OPTIMIZATION OF FINISHED FEED MIX FOR COMMERCIAL LAYERS AS A PANACEA FOR SCIENTIFIC ADVANCEMENT TOWARDS NATIONAL DEVELOPMENT.

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Abstract
Cost of feeding commercial layers is alarming in the recent decade. As such, this paper develops a minimization Linear Programming Model that will reduce the feeding cost without violating standard nutritional requirement. Five major finished layer mash, constrained with protein, carbohydrate, fat, fibre, calcium and phosphorus were selected from the south west region of Nigeria. Penalty method was employed to solve the model via TORA package and thereafter sensitivity analysis was carried out. The optimum solution was obtained at the 9th iteration with minimum cost of the mash to be #3,126 per bag (25kg) against #3,200 per bag which is currently been used by the farm under study. Therefore, for a better stabilization and profitable poultry industry, government should sensitize farmers about the benefits of Linear Programming model through integrated rural development scheme.
Keywords: *Objective function, Constraints, Nonnegativity condition. Decision variables, Basic variables.*

Introduction

Linear programming (LP) model is a mathematical tool to optimize (minimize/maximize) a linear function of two or more variables subject to some constraints on the variables. LP is a method by which limited resources are allocated, selected or evaluated to achieve an optimal result to particular objective.

Linear programming was first applied to animal compound feed industry in the mid -fifties. Since then, its application in minimizing cost formulation of feed for poultry has gained tremendous growth in most developed countries. However, minimum cost feed formulation for poultry is a recent innovation in developing countries like Nigeria.

Experience based judgement cannot be used to feed commercial layers for consistent results. Since, the body weight has to be regulated to maximize egg production. Therefore, some poultry famers in the south west of Nigeria prefer to go for finished layer mash rather than compounded layer mash. Since formulating layer mash ideally requires deep knowledge of several parameter such as the energy, protein level to be maintained in the diet, balancing the amino acid and electrolyte profile.

Egg producing hens have a great role to play in human diet. Therefore, rearing of commercial layers has become popular industry with tremendous growth to Nigeria GDP and employment opportunities to the youth (Adebayo and Adeola,2005; Okonkwo and Akubo,2001). It is therefore pertinent to protect and sustain the industry in terms of good nutritional requirement at minimum cost without jeopardizing the standard. It has been evaluated that feeding the commercial layers constitutes over 70% of total cost of egg production (Afolayan, 2008). With respect to this, this paper makes an attempt to solve the problem of high cost of feeding by applying linear programming techniques to get the minimum feed mix between five major finished layer mash.Application of this method is hoped to encourage both commercial and subsistence farmers to adopt LP in the development of low-cost feeds for their birds.

Modern poultry feed formulations are formulated under nutrient specification, which changes as more advances are made in poultry. Nutrient specification may include minimum or maximum level of nutrients. The ingredients in the formulation of poultry feed differ in content and importance in poultry diet. These ingredients both locally and internationally sourced fall within the classification based on the nutrients of which they have imperative content. Some researchers have highlighted the nutrients which are very crucial in formulation of poultry feed such as metabolizable energy, crude protein, crude fibre, fats and oil, vitamins, amino-acids and minerals (Kekeocha, 1984; Parr, 1988; Pond et al, 2005; and Godfreyetal, 2016).

In animal diet,Waugh in 1951 was the first researcher who attempted to solve the feed mix problem using mathematical programming. It happened when he figured out that linear programming (LP) method was best suited for solving animal diet problem. In his attempt, he honestly admitted that he was not an animal nutritionist, but

trying to look for the suitable method to lighten the animal nutritionist job. Since then, LP is used widely in modelling the animal feed problem (Alexander,2006; Htun,2005; Cander,1960; Chakeredza,2008; Engelbrecht,2008; Sirisatien,2009; Thomson,2001).The common objective in formulating the feed mix is to minimize cost while providing adequate nutrients to meet the needs of the animals.
In consequent, various type of methodologies have been proposed by researchers in this field.

