A COMPARISON OF TWO CLASSES OF CONSTRAINED OPTIMIZATION MODELS IN OPTIMIZING THE COST OF HUMAN CAPACITY BUILDING: A FOCUS ON TERTIARY INSTITUTION

BY

AKANBI, Olumuyiwa.O

Department of Mathematics & Statistics,
Federal Polytechnic, Ilaro,
Ogun State, Nigeria.

ABSTRACT

Training and re-training of academic staff and non-academic staff in Nigerian tertiary institution cannot be neglected in other to achieve the millennium goal. However, it is imperative that staff are trained at a minimum cost. The work exploits two mathematical programming models namely; Linear programming (LP) and integer Linear Programming Model (ILP) to minimize the cost of training both academic and non-academic staff in junior and senior categories. The study makes a comparison between the two methods. The result reflects that, some constraints are violated when optimal results of LP models are rounded off to nearest integer. However, all the constraints are satisfied at optimal level when ILP is applied. Therefore, the results obtained indicate that the ILP model is the best approach to the problem of personnel management in terms of efficiency and accuracy.

Keywords: *Linear Programming (LP), Integer Linear Programming (ILP), Branch and Bound method (B&B), Dual Simplex Method and Operations Research (OR).*

INTRODUCTION

One of the areas which has attracted much attention in the literature about the public sector efficiency during the last few decades is the educational sector. As such, all hands must be on desk to protect this sector during this period of economy recession where prudency is the only way out.

Most tertiary institutions in Nigeria make use of intuition or trial and error method to select number and the category of their staff to be sent for training (academic/professional) at a minimum cost within a specified period of time.

As such, these institutions will find it difficult to allocate limited resources within their capacity to ensure minimum cost on training programs.

Personnel management is the ability to manage problems relating to recruitment, selection, training and development of man power to different areas. Its Optimization is an area of wide research. Highly complex problem can be modeled and solved to optimality or near optimality using LP or ILP. This paper hopes to give the reader an idea about real world scenario that is modeled to differentiate between LP and ILP. The two classes of constrained optimization model considered in this work share the same general structure of optimization with restriction. Linear programming is the simplest of all and is still the most widely used type of constrained optimization model. Integer linear programming requires one or more of the variables to take on positive integer values and are harder to solve.

Any organization that wants to survive, must embrace operation research as one of the weapons in finding solutions to complex managerial decision. The theoretical techniques use verbal expression while quantitative methods make more use of mathematical symbols in arriving at solutions to model and analyze this decision taking problems. Among these quantitative technique are LP and ILP models.

Some researchers make it pertinent that the use of scientific methods, particularly LP and ILP in the allocation of scare resources is of vital importance for any enviable growth of the organization see Taha (2008), Johannes (2016), Richard (1991), Biniyam and Tizazu (2013), and the references therein.

Human resources of any organization are the largest factor of production. Any citadel of learning without creative mind and innovative personnel will bring zero growth to the nation. Therefore capacity building should be given adequate attention to achieve the millennium goal.

Linear programming can be viewed as part of a great revolutionary development which has given mankind the ability to state general goals and to lay out a path of detailed decisions to make in order to "best" achieve its goals when faced with practical situations of great complexity (Dantzig 2002).

Linear programming and its many extension have come into wide use. In academic circles, industries, military, business and others.

For convenience we define a pure integer problem as linear programs in which all the variables are integer. Otherwise the problem is a mixed integer problem.

More so, if all the variables in the optimal solution are allowed to take 0 or 1, such is referred to as 0-1 or standard discrete programming problem (Kalavathy, 2002). Meanwhile, our approach in this study is a pure integer linear programming techniques.

The significances of ILP are numerous, several occurring situations in business and industry that extend to planning models involve integer valued variables. In manufacturing, production is frequently scheduled in terms of batches, lots or runs. In allocation of goods, shipment must involve discrete number of trucks and in particular personnel management where numbers of staff should strictly assume positive integers.

Aim and Objectives.

The aim of this study is to make a position on the best constrained optimization model which stands as a veritable tool in dealing with personnel management problems.

Objectives of the study

The main objectives of the study are:

i. To develop a realistic model in minimizing cost of training the staff in tertiary institution.
ii. To consider the models as LP model
iii. To consider the models as ILP model.
iv. To compare the efficacy of ILP and LP in personnel management optimization.

LITERATURE REVIEW

Application of LP began in 1947, (in connection with the planning activities of the military) by George B. Dantzig, shortly after world war II and has been keeping the pace ever since with the extraordinary growth of computing power. Dantzig was fascinated by the work of Wassily Leontief who proposed in 1932 a large but simple matrix structure which he called the interindustry input-output model of the American Economy (Dantzig, 2002).

Linear programming technique is a very resourceful method in various fields. In the study by Snezana and Milorad (2009), they present a method for modeling and optimizing an industrial steam condensing system by linear programming techniques. LP is used to minimizing the total cost for energy net costs in steaming condensing systems.

Waheed (2012), demonstrates the use of linear programming methods as applicable in the manufacturing industry where KASMO industry limited, Osogbo, Nigeria was taken as a case study.

Kourosh, et al (2013), solve transportation problems using linear programming in Services Company. The paper reveals that an evaluation of 500 largest companies in the world showed that 85% of them have used linear programming.

Akinyele (2007), applies LP model based on integer programming to the determination of effective size of manpower to be engaged. His study also incorporate global constraints such as production capacity/demand rate and allowable time of operation into the model to reflect the reality activities in production organization in developing countries.

In the study by Agarana, Anake and Adeleke (2014), LP was applied to the management of loan portfolio of banks, where an answer is provided to the question of how to avoid possible occurrence of non-performing loans, bad and doubtful debts in banks.

In 2013, Mina, et al exploit LP to establish the optimal combination of production and the optimal allocation of human resources in a beverage company.

Integer linear programming began in 1958 by Gomory, unlike the earlier work on the travelling salesman problem (TSP) by Fulkerson (1954).

Land and Doig in 1960, introduced another method called Branch & Bound (B&B) which has turned out to be one of the most successful ways to solve practical ILP (Kurtz, 1992).Fulkerson and Johnson, (1954), on travelling salesman problem (TSP) provided a remarkable sources of idea for solving by hand combinational optimization problems including cutting, B&B and Lagrangian duality. Dantzing, Fulkerson and Johnson pioneered the idea of employing LP relaxation and valid inequalities to solve integer programs by solving (including a proof of optimality) a 49- city TSP in USA (Richard 1991). It is amazing that the three authors were able to find an optimal solution of such large TSP at instant and prove its optimality by manual computation. Biniyam and Tizazu (2013), worked on personnel scheduling using ILP model in which Avantis Blue-Nile Hotels, in Ethiopia serves as a case study. They used ILP to determine an optimal weekly shift schedule for the Hotel's engineering department personnel.

