

# Introducing Wind Energy Conversion System to Nigerian Power System: Application of Convex Optimization to Achieve Economic Dispatch

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## **Abstract**

*The Nigerian Power System produces less energy than required by the population of over 167 million and the production of energy from the conventional energy sources requires huge amount of money to be spent on fuel to power the generating units. Hence the need to introduce renewable energy production systems such as wind energy conversion system (WECS) that require very minimal running cost cannot be overemphasized. Determining the appropriate bus where such incoming WECS are to be connected to the existing power system network will make such a system very useful. This requires knowledge of output mix from existing and incoming generating units and the ability of the decision makers to ensure stability and minimal loss on the network since losses increase with power increase. In this work, the optimal and reduced cost of power production through wind energy conversion system (WECS) were determined using mathematical models taking into consideration those factors that affect the cost of generation of electric power and the effects of the losses on transmission lines, adopting semi-definite programming (SDP) with SeDuMi software. With the adopted technique, it was shown that it was possible to improve power production from the existing system of about 3500MW in Nigeria to about 4000MW, out of which 552.8MW was to be produced from optimally-placed two WECS (one in the north-western part and the other in the Jos axis) on the national grid.*

**Keywords:** Power system, optimization, Wind energy conversion system, Semi-definite programming, SeDuMi, Nigerian Power grid system.

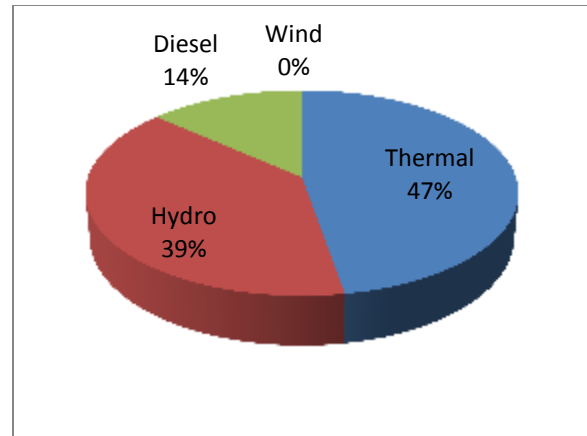
## 1.0 Introduction

### 1.1 State of Nigerian Electric Energy Sector

The Nigerian Power System (NPS) produces less energy than required by the population of over 167 million. Nigeria is well endowed with variety of renewable and non-renewable energy sources but electrical energy in the country is predominantly generated by conventional energy sources such as thermal and hydro. This account for about 86% of the power produced in the country as revealed in Figure 1.

The production of energy from the conventional energy sources requires huge amount of money to be spent on fuel to power the generating units. Hence the need to introduce renewable energy production systems such as wind energy conversion system (WECS) that require very minimal running cost. Figure 1 reveals that wind energy system is accounted 0.00%. This is because wind energy has not been really utilized in production of energy in the country, save for an inconsequential 5kW aero generator in Sayya Gidan, Gada, Sokoto State. Other sources of power in the country include diesel, and renewable energy resources such as solar, hydro, biomass, wind, etc. which are non-depletable as they are available on periodic/cyclic basis [1].

According to the Nigerian Energy Policy report from 2003, it is estimated that the population connected to the grid system is short of power supply over 60% of the time. Additionally, less than 40% of the population is even connected to the grid [2]. Several authors [1, 2, 3, 4, 5], have pointed to renewable energy sources such as wind energy as good alternatives to power production in the country.



**Fig. 1:** Proportions of Energy Sources in Nigeria

In addition, there is the current need for emissions from power stations reduction, that is, need for clean energy production. On a fundamental level, there is simply not enough electricity generated to support the entire population and less than about 14kwh per capita is available on the grid [3]. By this token, there is the need to incorporate renewable energy sources such as wind energy conversion system (WECS) to supplement the meager energy production in the country.