Vast literature of different approaches exists to minimize the cost of feed formulation. Researchers have engaged several mathematical techniques to provide solution to poultry feed formulation such astrial and error,Pearson's square method, goal programming, multi objectives goal programming, quadratic programming, nonlinear programming, pure integer programming, mixed integer programing and linear programming.
Wagner and Stanton (2014), used the Pearson's square method to balance animal rations. They found the nutritional requirement of an animal for a specific nutrient using Pearson's square. The Short coming of this method is that it cannot handle a situation where ingredients are many. A weighted goal programing DASH(Dietary Approaches to Stop Hypertention) diet model that minimizes the daily cost of the DASH eating plan as well as deviations of the diet nutrients content from the DASH diets tolerable to intake levels was presented by (Iwuji and Agwu, 2017).
In the thesis developed by Efeduma (2016). He determined the optimalfeed mix of broiler starter and finisher at least cost using linear programing technique. He used 10 feed ingredients and solved using an excel solver application to obtain optimal feed mix.

Oladokun and Johnson(2012), developed an optimization feed formulation model, using locally available feed ingredients, for the Nigeria poultry industry. They also carried out the sensitivity analysis to take a position on their model and the existing method on the farm under study.

Olorunfemi(2007), in his classical use of linear programming approach to least cost ration formulation for poultry, used a computer-based technique to investigate, analyze and indicate how best the available local ingredient can be combined effectively to formulate least cost ration for poultry. He concluded that utilization of diet containing fillers at 7.94% is cost effective and reduce cost of feeding by as much as 24.95%.
Samuel et al(2015). These authors employed linear programing to propose optimal formulation of the LP model which gives about 7.48% and 9.96% reduction in feed formation costs compared to the existing formulation in case of broiler starter and finisher. The proposed model also reduced the amount of fat contained in the existing ration but adding more flesh by significantly increasing the metabolizable energy needed for physiological structure of the birds.

Piyaratne, et al (2012). In their study, they focused on the development of linear model-based software with inclusion of digestible amino acid for least cost poultry ration formulation. Their model yielded ration with equalizing major nutrients requirement at the average inclusion level of commercial lysine 0.05% and methionine 0.02%. They realized that equal nutrient requirement gave up to 12 major nutrients with ideal amino acid profiles.
Godfrey, et al (2016), employed mixed integer programming to poultry feed ration optimization using the bat algorithm. In their research, they used findings of previous research to investigate the effects of moringa oleifera inclusion in poultry feed ration using the bat algorithm to obtain the optimum solution.
Nabasirye, et al (2011), demonstrated how to formulate a least cost diet using linear programming and discussed extensivelythe importance of proper interpretation of the sensitivity report on micro Excel solveroutput format.
Zhang and Roush (2002), applied a multiple objective programming (MOP) to the feed formulation process with the objectives of minimizing nutrient variance and minimizing ration cost, where 21 ingredients with 17 nutrients were included in the formulation. They concluded that MOP model is more flexible in providing appreciable solution than a traditional feed formulation.

Aim
To obtain an optimum layer feed mix with the sensitivity analysis on the five different finished layer mash.

Objectives

1. To develop a linear programming model that will satisfy necessary nutrient requirement of laying birds.
2. To determine the quantity of each of the finished feed in one bag of 25kg.
3. To Justify the proposed model with the existing formulation of the experimental farm.
4. To determine allowable variation in quantity of each layer mash.

Methodology

If the feasible region is a subset of the non-negative portion of Rn, define by linear equation and inequalities, and the objective function to be minimized or maximized is linear, then we have a linear programming problem. Therefore, a linear programming is the problem of maximizing or minimizing a linear function subject to a limited number of linear constraints. Such that:

$$Min/Max \sum_{j=1}^{n} c_j x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq (\leq)(=) b_i$$

$$L_k \leq x_j \leq U_K$$

With $x_j \geq 0$ for i = 1,2 …………m

j = 1,2 …………..n

Where x_j are the decision variables

c_j is the co-efficient of the objective function

a_{ij} is the co-efficient of the constraints.

bi is the boundary associated with the constraints.

L_k is the lower bound of x_j

U_k is the upper bound of x_j

Mathematical Formulation of the Model

With the seven constraints imposed on five set of finished layer mashwith respect to the nutrient requirement to be met, the following assumptions are made

1. Linearity: There exist a linear relationship between the output and the total quantity of each resource consumed.
2. Simple objective: The objective is a minimization type of one activity.
3. Certainty: All values and quantities are known with certainty.
4. Additivity: This means that the sum of resources used by different activities are equal to the total quantity of the resources used by each activity for all the resources.
5. Divisibility: Perfect divisibility of outputs and resources exist. (infinite divisibility).
6. Non-negativity: The decision variables are assumed to be either positive or zero.
7. Finiteness: The constraints and the variables are finite so that it can be programmed. Hence, a finite number of activities and constraints are employed.
8. Proportionality: The contribution of each variable to the final objective function is directly proportional to each variable.

