberg integration scheme cum shooting techniques is applied to yield the numerical solutions for the reduced equations. The findings are both displayed on graphs and tables showing the influences of various physical parameters on the dimensionless velocity, temperature, concentration and microrotation profiles. The obtained results compared favourably with the earlier reported data in the literature as special cases of this present research. The results show that the momentum and concentration boundary layer thicknesses fall with an increase in thermophoresis and chemical reaction parameters while the opposite occurs with a rise in the thermal conductivity parameter.
Keywords: Micropolar fluid; variable fluid properties; permeable surface; Soret-Dufour; Thermophoresis

1 Introduction

Various investigations on boundary layer flow and heat transfer involving Newtonian fluids have been conducted over the years. However, in the recent times the study of fluid flow and heat transfer characteristics of non-Newtonian fluids have received great attention from scientists and researchers owing to its practical usefulness both industrially and in engineering processes. Micropolar fluid is prominent among other non-Newtonian fluids because of its inherent attribute in modelling and simulating complex and complicated fluids which contain rigid, randomly oriented and bar-like particles. These fluids are made up of microstructural elements which cannot be effectively explained by the Navier-Stokes model. For instance, polymeric fluids, fluid suspensions, animal blood, lubricants, liquid crystals, colloidal fluids and so on (Qasim *et al.*, 2013; Aurangzaib *et al.*, 2016). Micropolar fluid as well as thermal micropolar model was developed by Eringen (1966, 1972), in this model, the field of microrotation and macro-velocity are coupled together, the fluids tend to display some microscopic effect leading to both translation and rotation of the fluid element. The possible applications of such fluids in engineering and industrial operations can be found in the bio-mechanic and chemical engineering, extrusion of polymer, slurry technologies, synovial lubrication, arterial blood flows, knee cap mechanics, a few of many (Lukaszewicz, 1999; Reena and Rana, 2009).

The study of combined heat and mass transfer is significant in engineering and manufacturing activities for instance for the design of relevant equipment, gas turbines, satellites and space vehicles, nuclear plants, electric transformers, etc. Many transport phenomenon take place in nature and industrial processes in the situation where heat transfer and mass transfer occur simultaneous due to a combined buoyancy influences of both thermal diffusion and diffusion of chemical species. These processes can be encountered in chemical processing industries such as food processing, groves of fruit trees and crop damage due to freezing, polymer production, etc. Studies of heat and mass transfer using micropolar fluid have possible applications in biomedical and engineering activities such as in the dialysis of blood in artificial kidney, blood flows, fluid flow in brain, flow in oxygenation, porous pipe design, design of filter (Reena and Rana, 2009; Olajuwon et al., 2014). Several authors (see Mohamed and Abo-Dahab, 2009; Pal and Chatterjee, 2010; Olanrewaju, *et al.*, 2013; Mishra *et al.*, 2016; Kumar, 2017; Fatunmbi and Fenuga, 2017) have investigated such studies with various parameters of interest, different geometry, boundary conditions and methods.

Many of the aforementioned researches were conducted with an assumption that the fluid thermo-physical properties are constant whereas these properties particularly, the fluid viscosity and thermal conductivity have been found to vary largely with changes in temperature. The internal heat generation due to friction as well as an increase in temperature may influence these thermo-physical properties, thus, the assumption of constant fluid properties becomes invalid. Specifically, the increase in temperature from 10^oC to 50^oC causes the viscosity of water to decrease by about 240%. Also, the viscosity of air is 0.6924×10^{-5} at $1000K$, 1.3289 at $2000K$, 2.286 at $4000K$ and 3.625 at $8000K$ (Cebeci and Bradshaw, 1984). Hence, the assumption of constant fluid properties may not yield accurate results as remarked by Postelnicu *et al.* [20]. For accurate prediction, therefore, it is essential to find out the possible effects of temperature-dependent thermophysical properties of the fluid on the heat transfer processes. The application of such study includes hot rolling, paper and textile production, process of wire drawing and drawing of plastic films. In view of the numerous applications of such studies, various researchers (see Mukhopadhyay, 2013; Salem, 2013; Thakur and Hazarika, 2014; Akinbobola and Okoya, 2015, Makinde *et al.*, 2016; Keimanesh and Aghanajafi, 2017, etc.) have reported the influence of variable fluid properties on Newtonian as well as non-Newtonian fluids under various boundary conditions and geometries.

Owing to the consequential applications of studies on heat and mass transfer with thermophoresis effects as highlighted above, the current research has set out to extend the study of Mondal *et al.* (2017) by investigating the influence of variable viscosity and thermal conductivity on thermophoretic mixed convection flow of an electrically conducting micropolar fluid passing an inclined permeable sheet in a porous medium with effects of first order homogenous chemical reaction, viscous dissipation and non-uniform heat source/sink. Specifically, the novelty of this work is in the inclusion of variable fluid properties as against constant fluid properties investigated by Mondal *et al.* (2017). Besides, the non-Newtonian micropolar fluid has been engaged as a working fluid as against the Newtonian fluid applied by those authors and in addition, the effects of the porous medium are also taken into consideration in this study. Possible application of this study can be found in biomedical engineering such as drug delivery and tissue implant and other areas of engineering such as in crude oil extraction, ground water hydrology, irrigation system, etc.