### 1.2 Incorporating WECS into the NPS

The 24-bus NPS network is depicted in Figure 2. It consists of 7 generating stations, 32 buses and the transmission voltage is 330kV and 132kV. The Nigerian Electricity Network comprises 11,000km transmission lines (330kV and 132kV), 24000km of sub-transmission line (33kV), 19000km of distribution line (11kV) and 22,500 substations. Few years ago, the energy losses on this power system network have been estimated to be 337.5 GWH [6]. This work seeks to design an economic dispatch system that shall greatly reduce such high losses.

Incorporating WECS into the NPS network requires the knowledge of where such incoming WECS are to be connected to the

existing power system network and adequate information as to what the resulting penetration might result into. The knowledge of the wind characteristics of the country is also essential. Adekoya, and Adewale (1992) and Matthew (2006) have shown that for a WECS to be viable and cost effective at a particular site, the site average wind speed should be between 4-6 m/s [7, 8]. Several works have dealt with this crucial aspect of power planning. The results indicate that the overall annual wind speed regime is low but few areas are promising locations. Such areas are the Jos, North Eastern and North Western axes of the country [3, 5, 9].

Thus, this work goes a step forward in implementing the suggestions of various authors by incorporating WECS into the Nigeria power system network and solving the network as a convex optimization problem to determine the optimal values for existing generators and newly added WECS. The points of entry of the incoming WECS are also determined.

## 2.0 Traditional Optimization Solutions

Some traditional methods reported in literature include the Lagrangian, method the weighted sum method [10, 11], lambda iteration, and the gradient method [12, 13]. Others are linear programming (LP), non-linear programming (NLP) and quadratic programming (QP) methods [13].

## 2.1 Semi-definite Optimization Solutions

More recently, semi-definite programming (SDP) was developed for solving optimization problems. SDP—linear matrix inequality (LMI) — uses Interior-Point Methods (IPMs) as an efficient way of solving linear programming. IPM's solve SDP problems as efficiently as they solve linear programming ones [14, 15, 16]. Other

methods such as non-sorting genetic algorithm (NSGA) and the micro genetic algorithm ( $\mu$ GA) have also been applied to power system solved as convex optimization and reported in literature [17, 18].

## 2.2 Solving Convex Optimization Using SDP

The convex optimization problem in this work is solved by employing SDP. It has been shown that convex programming problems, with quadratic cost and quadratic constraints, can be formulated as SDP programs [20]. SDP is a technique that can be used to solve convex optimization problems in polynomial time [21]. The solution requires that the variables' integer-value requirements are modeled as quadratic equations whose roots are determined as the desired integer values. Thus, an explicit integer Programming formulation of the problem is not necessary [22]. The SDP technique can only be used in problems where the intersection of constraints defines a convex set. By this way, a convergence to an optimum in polynomial time when IPMs are employed is ensured.