John, Ganesh, and Narayanan (2010), in their paper proposed a vendor selection model using ILP model for multi-product, multi-vendor environment. As such, their model is validated with a case study by implementing the model for Agricultural equipment whole sale company.

Christodoulos and Xiaoxia, (2005), reviewed the advances of mixed-integer linear programming (MILP) for the scheduling of chemical processing systems.

METHODOLOGY

A linear programming is the problem of maximizing (or minimizing) a linear function subject to a finite number of linear constraints.

$$\text{Min} \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to} \quad \sum_{j=1}^{n} c_j x_j \geq b_i (i = 1,2, \dots m)$$

$$\text{with} \quad x_j \geq 0 \text{ (j = 1,2, … n).}$$

if $x_j \geq 0$ *and* x_j *are integers then LP model becomes an ILP model.*

Mathematical formulation of the model

(i) Decision variables:
Let x_1 represent an individual in the senior category
Let x_2 represent an individual in the junior category

(ii) Objective function:
Minimize (z) = $c_1 x_1 + c_2 x_2$
Where c_1 *and* c_2 are the cost coefficients of training senior and junior staff respectively. c_1 *and* c_2 are taken as unity for simplicity and flexibility of the model.

(iii) Constraints:
The constraints for this work are basically the least time available for both academic and professional training. The available time for academic staff is 36 months (equivalent to minimum time for Ph.D degree) while non-academic is 18 months (equivalent to minimium time for professional & academics masters)
Therefore the general model governing the work is given as:

$$min \sum_{j=1}^{2} c_j x_j$$

Subject to:

$$\sum_{j=1}^{2} a_j x_j \geq bi \text{ (i = 1,2, … n)}$$

$$\text{With } x_j \geq 0 \text{ , } in\ case\ of\ ILP\ x_j \in \mathbb{Z}^+$$

(iv) Non negativity condition:

Case I

$x_1, x_2 \geq 0$

Case II

$x_1, x_2 \in \mathbb{Z}^+$

Case II renders the system as pure ILP models

The models

The models are developed according to Academic and Non-academic units.

Academic units are divided into four schools namely: School of Applied Science, School of Management Science, School of Engineering and School of Environmental Studies. Therefore, the work focuses on formulation, analysis and interpretation of five models as LP and ILP.

Let: Non-academic model = model I
School of Applied Science = model II
School of Management Studies = model III
School of Engineering = model IV
School of Environmental Studies = model V

Model Assumptions

i. The coefficient of the objective function is assumed to be one million naira.
ii. The time constraints for non-academic staff is assumed to be at least 18 months.
iii. The time constraints for academic model is assumed to be at least 36 months.
iv. The constraints & objective function are linear.

Valuation of the model

The model was validated with a polytechnic academic structure in Nigeria, where the non-academic staff are sub-divided into Rectory, Bursary, Library, Registry, Works & Services and Medical. We collected the data from Personnel Establishment department of a federal polytechnic to validate the models. Due to the agreement of confidentiality the name of the Polytechnic is withheld.

Academic staff are divided into various schools and departments.

Non Academic Units Model:

LP		ILP
Min (z) = $x_1 + x_2$		Min (z) = $x_1 + x_2$
Subject to:		
$27x_1 + 37x_2 \geq 18$	(Rectory)	$27x_1 + 37x_2 \geq 18$
$36x_1 + 10x_2 \geq 18$	(Bursary)	$36x_1 + 10x_2 \geq 18$
$20x_1 + 22x_2 \geq 18$	(Library)	$20x_1 + 22x_2 \geq 18$
$80x_1 + 23x_2 \geq 18$	(Registry)	$80x_1 + 23x_2 \geq 18$
$55x_1 + 50x_2 \geq 18$	(Works & Services)	$55x_1 + 50x_2 \geq 18$

$13x_1 + 15x_2 \geq 18$	(Medical)	$13x_1 + 15x_2 \geq 18$
with $x_{1,}x_2 \geq 0$		with $x_{1,}x_2 \geq 0$ & $x_{1,}x_2 \in \mathbb{Z}^+$

<u>ACADEMICS UNITS MODEL:</u>

School of Applied Science Model

LP		ILP
Min (z) $= x_1 + x_2$		Min (z) $= x_1 + x_2$
Subject to:		
$12x_1 + 14x_2 \geq 36$	(Food Technology Dept.)	$12x_1 + 14x_2 \geq 36$
$20x_1 + 23x_2 \geq 36$	(SLT Dept.)	$20x_1 + 23x_2 \geq 36$
$13x_1 + 15x_2 \geq 36$	(Maths & Stats Dept.)	$13x_1 + 15x_2 \geq 36$
$9x_1 + 8x_2 \geq 36$	(Comp. Science Dept.)	$9x_1 + 8x_2 \geq 36$
$10x_1 + 9x_2 \geq 36$	(OTM Dept.)	$10x_1 + 9x_2 \geq 36$
$9x_1 + 6x_2 \geq 36$	(HMT Dept.)	$9x_1 + 6x_2 \geq 36$
$7x_1 + 7x_2 \geq 36$	(Nutrition & Dietetics Dept.)	$7x_1 + 7x_2 \geq 36$
with $x_{1,}x_2 \geq 0$		with $x_{1,}x_2 \geq 0$ & $x_{1,}x_2 \in \mathbb{Z}^+$

School of Management Studies Model

LP		ILP
Min (z) $= x_1 + x_2$		Min (z) $= x_1 + x_2$
Subject to:		
$7x_1 + \geq 36$	(Bus. Admin. Dept.)	$7x_1 + \geq 36$
$8x_1 + 4x_2 \geq 36$	(Public Admin. Dept.)	$8x_1 + 4x_2 \geq 36$
$3x_1 + 4x_2 \geq 36$	(Insurance Dept.)	$3x_1 + 4x_2 \geq 36$
$14x_1 + 4x_2 \geq 36$	(Accountancy Dept.)	$14x_1 + 4x_2 \geq 36$
$10x_1 + 4x_2 \geq 36$	(Banking & Finance Dept.)	$10x_1 + 4x_2 \geq 36$
$9x_1 + 6x_2 \geq 36$	(GNS Dept.)	$9x_1 + 6x_2 \geq 36$
$7x_1 + 4x_2 \geq 36$	(Marketing Dept.)	$7x_1 + 4x_2 \geq 36$
with $x_{1,}x_2 \geq 0$		with $x_{1,}x_2 \geq 0$ & $x_{1,}x_2 \in \mathbb{Z}^+$