2 Mathematical Development of the Model

Considering a convective steady flow of an electrically conducting micropolar fluid on a two-dimensional semi-infinite permeable inclined sheet and embedded in a porous medium as described in Fig. 1. The sheet is inclined at an angle φ to the vertical, the x axis is measured along the sheet with y axis normal to it, a uniform magnetic field of strength B_0 is applied perpendicular to the flow direction while a uniform suction is applied at the sheet surface. The influences of first order chemical reaction, thermophoresis and Soret-Dufour are checked on the model. The heat source/sink is assumed to non-uniform, viscous dissipation effect is accounted for in the energy equation, the dynamic fluid viscosity and thermal conductivity are assumed to be temperature-dependent. Using the Boussinesq and boundary layer approximations with the aforementioned assumptions, the governing equations of the flow, heat and mass transfer are expressed as follows:

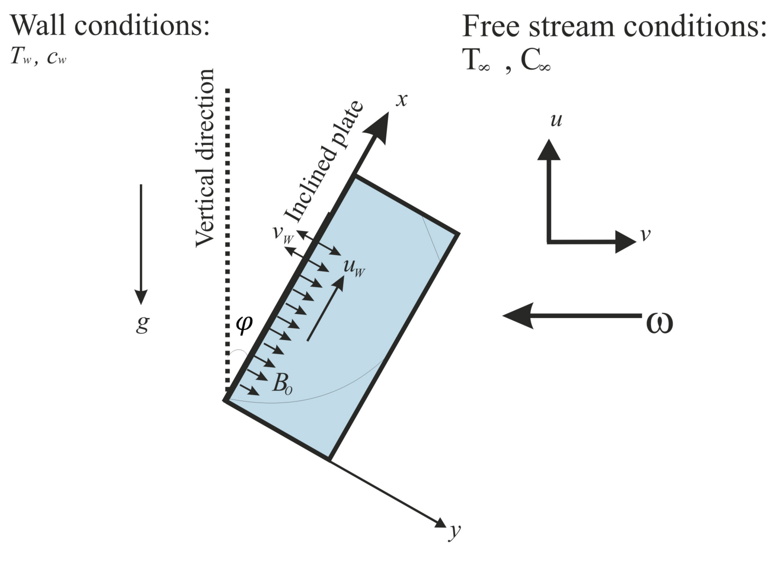

Fig. 1 The Sketch of the Physical Model

The Governing Equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right)+\frac{\mu_r}{\rho_\infty}\frac{\partial^2 u}{\partial y^2}+\frac{\mu_r}{\rho_\infty}\frac{\partial \omega}{\partial y}+g\beta_T\left(T-T_\infty\right)cos\varphi+g\beta_C\left(C-C_\infty\right)cos\varphi-\frac{\sigma B_0^2}{\rho_\infty}u-\frac{\mu(T)}{\rho_\infty k_p}u \tag{2}$$

$$u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2\omega}{\partial y^2}-\frac{\mu_r}{\rho j}\left(2\omega+\frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho_\infty c_p}\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right)+\frac{(\mu+\mu_r)}{\rho_\infty c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B^2}{\rho_\infty c_p}u^2+\frac{q^{'''}}{\rho_\infty c_p}+\frac{DmK_T}{CsC_p}\frac{\partial^2 C}{\partial \bar{y}^2}, \tag{4}$$

$$u\frac{\partial C}{\partial \bar{y}}+v\frac{\partial C}{\partial y}=Dm\frac{\partial^2 C}{\partial y^2}+\frac{DmK_T}{T_m}\frac{\partial^2 T}{\partial y^2}-k_r\left(C-C_\infty\right)-\frac{\partial}{\partial y}\left(V_T C\right) \tag{5}$$

The relevant boundary conditions for this study are as follows:

$$\begin{aligned}&u=U_\infty, v=V_w, \omega=-h\frac{\partial u}{\partial y}, T=T_w, C=C_w \quad at\ y=0,\\ &u=0, \omega\longrightarrow 0, T\longrightarrow T_\infty, C\longrightarrow C_\infty \quad \text{as} \quad y\to\infty.\end{aligned} \tag{6}$$

From the above governing Equations (1-5), u and v represent velocity components along x and y directions respectively, μ stands for dynamic viscosity while ν denotes kinematic viscosity with κ symbolizing thermal conductivity, ρ_∞ stands for the density of the ambient fluid whereas the vortex viscosity is denoted by μ_r while j symbolizes micro-inertial density, c_p is the specific heat at constant pressure and γ is the spin gradient viscosity. Also, T, T_w, T_∞ and ω describe the fluid temperature, temperature of the sheet, free stream temperature, microrotation component.