## 3.0 Methodology

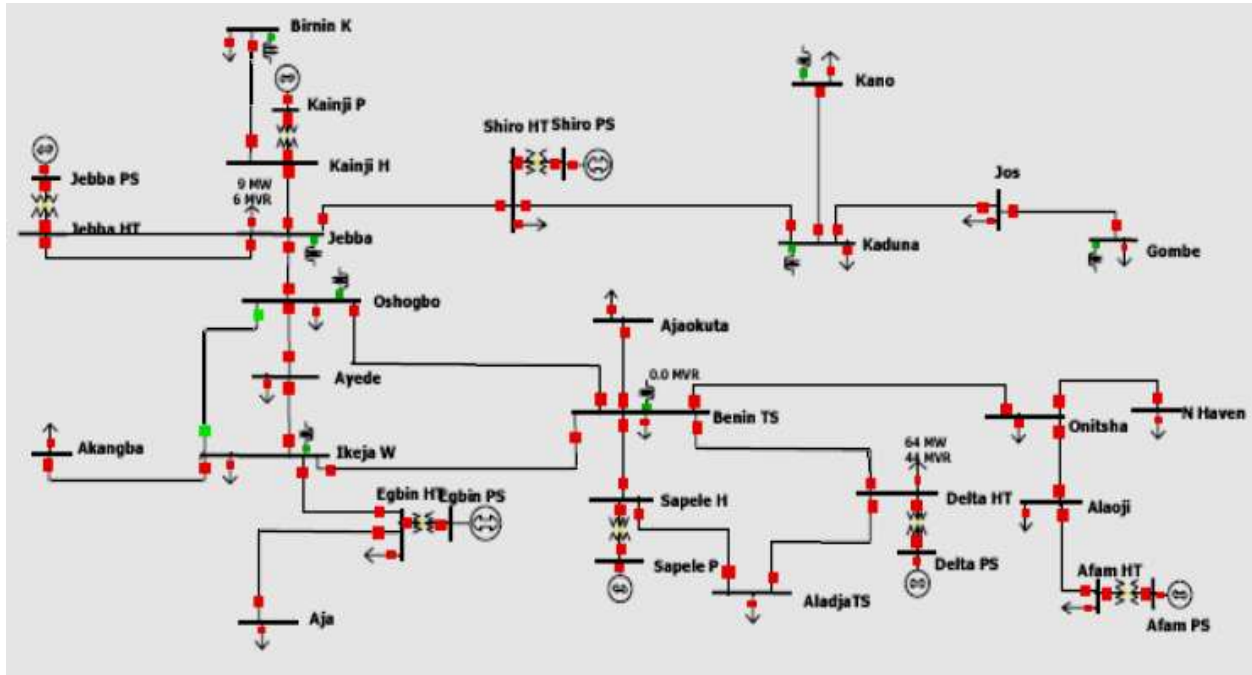
### 3.1 Problem Formulation

The NPS is considered as a convex system having the following objectives.

**Fuel cost objective.** The total fuel cost satisfying the total required demand for the seven existing generator is mathematically stated as follows [23, 24]:

$$C_i(p_i) = \alpha_i + \beta_i p_i + \gamma_i p_i^2. \quad (1)$$

where  $C$  : total fuel cost (\$/hr),  
 $\alpha_i, \beta_i, \gamma_i$  : fuel cost coefficients of thermal generator  $i$ ,  
 $p_i$  : power output by thermal generator  $i$ .



**Figure 2: Single Line Diagram of NPS Network**

**Transmission loss minimization objective.** Transmission losses on the network are modeled using the loss coefficients method (known as the B-coefficients) developed by Kron and adopted by Kirch Mayer [25]:

$$P_L = \sum_{i=1}^N \left( \sum_{j=1}^N P_{ij} B_{ij} P_j + \sum_{i=1}^N B_{oi} P_i + \sum_{i=1}^N B_{ooi} \right). \quad (2)$$

Here, the terms  $B_{ij}$  are referred to as the B coefficients. The expression determines the total power loss  $P_L$  as sum of a quadratic term, a linear term and a constant term.

### 3.2 Optimization Constraints

The constraints within which the optimization problem exists shall now be itemized and discussed. They represent the set of conditions that must be met for a sustainable power supply.

**Power balance constraint.** The total power generated must be sufficient to supply the total load demand and the transmission

losses and this can be expressed mathematically as:

$$\sum_{i=1}^N P_{gi} - P_D - P_L = 0. \quad (3)$$

$\sum_{i=1}^N P_{gi}$ : total power generated prior to wind generator being added (MW)

$P_D$ : total load demand (MW) and

$P_L$ : transmission losses without wind generator (MW).

**Maximum and minimum limits of power generation.** The power generated  $P_{gi}$ , by each generator is constrained between its minimum and maximum limits stated as:

$$\underline{p}_{gi} \leq p_{gi} \leq \overline{p}_{gi} \quad i = 1, \dots, n_g. \quad (4)$$

where  $\underline{p}_{gi}$ : minimum power generated by the  $i^{\text{th}}$  generator, and

$\overline{p}_{gi}$ : maximum power generated by the  $i^{\text{th}}$  generator.

**Renewable energy limit.** A WECS is limited in power generated by the available wind speed and Nigeria has a low wind speed regime. Thus, wind energy usage as a form of renewable energy is limited to only 30% of total power generated [26]. Hence an optimization constraint is required as follows:

$$P_W \leq 0.3 P_D. \quad (5)$$

where  $P_W$  is the total power produced by the WECS (MW)  
and  $P_D$  total load demand (MW).