School of Engineering Model

LP		ILP
Min (z) = $x_1 + x_2$		Min (z) = $x_1 + x_2$
Subject to:		
$21x_1 + x_2 \geq 36$	(Electrical/Electronic Dept.)	$21x_1 + x_2 \geq 36$
$22x_1 + 4x_2 \geq 36$	(Mechanical Engineering Dept.)	$22x_1 + 4x_2 \geq 36$
$13x_1 + x_2 \geq 36$	(Civil Engineering Dept.)	$13x_1 + x_2 \geq 36$
$9x_1 \geq 36$	(Computer Engineering Dept.)	$9x_1 \geq 36$
with $x_1, x_2 \geq 0$		with $x_1, x_2 \geq 0$ & $x_{1,}x_2 \in \mathbb{Z}^+$

School of Environmental Studies Model

LP		ILP
Min (z) = $x_1 + x_2$		Min (z) = $x_1 + x_2$
Subject to:		
$9x_1 + 10x_2 \geq 36$	(Surveying & Geoinformatics. Dept.)	$9x_1 + 10x_2 \geq 36$
$8x_1 + 7x_2 \geq 36$	(Architecture Dept.)	$8x_1 + 7x_2 \geq 36$
$6x_1 + 8x_2 \geq 36$	(Quantity Surveying Dept.)	$6x_1 + 8x_2 \geq 36$
$11x_1 + 9x_2 \geq 36$	(Urban & Regional Planning Dept.)	$11x_1 + 9x_2 \geq 36$
$7x_1 + 7x_2 \geq 36$	(Estate Management Dept.)	$7x_1 + 7x_2 \geq 36$
$10x_1 + \geq 36$	(Building Technology Dept.)	$10x_1 + \geq 36$
with $x_{1,}x_2 \geq 0$		with $x_1, x_2 \geq 0$ & $x_{1,}x_2 \in \mathbb{Z}^+$

DATA ANALYSIS

The work exploits two techniques Dual simplex and Branch & Bound techniques to obtain solutions to our LP and ILP models respectively via TORA Mathematical package.

Results: See the appendix for the output of the work sheets

Table 1: Optimal results of different models.

Model	LP Optimum Solution	ILP Optimum Solution

1	Min (z) = 1.23 $x_1 = 0.22$ $x_2 = 1.01$	Min (z) = 2 $x_1 = 1$ $x_2 = 1$
2	Min (z) = 5.14 $x_1 = 1.71$ $x_2 = 3.43$	Min (z) = 6 $x_1 = 2$ $x_2 = 4$
3	Min (z) = 10.29 $x_1 = 5.14$ $x_2 = 5.14$	Min (z) = 11 $x_1 = 6$ $x_2 = 5$
4	Min (z) = 4 $x_1 = 4$ $x_2 = 0$	Min (z) = 4 $x_1 = 4$ $x_2 = 0$
5	Min (z) = 5.40 $x_1 = 3.60$ $x_2 = 1.80$	Min (z) = 6 $x_1 = 4$ $x_2 = 2$

Table 2: Comparison table between approximated optimal result of LP and ILP

MODELS	**LP**					**ILP**		
Model I	Z	x_1		x_2		x_1	x_2	Z
		Actual Value	Rounded Value	Actual Value	Rounded Value	Actual Value	Actual Value	
	1.23	0.22	0	1.01	1	1	1	2
Model II	5.14	1.71	2	3.43	3	2	4	6
Model III	10.29	5.14	5	5.14	5	6	5	11
Model IV	4	4	4	0	0	4	0	4
Model V	5.40	3.60	4	1.80	2	4	2	6

INTERPRETATION OF RESULTS

The various models output are presented in table 1. The approximated optimal value of IL is presented in table 2 to conform to reality (human beings cannot assume a fractional value)

1. Considering the LP models

From model 1, No senior staff is expected to be sent for training to minimize the cost of training at ₦1.23m. From model II, 2 senior staff and 3 junior staff can be sent for training to minimize the cost at ₦5.14m. From model III, equal number of senior and junior staff can be sent for training ($x_1 = x_2 = 5$) at ₦10.29m. From model IV, 4 senior staff can be sent for training and no junior staff should be sent for training to optimize cost of training at ₦4m. From model V, 4 senior staff can be sent for training and 2 from the junior cadre at a cost of ₦5m.

2. Considering the ILP models:

The same number of senior and junior staff ($x_1 = x_2 = 1$) can be sent for training at a minimum cost of ₦2m. From model II, 2 senior staff and 4 junior staff can be sent for training at a cost of N6m. From model III, 6 senior staff and 5 junior staff can be sent for training at an optimal cost of N11m. From model III, 4 senior and no junior staff should be sent for training in order to minimize the cost at N4m. From model IV, 4 senior and 2 junior staff should be sent for training at a minimum cost of N6m

CONCLUSION

The work successfully established the models of optimizing the cost of building human capacity in the citadel of learning. Furthermore, the study obtained the optimal solution to the five models with the help of Dual Simplex Method (LP) and Branch & Bound Technique (ILP).

With critical observation from our findings, in the case of **model I;** when this model is taken as (LP) we obtained the optimum solution to be min (z) = 1.23, x_1= 0.22, x_2 =1.01. Since we are dealing with human beings, the results of our decision variables x_1 = 0.22 & $x_1 = 1.01$ are meaningless (fractional part of staff cannot be obtained as a living being). Hence we are forced to round them off to nearest integer i.e. $x_1 = 0, x_2 = 1$. With these rounded off values, we observed that the rounded values of x_1 *and* x_2 violate all the constraints except the first constraint. This implies that our solution is not a good optimal solution. Taken the model as ILP, we obtained solution given as min (z) = 2, $x_1 = 1, x_2 = 1$, in all ramifications values x_1 and x_2 satisfy all the constraints. Hence one junior and one senior staff should be sent for training within the cost implication.

Similar observation was made in **Model II,** If taken as LP; we have min (z) = 5.14, $x_1 = 1.71$ *and* x_2= 3.43, if these values are rounded to nearest integer i.e. x_1= 2 and x_2 = 3. The seventh constraint is going to be violated (hence not a good solution). But if the model is treated as ILP we shall obtain an optimum solution that satisfies all the constraints i.e. min (z) = 6, x_1 = 2, x_2 = 4.

Also another typical observation was made in **Model III**; If examined as LP, we have min (z) = 10.29, x_1 = 5.14 and x_2 = 5.14, rounding off gives x_1 = 5 and x_2 = 5. These value violate the third constraint. But if the model is viewed as ILP we have optimum solution as min (z) = 10, x_1 = 4, x_2 = 6, which automatically satisfies all the constraints.