The suction/injection term is denoted by V_w, σ is the electrical conductivity. Furthermore, β_T, β_C, k_r, k_p, Dm, Cs and Tm are coefficient of thermal expansion, coefficient of solutal expansion, chemical rate of reaction, permeability of the porous medium, mass diffusivity, fluid susceptibility and mean fluid temperature in that order. The parameter h appearing in equation (6) is a boundary parameter having characteristics $0\le h\le 1$. At the lower limit $h=0$, the component of microrotation ω implies $\omega=0$, this situation is described by Jena and Mathur (1981) as a strong concentration of the micro-particles at the wall implying that the micro-particles near the wall are incapable of rotation. On the other hand, Ahmadi (1976) pointed out that when $h=1/2$, there is a weak concentration such that effect of the microstructure is negligible in the neighbourhood of the boundary. However, in modelling the turbulent boundary layer situations the upper limit of h corresponds to $h=1$ as reported by Peddieson (1972). The deviation of the micropolar fluid characteristics from the classical Newtonian fluid model can be measured by the size of the vortex viscosity parameter μ_r, thus, when $\mu_r=0$, then equations (2) and (4) are decoupled from equation (3). In that case, the model in the current study alongside the results obtained correspond to that of Newtonian fluid model. We have also assumed that $V_w=V_0x^{-1}$, $\sigma=\sigma_0x^{-1}, \beta_T=\beta_0x^{-1}, \beta_C=\beta_0^\star x^{-1}, k_r=k_1x^{-1}k_p=k_0x$ where $V_0, \sigma_0, beta_0, \beta_0^\star, k_1$ and k_0 are constants (Ishak, 2010;, Makinde, 2012, Parida *et. al.*, 2015).

The non-uniform heat source/sink q''' in equation (4) is expressed as (see Olajuwon *et al.* 2013; Pal and Mondal,2014 and Das *et al.*, 2015)

$$q'''=\frac{\kappa U_0}{2x\nu}\left[\alpha\left(T_w-T_\infty\right)e^{-\eta}+\beta\left(T-T_\infty\right)\right] \tag{7}$$

with α and β being the space and heat dependent source/sink respectively. When $\alpha>0$ and $\beta>0$ then heat is generated whereas heat is absorbed when $\alpha<0$ and $\beta<0$.

The variation of the viscosity with temperature is taken to be in nonlinear manner as described in equation (8), (Lai and Kulacki, 1989; Makinde, 2010; Akinbobola & Okoya, 2015)

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+A\left(T-T_\infty\right)\right]=B\left(T-T_r\right), \tag{8}$$

with

$$B=\frac{A}{\mu_\infty}, T_r=T_\infty-\frac{1}{A}, \tag{9}$$

where A is a constant corresponding to the fluid thermal property, μ_∞ indicates the ambient fluid viscosity, B and T_r are constants. The thermal conductivity κ is supposed to vary with temperature in an approximately linear manner as described in equation (10) (see Chiam, 1997; Oahimire and Olajuwon, 2013).

$$\kappa\left(T_w-T_\infty\right)=\kappa_\infty\left[\left(T_w-T_\infty\right)+\delta\left(T-T_\infty\right)\right], \tag{10}$$

where κ_∞ indicates the ambient thermal conductivity, δ stands for the variable thermal conductivity parameter. Meanwhile, Mahmoud (2012) reported that the range of values of $0\le\delta\le 6$ is applicable for air while $0\le\delta\le 0.12$ is appropriate for water and $-0.1\le\delta\le 0.12$ for lubrication oils.

The thermophoretic velocity is expressed as

$$V_T = -\frac{k_t^\star}{T_{ref}}\frac{\partial T}{\partial y}. \tag{11}$$

The thermophoresis coefficient $k_t^\star$ is described as

$$k_t^\star = \frac{2Cs\left(\kappa/k_s + C_t K_n\right)\left(C_1 + C_2 e^{C_3/K_n}\right)}{\left(1+3C_m K_n\right)\left(1+2\kappa/k_s + 2C_t K_n\right)}. \tag{12}$$

Here C_1, C_2, C_3, C_m and C_t are constants while κ and k_s are represent the fluid thermal conductivity and the diffused particles thermal conductivity respectively whereas K_n is the Knudsen number.
The thermophoretic parameter τ is defined as

$$\tau = -\frac{k_t^\star\left(T_w - T_\infty\right)}{T_{ref}}. \tag{13}$$

Typical value of τ are 0.01, 0.05 and 0.1 corresponding to the approximate values of $-k_t^\star\left(T_w - T_\infty\right)$ equal to $3K, 15K$ and $30K$ for $T_{ref} = 300K$. In line with previous authors Rahman *et al.* (2012) and Mondal *et al.* (2017), the following similarity variables (14) are used to transmute the governing equations

$$\begin{aligned}&\eta = y\left(\frac{U_\infty}{2x\nu_\infty}\right)^{1/2},\ \psi = \left(2U_\infty x\nu_\infty\right)^{1/2} f(\eta),\ \omega = U_\infty\left(\frac{U_\infty}{2x\nu_\infty}\right)^{1/2} g(\eta),\ \gamma = \left(\mu_\infty + \frac{\mu_r}{2}\right) j, j = \frac{\nu_\infty x}{U_\infty},\\ &\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} = \frac{T - T_r}{T_w - T_\infty} + Q,\ Q = \frac{T_r - T_\infty}{T_w - T_\infty}, u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}.\end{aligned} \tag{14}$$