### 3.3 Multi-Objective Formulation

The multi-objective Economic Dispatch optimization problem is therefore formulated as:

$$\text{Minimize } [C, P_L]. \quad (6)$$

subject to:

$$\sum_i p_{gi} = p_D - P_L \text{ (power balance);} \quad (7)$$

$$\underline{p}_{gi} \leq p_{gi} \leq \overline{p}_{gi} \quad i = 1, \dots, n_g \text{ (power limit);} \quad (8)$$

$$P_W \leq 0.3 P_D \text{ (renewable energy limit).} \quad (9)$$

$P_g$  is a vector representing the generator outputs combined and is represented as follows:

$$P_g = [P_{g_1}, P_{g_2}, \dots, P_{g_N}]^T. \quad (10)$$

$P_L$  is the total power loss from all the thermal generating units and is formulated as:

$$P_L(P_g) = P_{g_i}^T [B] P_{g_i} + P_{g_i}^T B_0 + B_{00}. \quad (11)$$

where  $[B]$ ,  $B_0$  and  $B_{00}$  are the B-loss coefficients defined in (2).

## 4.0 Results and Analysis

The optimization software is employed in solving equations (1) through (4) based on the formulations (6) through (11) in determining the quantitative size of power injectable to the NPS network from the WECS to optimize the system performance against losses and power demand on it. The data used as input are as shown in Table 1. The optimal and reduced cost is determined by the software and shown in the second column of Table 2.

The SDP solution was run iteratively within the boundary of  $0$  to  $\pi/2$ . Provided in Table 1 are the existing thermal and hydro generators' cost coefficients. The minimum power at any instant of time is not expected to be lower than 1909.5MW and the overall upper limit is put at 3783MW.

The SDP optimization approach requires that each of the generating units (thermal, hydro and WECS) produces the value of power indicated for each on Table 2. When the values obtained for each of the generating units of thermal and hydro are compared with previous works [17, 18], it was observed that the values were very similar and the losses reduction in the SDP method is substantial. This is as presented in Table 2.

It can be observed on the table that the *SeDuMi*<sup>1</sup> [27] (SDP) solution has a power loss a little above the other two solutions compared. This is because the total power produced by the SDP solution method is also higher than those from the other two solutions. A summation of equivalent units (units 1–7) revealed that *SeDuMi* provided a total power of 3711.2MW while NSGA-II

<sup>1</sup> SeDuMi was developed by Jos Storm (1998-2003) and Imre Pólik (since 2005 after the death of Jos Storm) and is available from <http://sedumi.ie.lehigh.edu>.

**Table 1: Existing Generators' (Thermal and Hydro) Cost Coefficients for NPS [17]**

| Generating unit | $\alpha(x150)$<br>( $\text{N/hr}$ ) | $\beta(x150)$<br>( $\text{N/MWhr}$ ) | $\gamma(1.5x10^{-2})$<br>( $\text{N/MW}^2\text{hr}$ ) | $P_G^{\min}$<br>(MW) | $P_G^{\max}$<br>(MW) |
|-----------------|-------------------------------------|--------------------------------------|---|----------------------|----------------------|
| Egbin           | 12787.0                             | 13.1                                 | 0.031   | 275.00               | 1100.0               |
| Sapele          | 6929.0                              | 7.84                                 | 0.13  | 137.50               | 550.0                |
| Delta           | 525.74                              | -6.13                                | 1.20  | 75.00                | 300.0                |
| Afam            | 1998.0                              | 56.9                                 | 0.092   | 135.00               | 540.0                |
| Shiroro         | 19110.0                             | 0.00                                 | 0.00  | 489.00               | 491.0                |
| Kainji          | 13650.0                             | 0.00                                 | 0.00  | 349.00               | 351.0                |
| Jebba           | 17650.0                             | 0.00                                 | 0.00  | 449.00               | 451.0                |
| <b>Total</b>    |                                     |                                      |   | <b>1909.50</b>       | <b>3783.0</b>        |