Though models IV and V after rounding off to nearest integer do not violate any of the constraints. This is not always the case. In conclusion, it worth noting that the rounded values in **model I**, **Model II** and **Model III** do not give the exact objective when substituted into the objective function. In this research work, the best approach to personnel management model is the ILP model because it alleviates this computational burden dramatically, since B & B method embraces an intelligent search procedure designed to reach optimum integer solution without rounding off the result. Therefore, we recommend that a viable approach to solving personnel management of the type investigated in this work is ILP.

References

Abara, J. (1989). Applying Integer Programming to the fleet Assignment Problem. *Interfaces 19, (4, 20-28*.

Agarana, M.C., Anake, T.A. and Adekele, O.J. (2014). Application of linear programming model to unsecured loans and bad debt risk control in banks. *International journal of management, information technology and engineering (2) issue 7, 93-102.*

Akinyele Taiwo (2007). Determination of the optimal manpower size using linear programming model. *Research journal of Business Management (1) 30-36.*

Benedict, I. & Uzochukwu, A. (2012). Optimizing Profit with the Linear Programming Model: A focus on Golden Plastic Industry Limited, Enugu, Nigeria. *Interdisciplinary Journal of Research sin Business 2, Issue 2, 37-49)*

Biniyam, K. and Tizazu, A. (2013). Personnel Scheduling using an integer programming model an application at Avanti Blue-Nile Hotels. *(Springer plus 2013, 2:333)*

Christodoulos, A. and Xiaoxia, L. (2005). Mixed integer linear programming in process scheduling modeling, algorithms and applications. *Annals of operations research 139, 131-162*

Dantzig, G, (2002). Linear Programming *(INFORMS No 1, 42-47)*

Dantzig, G. Fulkerson, D. and Johnson, S. (1954). Solution of a Large-Scale Traveling-Salesman Problem *(Operations Research 2, 393-410)*.

Everette, A. (1993). Production and Operations Management: Concepts, Models and Behaviour. *PHI Learning, New Delhi***.**

Grosskopf, S., Chung, Y. and Fare, R. (1997). Productivity and undesirable output: a directional distance function approach. *Journal of Environmental Management 51, 229-240.*

Johannes, B. (2016). AIMMS Optimization Modeling *(Bellenue, U.S.A.)*

John Rajan, Ganesh, K. and Narayanan, K. (2010). Supply chain Application of ILP model for vendor selection in a two stage. *Proceedings of the 2010 International Conference on Industrial Engineering and Operations Management Dhaka, Bangladesh, January 9-10.*

Kalavaty, S. (2002). Operation Research 2nd edition Vikas Publishing House PVT Limited.

Kourosh Rajeoyan, Farhang Khalaghdoost Nejati, Reza Hajah (2013). Using Linear Programming in Solving the Problem of Services Company's cost. *Singaporean journal of Business Economics and Management Studies*.

Martin, G. and George, L. (2008). George Dantzig's Contributions to integer programming. *Discrete optimization 5, 168-173.*S

Mina, A., Yeyhameh, B. Maryam, I. and Zeynab, N. (2013). Optimal allocation of Human Resources by Using Linear Programming in the Beverage Company. *Universal Journal of Management and Social Sciences 3, No 5 5-12*

Musah, S. and Abdul-Rahaman (2014). Modeling the problem of profit optimization of Bank & Tamale as linear programming problem. *Scientific and Academic publishing 4(1): 22-40*

Snezana Dragicevic and Milorad Bojic (2009). Application of linear programming in energy management. *Serbian Journal of Management 4(2) 227-238.*

Waheed Babatunde (2012). Profit maximization in a product mix company using linear programming. *European Journal of Business and Management 4(17)*

APPENDIX

List of academic and non-academic staff:

Units	Department	Senior Staff	Junior Staff
Non Academic unit	Rectory Bursary Library Registry Works & Services Medical	27 36 20 80 55 13	37 10 22 23 50 15
Academic unit: School of Applied Science	Food Technology SLT Maths & Statistics Computer Science Office Tech & Mgt. Hospitality Mgt. Nutrition & Dietetic	12 20 13 9 10 9 7	14 23 15 8 9 6 7
Academic unit: School of Mgt. Studies	Business Adm. Public Adm. Insurance Accountancy Banking & Finance General Studies Marketing	7 8 3 14 10 9 7	- 4 4 4 4 6 4
Academic unit School of Engineering	Electrical/Elect.Engineering Mechanical Engineering Civil Engineering Computer Engineering	21 22 13 9	1 4 1 -
Academic unit School of Environmental Studies	Surveying&Geo-Info. Architectural Design URP Estate Magt. Building Tech.	9 8 6 11 7 10	10 7 8 9 7 -

TORA

LINEAR PROGRAMMING

TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Saturday, October 15, 2016 15:03

SIMPLEX TABLEAU - (Dual Simplex Method)

Title: Model IV (Minimize)

Steps for generating NEXT tableau from CURRENT one:

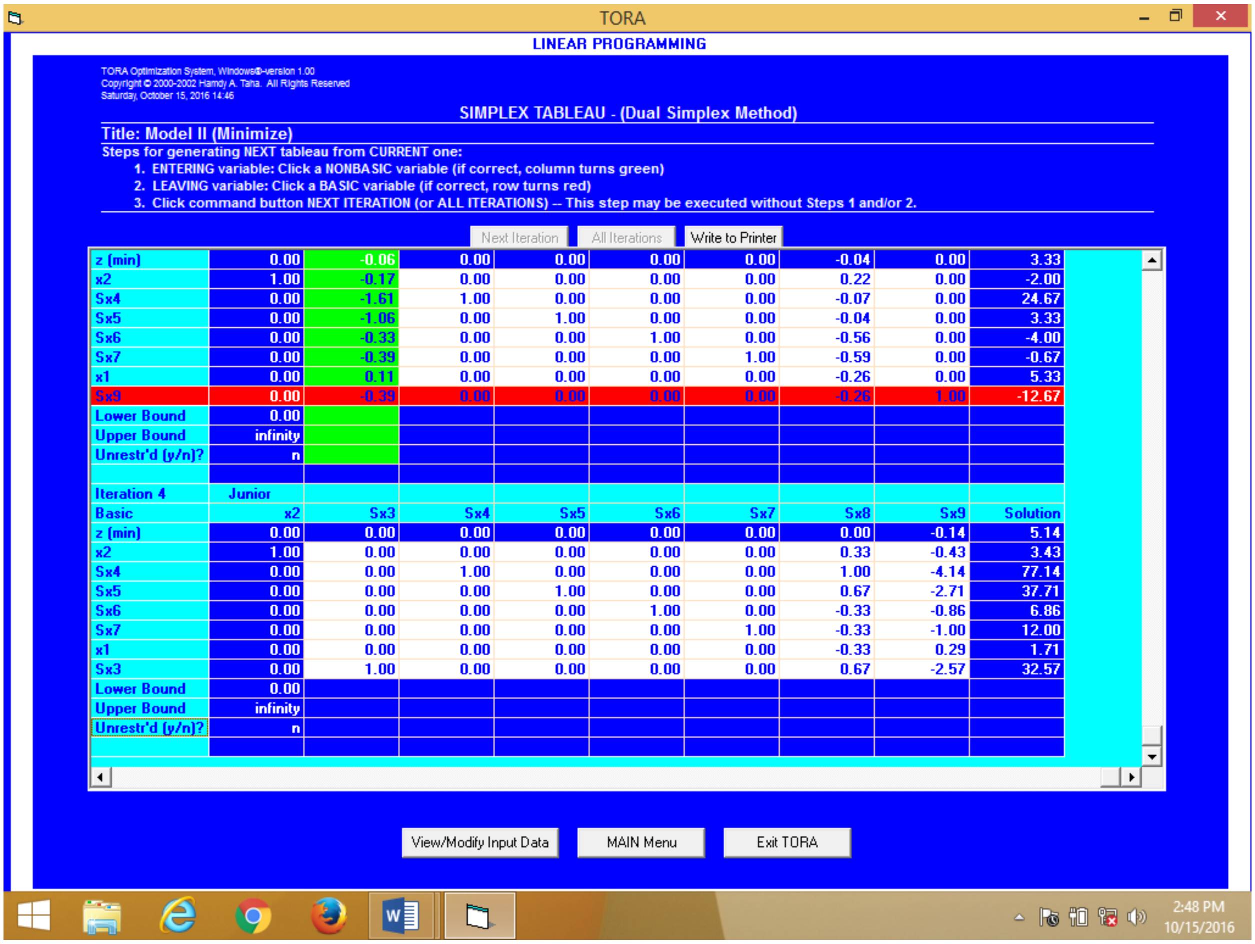
1. ENTERING variable: Click a NONBASIC variable (if correct, column turns green)
2. LEAVING variable: Click a BASIC variable (if correct, row turns red)
3. Click command button NEXT ITERATION (or ALL ITERATIONS) -- This step may be executed without Steps 1 and/or 2.

Next Iteration | All Iterations | Write to Printer

Sx6	-9.00	0.00	0.00	0.00	0.00	1.00	-36.00
Lower Bound	0.00	0.00					
Upper Bound	infinity	infinity					
Unrestr'd (y/n)?	n	n					
Iteration 2	Senior	Junior					
Basic	x1	x2	Sx3	Sx4	Sx5	Sx6	Solution
z (min)	0.00	-0.95	-0.05	0.00	0.00	0.00	1.71
x1	1.00	0.05	-0.05	0.00	0.00	0.00	1.71
Sx4	0.00	-2.95	-1.05	1.00	0.00	0.00	1.71
Sx5	0.00	-0.38	-0.62	0.00	1.00	0.00	-13.71
Sx6	0.00	0.43	-0.43	0.00	0.00	1.00	-20.57
Lower Bound	0.00	0.00					
Upper Bound	infinity	infinity					
Unrestr'd (y/n)?	n	n					
Iteration 3	Senior	Junior					
Basic	x1	x2	Sx3	Sx4	Sx5	Sx6	Solution
z (min)	0.00	-1.00	0.00	0.00	0.00	-0.11	4.00
x1	1.00	0.00	0.00	0.00	0.00	-0.11	4.00
Sx4	0.00	-4.00	0.00	1.00	0.00	-2.44	52.00
Sx5	0.00	-1.00	0.00	0.00	1.00	-1.44	16.00
Sx3	0.00	-1.00	1.00	0.00	0.00	-2.33	48.00
Lower Bound	0.00	0.00					
Upper Bound	infinity	infinity					
Unrestr'd (y/n)?	n	n					

View/Modify Input Data | MAIN Menu | Exit TORA

3:04 PM
10/15/2016



TORA

LINEAR PROGRAMMING

TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Saturday, October 15, 2016 14:46

SIMPLEX TABLEAU - (Dual Simplex Method)

Title: Model II (Minimize)

Steps for generating NEXT tableau from CURRENT one:

1. ENTERING variable: Click a NONBASIC variable (if correct, column turns green)
2. LEAVING variable: Click a BASIC variable (if correct, row turns red)
3. Click command button NEXT ITERATION (or ALL ITERATIONS) -- This step may be executed without Steps 1 and/or 2.

Next Iteration | All Iterations | Write to Printer

z (min)	0.00	-0.06	0.00	0.00	0.00	0.00	-0.04	0.00	3.33
x2	1.00	-0.17	0.00	0.00	0.00	0.00	0.22	0.00	-2.00
Sx4	0.00	-1.61	1.00	0.00	0.00	0.00	-0.07	0.00	24.67
Sx5	0.00	-1.06	0.00	1.00	0.00	0.00	-0.04	0.00	3.33
Sx6	0.00	-0.33	0.00	0.00	1.00	0.00	-0.56	0.00	-4.00
Sx7	0.00	-0.39	0.00	0.00	0.00	1.00	-0.59	0.00	-0.67
x1	0.00	0.11	0.00	0.00	0.00	0.00	-0.26	0.00	5.33
Sx9	0.00	-0.39	0.00	0.00	0.00	0.00	-0.26	1.00	-12.67
Lower Bound	0.00								
Upper Bound	infinity								
Unrestr'd (y/n)?	n								

Iteration 4	Junior								
Basic	x2	Sx3	Sx4	Sx5	Sx6	Sx7	Sx8	Sx9	Solution
z (min)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.14	5.14
x2	1.00	0.00	0.00	0.00	0.00	0.00	0.33	-0.43	3.43
Sx4	0.00	0.00	1.00	0.00	0.00	0.00	1.00	-4.14	77.14
Sx5	0.00	0.00	0.00	1.00	0.00	0.00	0.67	-2.71	37.71
Sx6	0.00	0.00	0.00	0.00	1.00	0.00	-0.33	-0.86	6.86
Sx7	0.00	0.00	0.00	0.00	0.00	1.00	-0.33	-1.00	12.00
x1	0.00	0.00	0.00	0.00	0.00	0.00	-0.33	0.29	1.71
Sx3	0.00	1.00	0.00	0.00	0.00	0.00	0.67	-2.57	32.57
Lower Bound	0.00								
Upper Bound	infinity								
Unrestr'd (y/n)?	n								

View/Modify Input Data | MAIN Menu | Exit TORA

2:48 PM
10/15/2016

TORA

LINEAR PROGRAMMING

TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Saturday, October 15, 2016 14:54

SIMPLEX TABLEAU - (Dual Simplex Method)

Title: Model III (Minimize)

Steps for generating NEXT tableau from CURRENT one:

1. ENTERING variable: Click a NONBASIC variable (if correct, column turns green)
2. LEAVING variable: Click a BASIC variable (if correct, row turns red)
3. Click command button NEXT ITERATION (or ALL ITERATIONS) -- This step may be executed without Steps 1 and/or 2.