Furthermore, by making use of quantities in equation (14) and taking cognizance of Eqs. (7-13) the governing Eqs. (2-5) yield the underlisted equations:

$$\left(\frac{Q}{Q-\theta} + K\right) f''' + \frac{Q}{\left(Q-\theta\right)^2}\theta' f'' + Kg' + ff'' + Gr\theta\cos\varphi + Gc\phi\cos\varphi - \left[Da\left(\frac{Q}{Q-\theta}\right) + M\right] f' = 0, \tag{15}$$

$$\left(1 + K/2\right) g'' + 2\left(f'g + fg'\right) - 2K\left(2g + f''\right) = 0, \tag{16}$$

$$\left(1+\delta\theta\right)\theta'' + \delta\theta'^2 + Prf\theta' + \left(\frac{Q}{Q-\theta} + K\right) PrEcf''^2 + PrMEcf'^2 + \left(1+\delta\theta\right)\left(\alpha e^{-\eta} + \beta\theta\right) + PrDu\phi'' = 0. \tag{17}$$

$$\phi'' + Sc\left(f - \tau\theta'\right)\phi' - Sc\zeta\phi + Sc\left(Sr - \tau\phi\right)\theta'' = 0. \tag{18}$$

Also, the boundary conditions become

$$\begin{aligned}&f'(0) = 1,\ f(0) = fw,\ g(0) = -hf'',\ \theta(0) = 1,\ \phi(0) = 1\\ &f'(\infty) = 0\ g(\infty) = 0,\ \theta(\infty) = 0,\ \phi(\infty) = 0.\end{aligned} \tag{19}$$

With

$$\begin{aligned}&fw = \frac{-\sqrt{2}V_0}{\left(\sqrt{U_\infty\nu_\infty}\right)}, M = \frac{2\sigma_0 B_o^2}{\rho_\infty U_\infty}, Gr = \frac{2xg\beta_T\left(T_w - T_\infty\right)}{U_\infty^2}, Gc = \frac{2xg\beta_C\left(C_w - C_\infty\right)}{U_\infty^2},\\ &Q = -\frac{1}{A\left(T_w - T_\infty\right)}, K = \frac{\mu_r}{\mu_\infty}, Ec = \frac{U_\infty^2}{cp\left(T_w - T_\infty\right)}, Pr = \frac{\mu_\infty c_p}{\kappa_\infty}, Da = \frac{2\nu_\infty}{k_o U_\infty},\\ &Du = \frac{DmK_T}{CsCp\nu_\infty}\left(\frac{C_w - C_\infty}{T_w - T_\infty}\right), Sr = \frac{DmK_T}{Tm\nu_\infty}\left(\frac{T_w - T_\infty}{C_w - C_\infty}\right), Sc = \frac{\nu_\infty}{Dm}, \zeta = \frac{2k_1}{U_\infty}\end{aligned} \tag{20}$$

Where the viscosity variation parameter is indicated by Q, the material (micropolar) parameter is denoted as K, the suction/injection parameter is symbolized as fw with $fw > 0$ indicating suction while $fw < 0$ relates to injection whereas an impermeable sheet is described when $fw = 0$. Moreso, M depicts the Magnetic field parameter while α symbolizes space-dependent source/sink whereas temperature-dependent heat source/sink is designated by β, the thermal Grashof number is indicated by Gr, the solutal Grashof number is resented by Gc, Du symbolizes Dufour number, Sr indicate Soret number, ζ connotes chemical reaction rate, δ denotes thermal conductivity parameter and the Eckert number is represented as Ec whereas the differentiation is done with respect to η and Pr indicates the Prandtl number.
The relevant quantities of engineering interest are the skin friction coefficient, Nusselt and Sherwood numbers as respectively given in Eq. (21) in that order.

$$C_{fx} = \frac{\tau_w}{\rho_\infty u_w^2},\ Nu_x = \frac{xq_w}{\kappa_\infty\left(T_w - T_\infty\right)}, Sh_x = \frac{xq_m}{Dm\left(C_w - C_\infty\right)}, \tag{21}$$

with τ_w being shear stress, q_w heat flux at the surface and q_m is the mass flux at the surface such that

$$\tau_w = \left[(\mu(T) + \mu_r) \frac{\partial u}{\partial y} + \mu_r N \right]_{y=0}, \quad q_w = - \left[\kappa \frac{\partial T}{\partial y} \right]_{y=0}, q_m = - \left[Dm \frac{\partial C}{\partial y} \right]_{y=0} \tag{22}$$

in view of equations (14) and (20), the skin friction coefficient yields

$$C_{fx} = \left[\frac{Q}{Q - \theta} + (1 - h) K \right] Re_x^{-1/2} f''(0), \tag{23}$$

while the Nusselt and Sherwood numbers respectively becomes

$$Nu_x = -Re_x^{1/2} \theta'(0), Sh_x = -Re_x^{1/2} \phi'(0) \tag{24}$$

3 Method of Solution

Due to the highly nonlinearity of the system of equations (15-18) together with the boundary conditions (19), a numerical solution has been sought in this study via shooting technique in company with Runge-Kutta-Fehlberg integration algorithm based on computer algebra symbolic Maple 2016 package. To check for the accuracy and validity of the numerical code employed in this study, the computed values of the skin friction coefficient, Nusselt number as well as the Sherwood number have been cross-checked with existing results of Alam *et al.* (2006) and Mondal *et al.* (2017) in the absence of $K, \delta, \tau, \alpha, \beta, \zeta, \varphi, Ec$ and Da. These values are recorded in Table 1 and has been found to be in excellent relationship with those previous authors.