Mathematically, in algebraic terms our model has the form below:

$$Minimize\ (Z) = \sum_{j=1}^{n} c_j x_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(i)$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq (\leq, =) b_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(ii)$$

$$\sum_{j=1}^{n} x_j = q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(iii)$$

$$l_i q \leq \sum_{j=1}^{n} a_{ij} x_j \leq u_i q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(iv)$$

$$x_j \geq 0 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(v)$$

Where Z is the total cost of the feed per bag.

C_j is the cost of finished feed j.

x_j is the quantity of finished feed j.

a_{ij} is the quantity of nutrient *i*in finished feed j.

bi is the required amount of nutrient *i*in the feed.

q is the weight value of the feed.

l_i is the lower limits of finished feed j.in a bag of feed.

u_i is the upper limits of finished feed j in a bag of feed.

The inclusion of (iv) is necessary because we need to set an upper limit due to undesirable characteristics or simply to avoid unevenness of nutrients present in each feed especially if this mash costs less than other mash and lower limits due to some desirable characteristics.

The model is developed to consider five major finished layer mash namely; Topfeed, Animal care, Chikun, Hybrid and Corner stone layer mash. These feeds are constrained with some selected nutrients requirement viz; crude protein, metabolizable energy, ether extract(oil), crude fibre, calcium and available phosphorus. Methionine, lysine,etc. are not considered because they are into regulatory use.

Decision variables:

Let x_1 be one unit(kg) of the quantity of Animal care=A

Let x_2 be one unit(kg) of the quantity of Topfeed =B

Let x_3 be one unit(kg) of the quantity of Chikun =C

Let x_4 be one unit(kg) of the quantity of Hybrid =D

Let x_5 be one unit(kg) of the quantity of Cornerstone =E

The case study farm "Morac farms Ltd", Ogun- state, Nigeria, has an existing finished layer feed that cost #3200 per bag. The table below summarizes the cost of each Mash and nutritional constituent.

Table 1: Nutrient composition of the feeds.

Feed Type	Price in #(kg)	Crude Protein %	Energy ME(kcal)	Ether (Oil)%	Crude fibre %	Calcium %	Available Phosphorous %
X_1	124	16.5	2500	5	6	3.5	0.41
X_2	130	18	2500	5	6	3.6	0.45
X_3	127	16.5	2650	4.4	5	3.75	0.45
X_4	126	17	2700	4.5	5.5	3.5	0.4
X_5	128	16	2600	4.0	5	4	0.37

The information in table 1 together with nutrient requirement can thus be expressed in LP form as given below:

$\text{Min}(z)= 124x_1 + 128x_2 + 127x_3 +126x_4 + 122x_5.$

subject to

$x_1 + x_2 + x_3 + x_4 + x_5 = 25$ (demand requirement)

$16.5 x_1 +18 x_2 +16.5 x_3 +17 x_4 +16 x_5 \geq 18.5$ (crude protein)

$2500x_1 +2500 x_2 +2650 x_3 +2700 x_4 +2600 x_5 \geq 2750$ (metabolizable energy)

$5.0x_1 +5.0x_2 +4.4x_3 + 4.5x_4 + 4.0x_5 \geq 6$ (Ether extract)

$6x_1 +6 x_2 +5.5 x_3 +5x_4 + 4x_5 \leq 7$ (Crude fibre)

$3.5x_1 + 3.6x_2 + 3.75x_3 + 3.5x_4 + 4x_5 \geq 4.4$ (Calcium)

$0.41x_1 + 0.45x_2 + 0.45x_3 + 0.4x_4 + 0.37x_5 \geq 0.6$(Available Phosphorus)

With $x_1, x_2, x_3, x_4, x_5 \geq 0$

The following upper and lower limit are also imposed on the feeds

x_1 is between 4-6kg

x_2 is between 4-6kg

x_3 is between 4- 6kg

x_4 is between 4-6kg

x_5 is between 4-6kg

Analysis and Discussion

Infeasibility occurred during the analysis, this makes the model to be extended to elastic programming or what we refer to as elastic filter. Subsequently the proposed model was solved by TORA package, the sensitivity analysis was carried out and constrained that brought about infeasibility identified.