Next Iteration | All Iterations | Write to Printer

z (min)	-1.00	-0.14	0.00	0.00	0.00	0.00	0.00	0.00	5.14
x1	0.00	-0.14	0.00	0.00	0.00	0.00	0.00	0.00	5.14
Sx4	-4.00	-1.14	1.00	0.00	0.00	0.00	0.00	0.00	5.14
Sx5	-4.00	-0.43	0.00	1.00	0.00	0.00	0.00	0.00	-20.57
Sx6	-4.00	-2.00	0.00	0.00	1.00	0.00	0.00	0.00	36.00
Sx7	-4.00	-1.43	0.00	0.00	0.00	1.00	0.00	0.00	15.43
Sx8	-6.00	-1.29	0.00	0.00	0.00	0.00	1.00	0.00	10.29
Sx9	-4.00	-1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Lower Bound	0.00								
Upper Bound	infinity								
Unrestr'd (y/n)?	n								

Iteration 3	Junior								
Basic	x2	Sx3	Sx4	Sx5	Sx6	Sx7	Sx8	Sx9	Solution
z (min)	0.00	-0.04	0.00	-0.25	0.00	0.00	0.00	0.00	10.29
x1	0.00	-0.14	0.00	0.00	0.00	0.00	0.00	0.00	5.14
Sx4	0.00	-0.71	1.00	-1.00	0.00	0.00	0.00	0.00	25.71
x2	1.00	0.11	0.00	-0.25	0.00	0.00	0.00	0.00	5.14
Sx6	0.00	-1.57	0.00	-1.00	1.00	0.00	0.00	0.00	56.57
Sx7	0.00	-1.00	0.00	-1.00	0.00	1.00	0.00	0.00	36.00
Sx8	0.00	-0.64	0.00	-1.50	0.00	0.00	1.00	0.00	41.14
Sx9	0.00	-0.57	0.00	-1.00	0.00	0.00	0.00	1.00	20.57
Lower Bound	0.00								
Upper Bound	infinity								
Unrestr'd (y/n)?	n								

View/Modify Input Data | MAIN Menu | Exit TORA

2:55 PM
10/15/2016

TORA

LINEAR PROGRAMMING

TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Saturday, October 15, 2016 15:08

SIMPLEX TABLEAU - (Dual Simplex Method)

Title: Model V (Minimize)

Steps for generating NEXT tableau from CURRENT one:

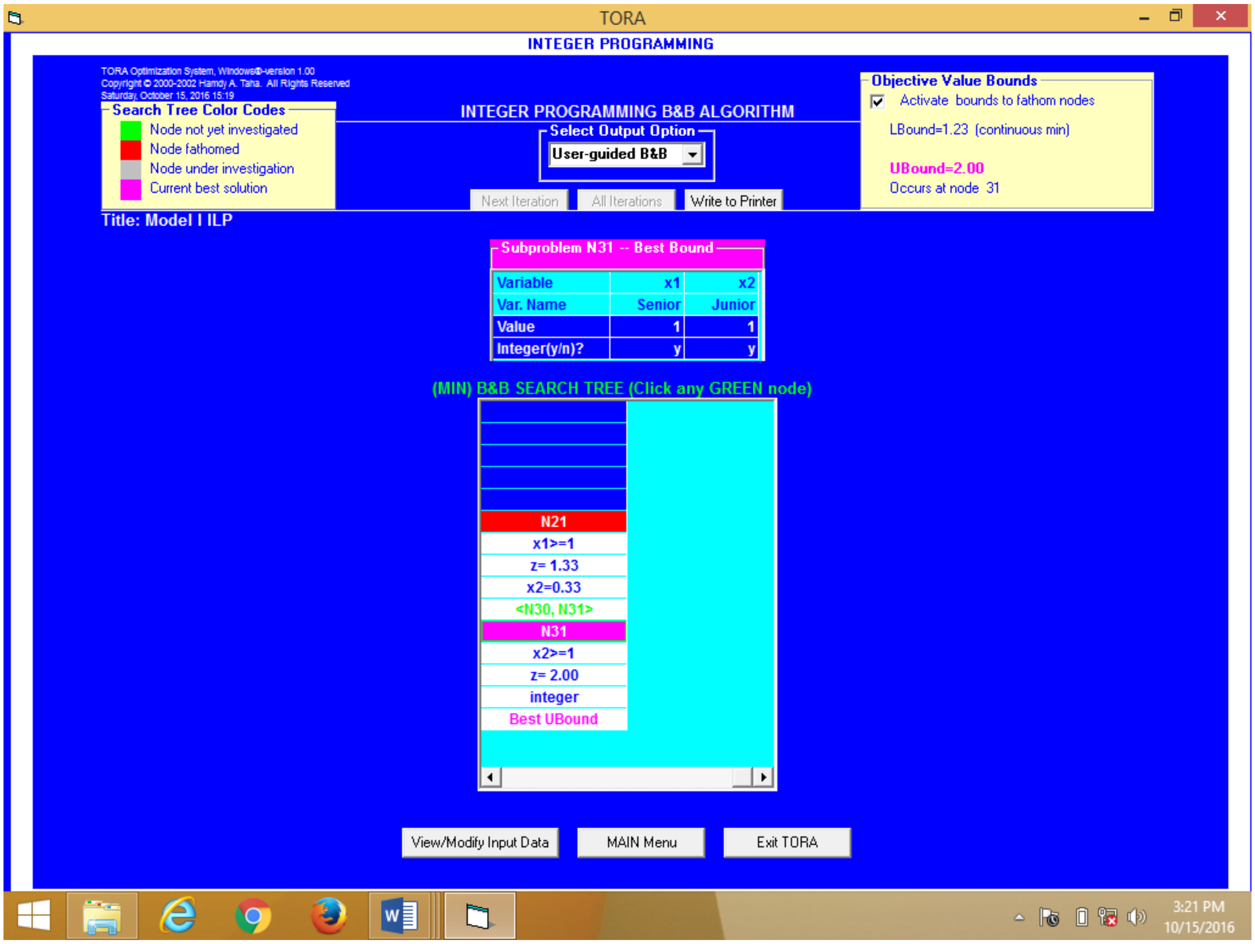
1. ENTERING variable: Click a NONBASIC variable (if correct, column turns green)
2. LEAVING variable: Click a BASIC variable (if correct, row turns red)
3. Click command button NEXT ITERATION (or ALL ITERATIONS) -- This step may be executed without Steps 1 and/or 2.