4 Results Analysis and Discussion

To gain more insight into the behaviour of the controlling parameters in the flow field, various graphs have been included corresponding to the dimensionless velocity, microrotation, temperature and concentration fields as well as on the skin friction coefficient, Nusselt and Sherwood numbers.

Table 1: Comparison of values of C_{f_x} and Nu_x with variations in Du and Sr when $Gr = 10.0; Gc = 4.0; fw = 0.5; M = 0.3; Pr = 0.71; Sc = 0.22$ and $\zeta = \tau = K = Ec = \delta = \alpha = \beta = 0, Q \longrightarrow \infty$

Du	Sr	Alam et al		Mondal et al		Present	
		C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x
0.030	2.0	6.2285	1.1565	6.238707	1.151932	6.229908	1.156453
0.037	1.6	6.1491	1.1501	6.160867	1.144651	6.150610	1.150059
0.050	1.2	6.0720	1.1428	6.087934	1.135752	6.073556	1.142710
0.075	0.8	6.0006	1.1333	6.023187	1.123219	6.002251	1.133220
0.150	0.4	5.9553	1.1157	5.996934	1.096560	5.957028	1.115613

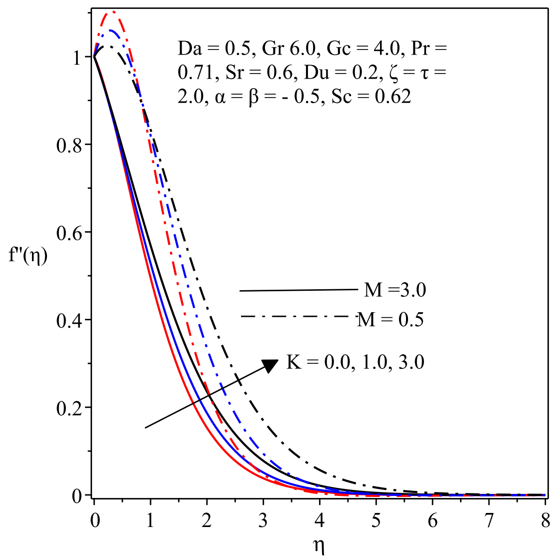

Fig. 2 Velocity field for variation in K & M

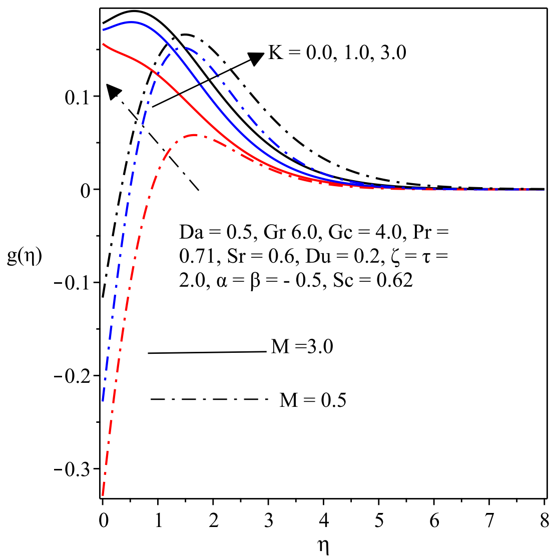

Fig. 3 The impact of K & M on microrotation profiles

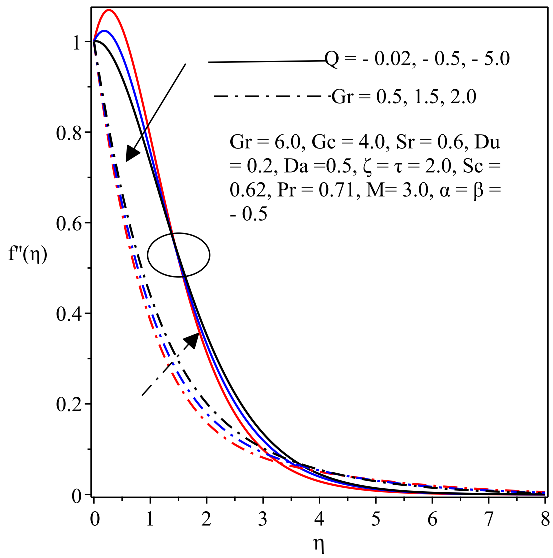

Fig. 4 Velocity field for variation of *Gr* & *Q*

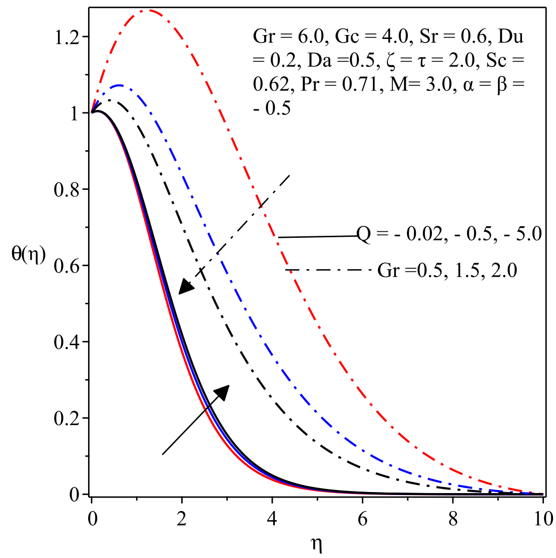

Fig. 5 The impact of *Gr* & *Q* on temperature profiles