**Table 2: Computational Results Compared with Previous Solutions**

| Generating Unit  | SeDuMi<br>SDP(MW) | NSGA-II<br>(MW) | $\mu$ GA<br>(MW) | $P_G^{\min}$ (MW) | $P_G^{\max}$ (MW) |
|------------------|-------------------|-----------------|------------------|-------------------|-------------------|
| Egbin            | 870.5316          | 1024.68         | 1011.39          | 275.00            | 1100.0            |
| Sapele           | 406.9654          | 284.70          | 173.28           | 137.50            | 550.0             |
| Delta            | 192.9820          | 75.00           | 111.18           | 75.00             | 300.0             |
| Afam             | 398.0490          | 177.05          | 261.43           | 135.00            | 540.0             |
| Shiroro          | 489.9436          | 490.00          | 490.00           | 489.00            | 491.0             |
| Kainji           | 349.9447          | 350.00          | 350.00           | 349.00            | 351.0             |
| Jebba            | 449.9426          | 450.00          | 450.00           | 449.00            | 451.0             |
| WECS 1           | 276.4006          | –               | –                | –                 | –                 |
| WECS 2           | 276.4006          | –               | –                | –                 | –                 |
| <b>Fuel Cost</b> | <b>31.88</b>      | <b>28.32</b>    | <b>24.55</b>     |                   |                   |

and  $\mu$ GA each gave only a total of 2851.43MW and 2847.28MW respectively. The total optimal power generated by the SDP method is sufficient to supply the peak power demand of 3419 MW and a loss of 184.31MW with a surplus of 107.89MW. When the existing generating units produce the optimal power indicated and the two WECS unit are added with each producing up to 276.4MW the objectives of the optimization shall be met.

## 5.0 Conclusions

A practical optimal solution has been provided for the NPS network. For the optimal solution to be adopted, most of the

existing power stations have to up their power production. This is advantageous since the present lower production has resulted in high losses on the system. Higher production can prove to be advantageous if the optimal solution is employed since the losses shall be reduced.

Further, the incoming two WECS have to very buoyant. The optimal power expected from each is 276.4MW. This is not small a power to come by in WECS considering the generally low speed regime in the country. Therefore, a very large wind farm shall be required at each location.

For the NPS to maintain the minimal loss of 276.4MW, the two incoming WECS must be connected at points where little or no loss is introduced to the existing grid. For the two WECS to be connected, it is recommended that they are installed one at the north-western axis (Sokoto or Katsina state) and the other one at Jos to maintain optimal results. By this way the existing buses at Birnin Kebbi and Jos shall be utilized for the proposed Sokoto/Katsina and Jos WECS respectively with little or no extension made to the busses. This shall improve power production from the existing system of about 3500MW in Nigeria to about 4000MW, out of which 552.8MW was to be produced from optimally-placed two WECS.

## References

1. **A.S. Sambo**, *Solar Energy and Other Renewable Energy Options for Sustainable Development*. Kaduna, Nigeria.: Paper presented at the (NBTE) photovoltaic training 2010 organized by the National Board for Technical Education , (2010).
2. **J. K. Okoye**, *Background study on water and energy issues in Nigeria to inform the national consultative conference on dams and development* (2007).
3. **Z.A. Adetona**, *Tackling the Challenges of Power Generation in Nigeria through Empirical Data Analysis*. Proceedings of the Faculty of Technology, Obafemi Awolowo University Ile-Ife, Conference, (2011) pp. 310-316.
4. **O.I. Okoro, E. Chikuni, and P. Govender**, *Prospects of Wind Energy in Nigeria*. Cape Town: Proc. 15th International Conference on the Domestic Use of Energy, (2007) pp. 107-111.
5. **M. S. Adaramola, and O. M. Oyewola**, *Wind Speed Distribution and Characteristics in Nigeria*. ARPN Journal of Engineering and Applied Sciences, (2011).
6. **O.S. Onohaebi, and P.A. Kuale**, *Estimation of Technical Losses in Nigerian 330kV*. Int. J. Elect. Power Eng., (2007) pp. 421-431. Vol. 1(4).
7. **L.O. Adekoya, and A.A. Adewale**, *Wind energy potential in Nigeria*. Renewable Energy, (1992) pp. 35-39.
8. **Mathew, S.** *Wind Energy Fundamentals, Resource Analysis, and Economics*. Springer, 2006.
9. **Sambo, A.S.** *Strategic Developments in Renewable Energy in Nigeria*. International Association for Energy Economics, (2009).
10. **J. S. Dhillon, S. C. Parti, and D. P. Kothari**, Stochastic Economic Emission Load Dispatch, *Electric Power Syst. Res.*, vol. 26, (1993) pp. 179-186.
11. **C. S. Chang, K. P. Wong, and B. Fan**, Security-Constrained Multi-objective Generation Dispatch using Bi-criterion Global Optimization, Proc. Inst. Elect. Eng. - Gen. Transm. Dist., vol. 142(4), (1995) 406-414.
12. **P. Attaviriyanupap, H. Kita, E. Tanaka and J. Hasegawa**, A hybrid EP and SQP for dynamic economic dispatch with nonsmooth fuel cost function, IEEE Trans. Power Syst., (2002) pp. 411–416.
13. **H. Chowdhury, and S. Rahman**, A review of recent advances in economic dispatch, IEEE Trans. Power Syst, 5 (4) 1990, pp. 1248–1259.