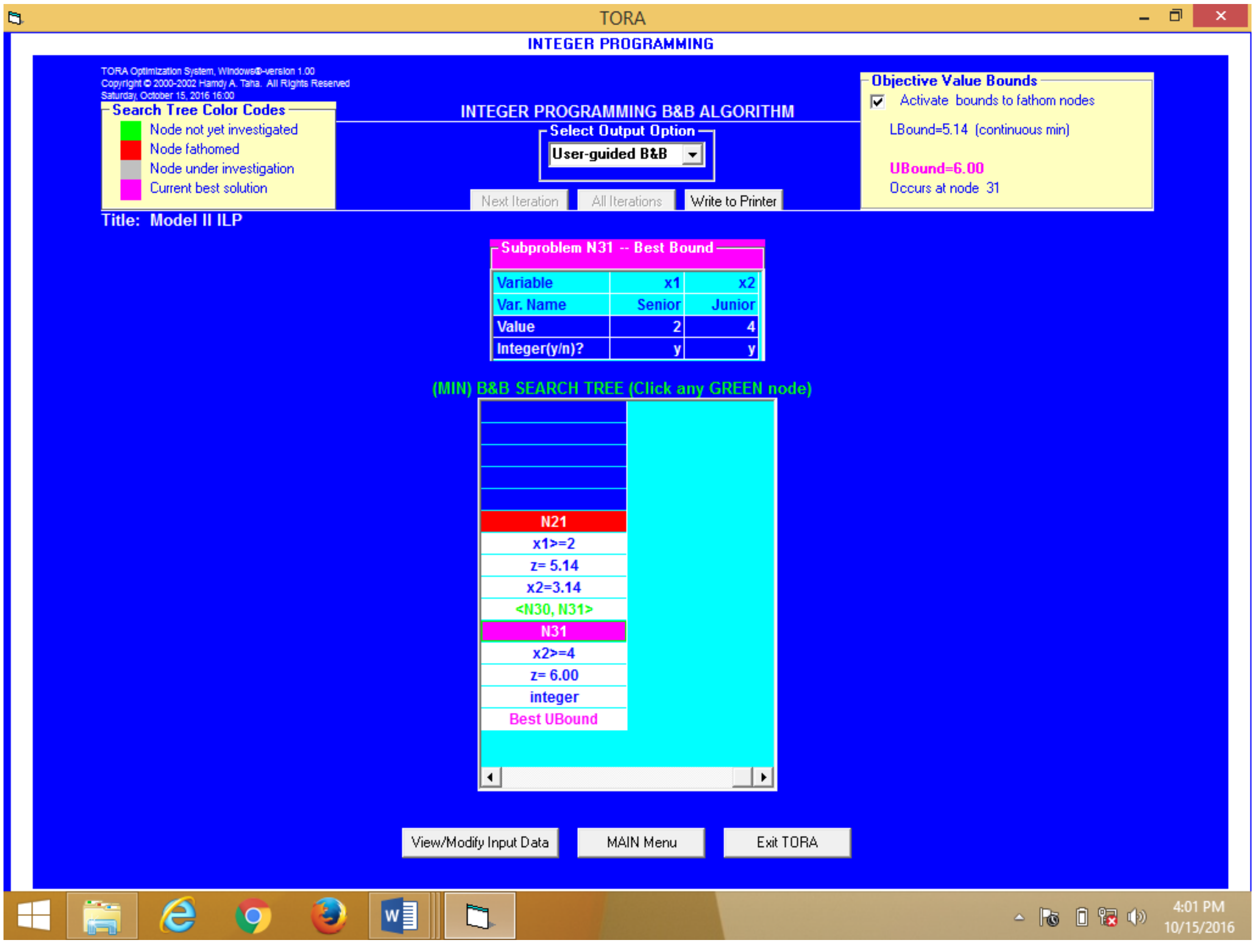
Next Iteration | All Iterations | Write to Printer

Iteration 3	Senior	Junior							
Basic	x1	x2	Sx3	Sx4	Sx5	Sx6	Sx7	Sx8	Solution
z (min)	0.00	0.00	-0.10	0.00	0.00	0.00	0.00	-0.01	3.96
x2	0.00	1.00	-0.10	0.00	0.00	0.00	0.00	0.09	0.36
Sx4	0.00	0.00	-0.70	1.00	0.00	0.00	0.00	-0.17	-4.68
Sx5	0.00	0.00	-0.80	0.00	1.00	0.00	0.00	0.12	-11.52
Sx6	0.00	0.00	-0.90	0.00	0.00	1.00	0.00	-0.29	6.84
Sx7	0.00	0.00	-0.70	0.00	0.00	0.00	1.00	-0.07	-8.28
x1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.10	3.60
Lower Bound	0.00	0.00							
Upper Bound	infinity	infinity							
Unrestr'd (y/n)?	n	n							
Iteration 4	Senior	Junior							
Basic	x1	x2	Sx3	Sx4	Sx5	Sx6	Sx7	Sx8	Solution
z (min)	0.00	0.00	0.00	0.00	-0.13	0.00	0.00	-0.03	5.40
x2	0.00	1.00	0.00	0.00	-0.13	0.00	0.00	0.08	1.80
Sx4	0.00	0.00	0.00	1.00	-0.88	0.00	0.00	-0.28	5.40
Sx3	0.00	0.00	1.00	0.00	-1.25	0.00	0.00	-0.15	14.40
Sx6	0.00	0.00	0.00	0.00	-1.13	1.00	0.00	-0.43	19.80
Sx7	0.00	0.00	0.00	0.00	-0.88	0.00	1.00	-0.18	1.80
x1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.10	3.60
Lower Bound	0.00	0.00							
Upper Bound	infinity	infinity							
Unrestr'd (y/n)?	n	n							