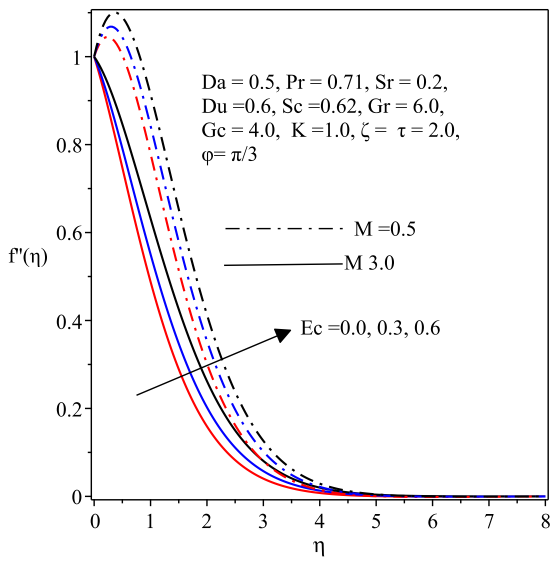

Fig. 6 The influence of *Ec* & *M* Velocity profiles

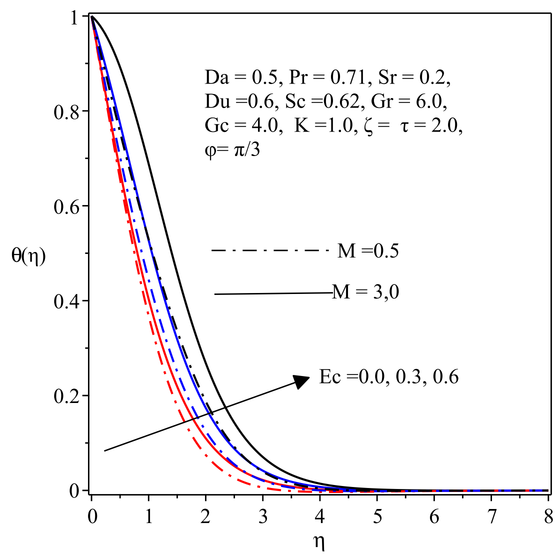

Fig. 7 The variation of *Ec* & *M* on temperature

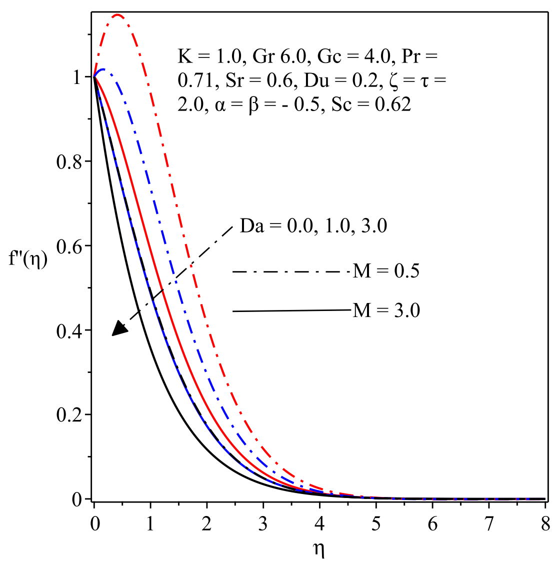

Fig. 8 Effects of *Da* & *M* on velocity field

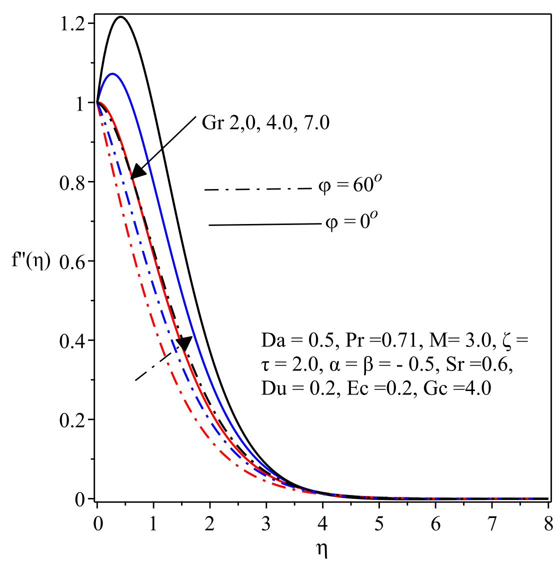

Fig. 9 The impact of *Gr* & φ on velocity

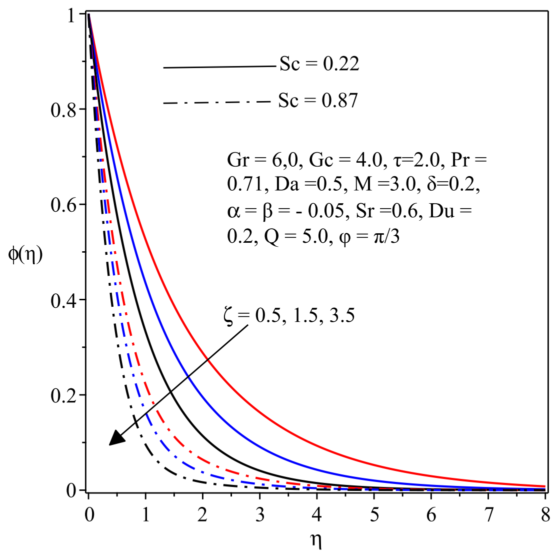

Fig. 10 Influence of ζ on concentration

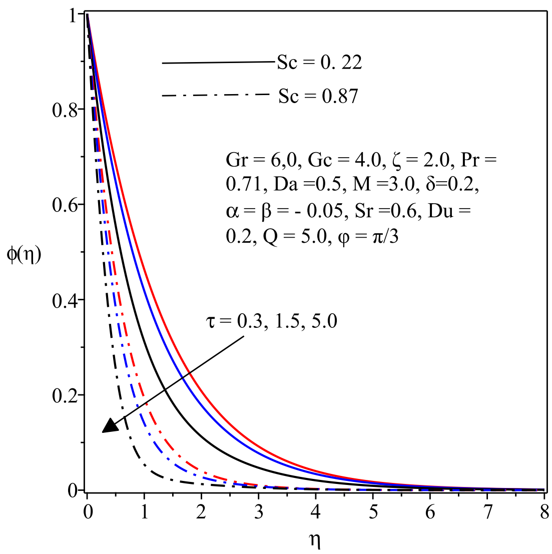

Fig. 11 The impact of τ on concentrations

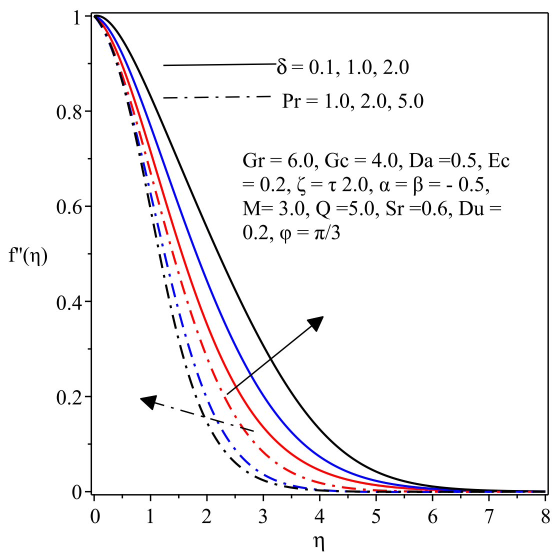

Fig. 12 Velocity profiles for variation in Pr & δ