14. **M. Madrigal and V.H. Quintana**, Semidefinite programming relaxations for 0,1–power dispatch problems. In Proc. IEEE–PES 1999 Summer Meeting Conference, Edmonton, Al. Canada, July 18-22, (1999) pages 697–702.
15. **F. E Alizadeh**, Combinatorial optimization with interior point methods and semi-definite matrices. University of Minnesota, 1991.
16. **Y.E. Nesterov, and A.S. Nemirovskii**, Interior Point Polynomial Algorithms in Convex Programming. Philadelphia, PA. : SIAM, 1994).
17. **G. A. Bakare, U.O. Aliyu, and G.K. Venayagamoorthy**. *Genetic Algorithms Based Economic Dispatch with Application to Coordination of Nigerian Thermal Power Plants*. IEEE Power Eng. Society General Meeting, 12-16 June, 2005. pp. 551-556. Vol. 1.
18. **K. O. Alawode, and A. M. Jubril**, *Multi-objective Optimal Power Dispatch for the Nigerian Power Network*. Proceedings of the Faculty of Technology, Obafemi Awolowo University Ile-Ife, Conference (2010).
19. **S. P. Boyd, and L. Vandenberghe**, *Semi-definite Programming*. SIAM REVIEW, March 1996.
20. **C. Helmberg**, *Semidefinite Programming for Combinatorial Optimization*. Berlin, Germany : Konrad-Zuse-Zentrum für Information stechnik Berlin, TakustraBe, 2000. 7, D-14195.
21. **L. Wolsey**, *Integer Programming*. John Wiley & Sons, 1998.
22. **R. Fuentes, and V. Quintana**, *Semidefinite Programming: A Practical Application to Hydro -Thermal Coordination*. Sevilla : 14th PSCC, (2002) pp. 24-28.
23. **R. T. F. Ah King, H. C. S. Rughooputh, and K. Deb**, *Evolutionary multi-objective environmental/economic dispatch: Stochastic versus deterministic approaches*. Guanajuato, Mexico : Proceedings of the Third International Conference on Evolutionary Multi-Criterion Optimization (EMO-2005) (2005) pp. 677-691.
24. **R. Yokoyama, S. H. Bae, T. Morita and H. Sasaki**, , *Multi-objective Optimal Generation Dispatch based on Probability Security Criteria*. IEEE Transactions on Power Systems, (1988) Vol.3, No. 1 pp. 317-324.
25. **H. Sadaat**, *Power System Analysis*. International edition. Boston: WCB/McGraw Hill, 1999.
26. **S. Brini, H.H. Abdallah, and A. Ouali**, *Economic Dispatch for Power System included Wind and Solar Thermal Energy* . Issue 14. Leonardo Journal of Sciences, 2009. pp. 204-220. Vols. January-June.
27. **SeDuMi SDP solver**:  
<http://sedumi.ie.lehigh.edu>