Summary of output

Table 2: Objective value =#3,126

Variable	Value	Objective Coefficient	Objective Value Contribution
X_1	6.00	124.00	744.00
X_2	4.00	128.00	512.00
X_3	4.00	127.00	508.00
X_4	5.00	126.00	630.00
X_5	6.00	122.00	732.00

Table 3: Limitations

Constraint	RHS	Slack(-)/Surplus(+)
1	25	0.00
2	18.5	399.50(-)
3	2750	61950.00(-)
4	6.0	108.10(-)
5	7.0	124.00(-)
6	4.4	87.50(-)
7	0.60	9.68(-)
LB; X_1	4.0	2.00(+)
UB;X_1	6.0	0.00
LB; X_2	4.0	0.00
UB;X_2	6.0	2.00(-)
LB; X_3	4.0	0.00
UB;X_3	6.0	2.00(-)
LB; X_4	4.0	1.00(+)
UB;X_4	6.0	1.00(-)
LB; X_5	4.0	2.00
UB;X_5	6.0	0.00

Table 4: Sensitivity Analysis

Variable	Current Obj coefficient	Min. Obj coefficient	Max. Obj coefficient	Reduced Cost
X_1	124.00	$-\infty$	126.00	-2.00
X_2	128	126	∞	-2.00
X_3	127	126	∞	-1.00
X_4	126.00	124.00	127	0.00
X_5	122	$-\infty$	126	-4.00

Table 5: Limitation

Constraint	Current RHS	Min. RHS	Max. RHS	Dual Price
1	25	24	26	126
2	18.5	$-\infty$	418	0.00
3	2750	$-\infty$	64700	0.00
4	6.00	$-\infty$	114.10	0.00
5	7.00	$-\infty$	131	0.00
6	4.40	$-\infty$	91.90	0.00
7	0.60	$-\infty$	10.28	0.00
LB; x_1	4.0	0.00	6.00	0.00
UB;x_1	6.0	5.00	7.00	-2.00
LB; X_2	4.0	3.00	5.00	2.00
UB;X_2	6.0	4.00	∞	0.00
LB; X_3	4.0	3.00	5.00	1.00
UB;X_3	6.0	4.00	∞	0.00

LB; X_4	4.0	0.00	5.00	0.00
UB;X_4	6.0	5.00	∞	0.00
LB; X_5	4.0	0.00	6.00	0.00
UB;X_5	6.0	5.00	7.00	-4.00

From the analysis above, optimal solution is obtained as #3,126.00 with corresponding value of our decision variables to be $X_1 = 6kg. X_2 = 4kg. X_3 = 4kg. X_4 = 5kg. X_5 = 6kg.$ These variables have different contributions to the objective function i.e #744, #512, #508, #630 and #732 respectively. This implies that X_1 $has\ 24\%\ of\ the\ total\ cost\ of\ the\ feed\ , other\ types\ of\ feed\ follow\ the\ same\ suit\ with\ X_2 = 17\%. X_3 = 16\%. X_4 = 20\%. X_5 = 23\%$

This model is flexible to the extent that the input data of the model can change within certain limits without causing the optimum solution to change. Sensitivity analysis reveals that the optimum solution remains unchanged within the range of objective coefficient in table 4 which implies that the optimum solution obtained is robust. The range of nutritional requirement that would permit a feasible solution is shown on table 5. The Dual prices which are zero have no economic importance on the feasibility range.

Conclusion

Controlling feed cost is germane to the sustainability of Nigeria poultry industry. This paper develops a LP model as an effective tool to determine balanced feed mix at a minimum cost and effective use of limited resources. The proposed model progresses to elastic programming which is the recent area of research in LP. Due to the flexibilityof the model proposed, it can be extended to model of more than five decision variables and seven constraints.

The sensitivity analysis of this model is capable of handling problematic condition such as unstable market prices and nutrient variations which is prevalent in Nigeria. Therefore,the model also proposed the inclusion of 6kg of Animal care, 4kg of Top, 4kg of Chikun, 5kg of Hybrid and 6kg of Cornerstone in a bag of mash at a total cost of #3126 against #3200 which is the existing cost per bag of the local farm under study. Therefore, the farmers are advised to embrace LP as veritable tool to solve their feeding problems.

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