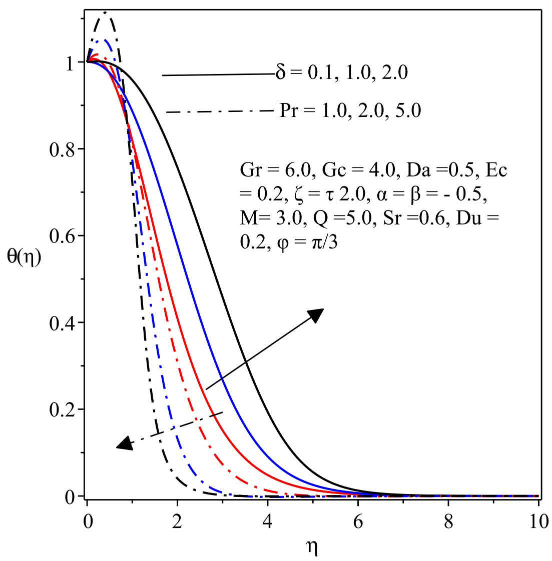

Fig. 13 Temperature profiles for variation in Pr & δ

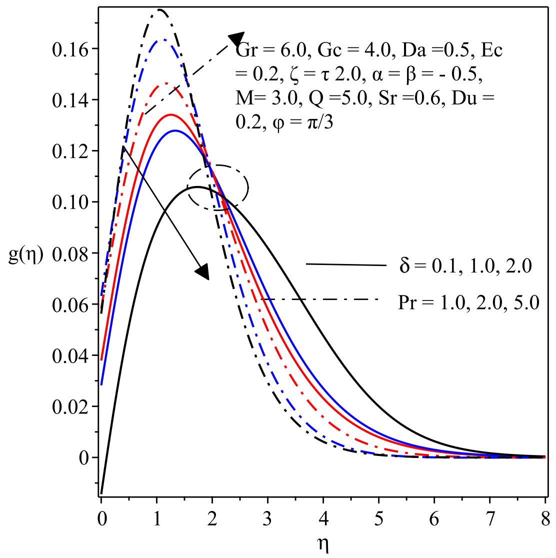

Fig. 14 Effect of Pr & δ on microrotation

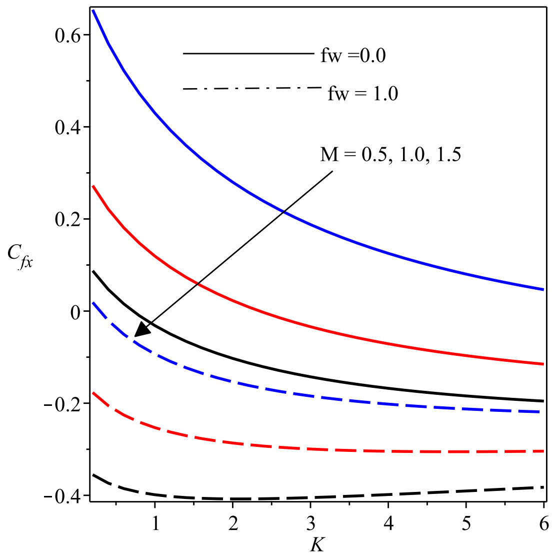

Fig. 15 Effects of M & K on C_{fx}

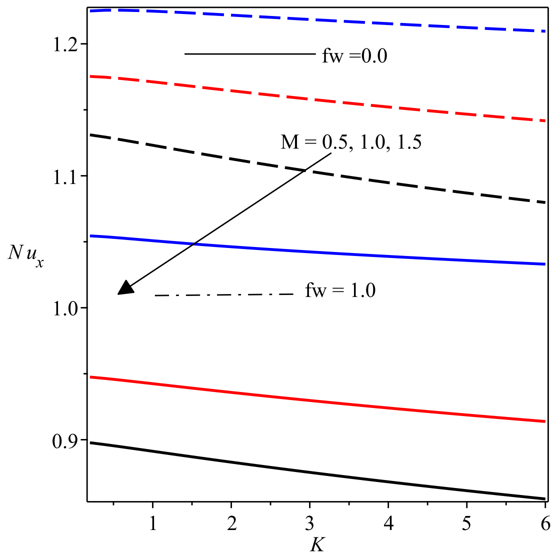

Fig. 16 Variation of K & M on Nu_x

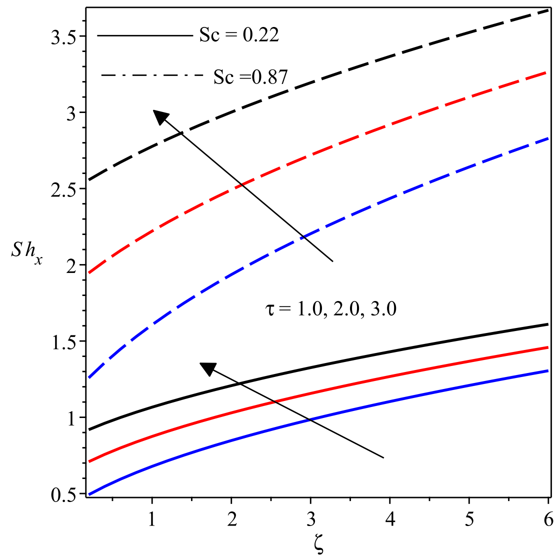

Fig. 17 Reaction of Sh_x for variation in τ & ζ

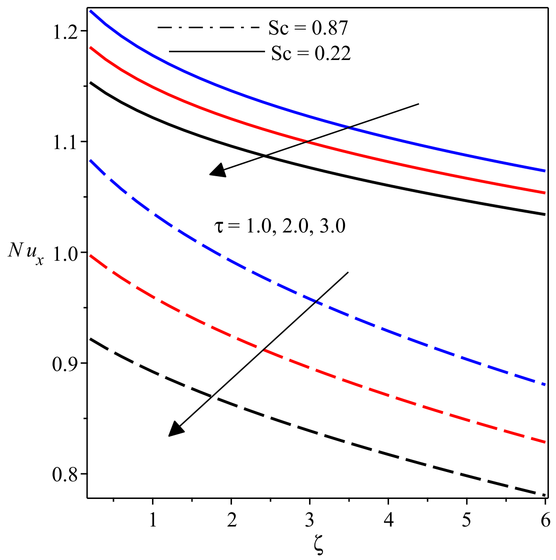

Fig. 18 Variation of τ & ζ on Sh_x

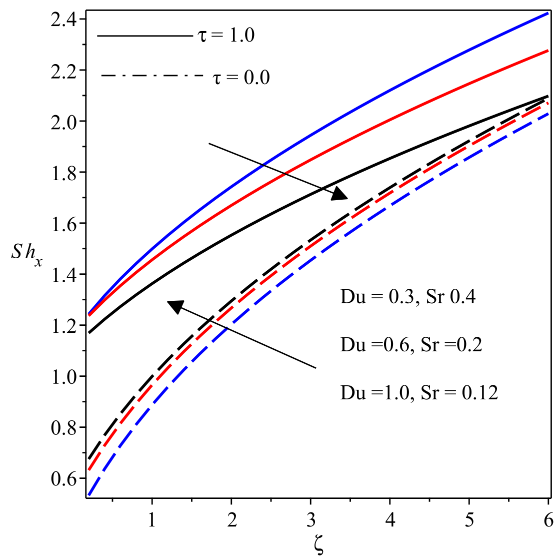

Fig. 19 Response of Sh_x to variation in Du/Sr & ζ

5 Conclusions

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