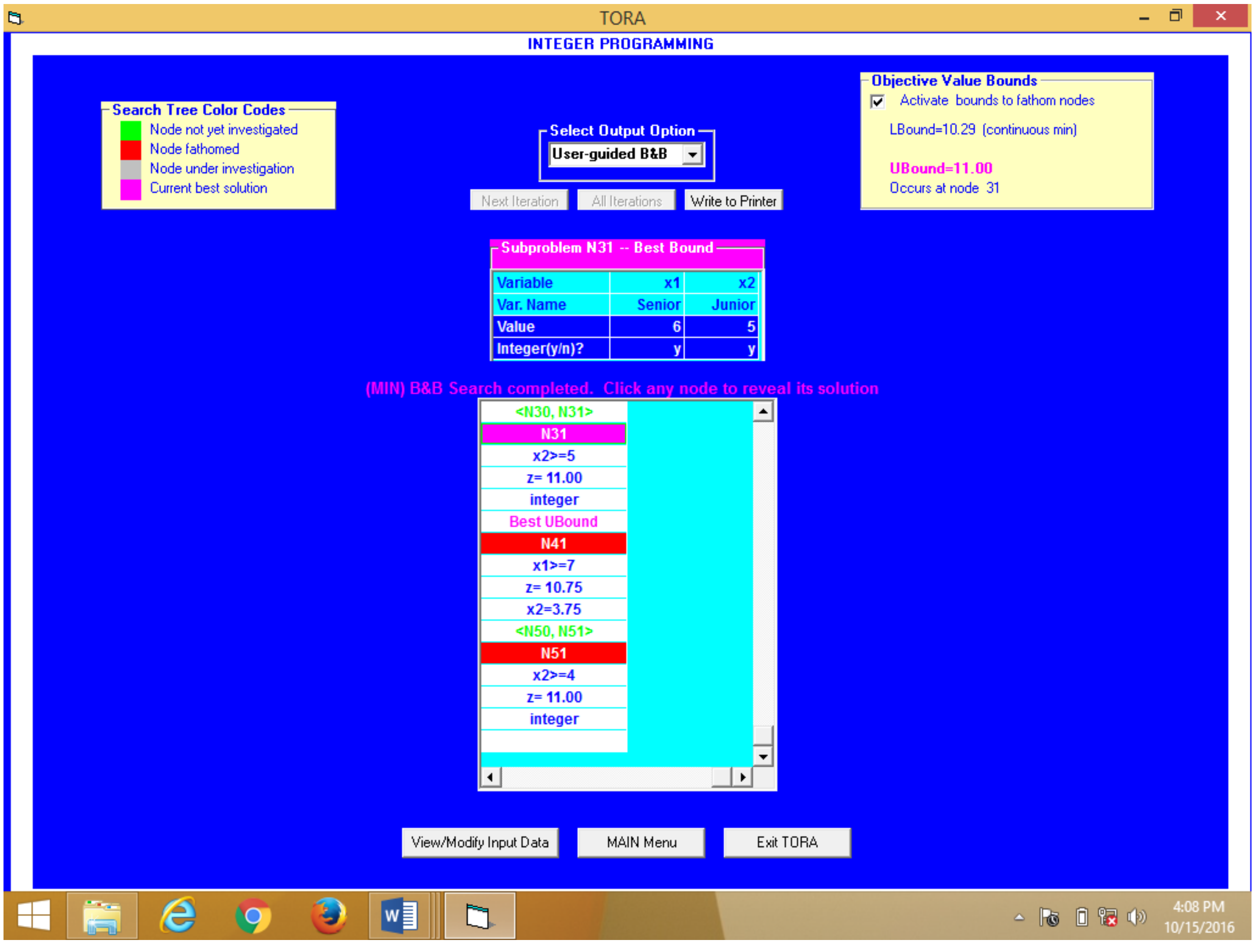
View/Modify Input Data | MAIN Menu | Exit TORA

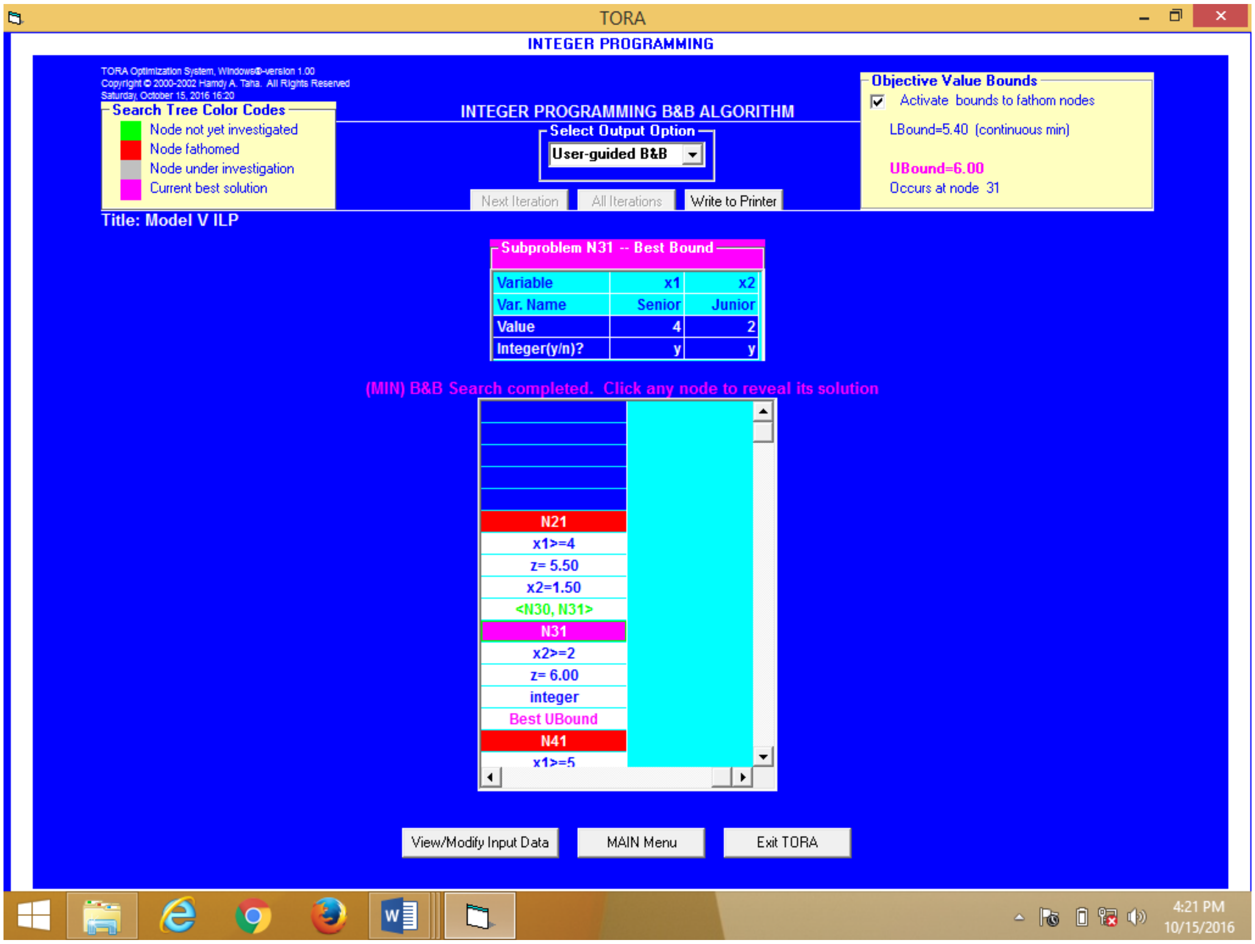
3:09 PM
10/15/2016

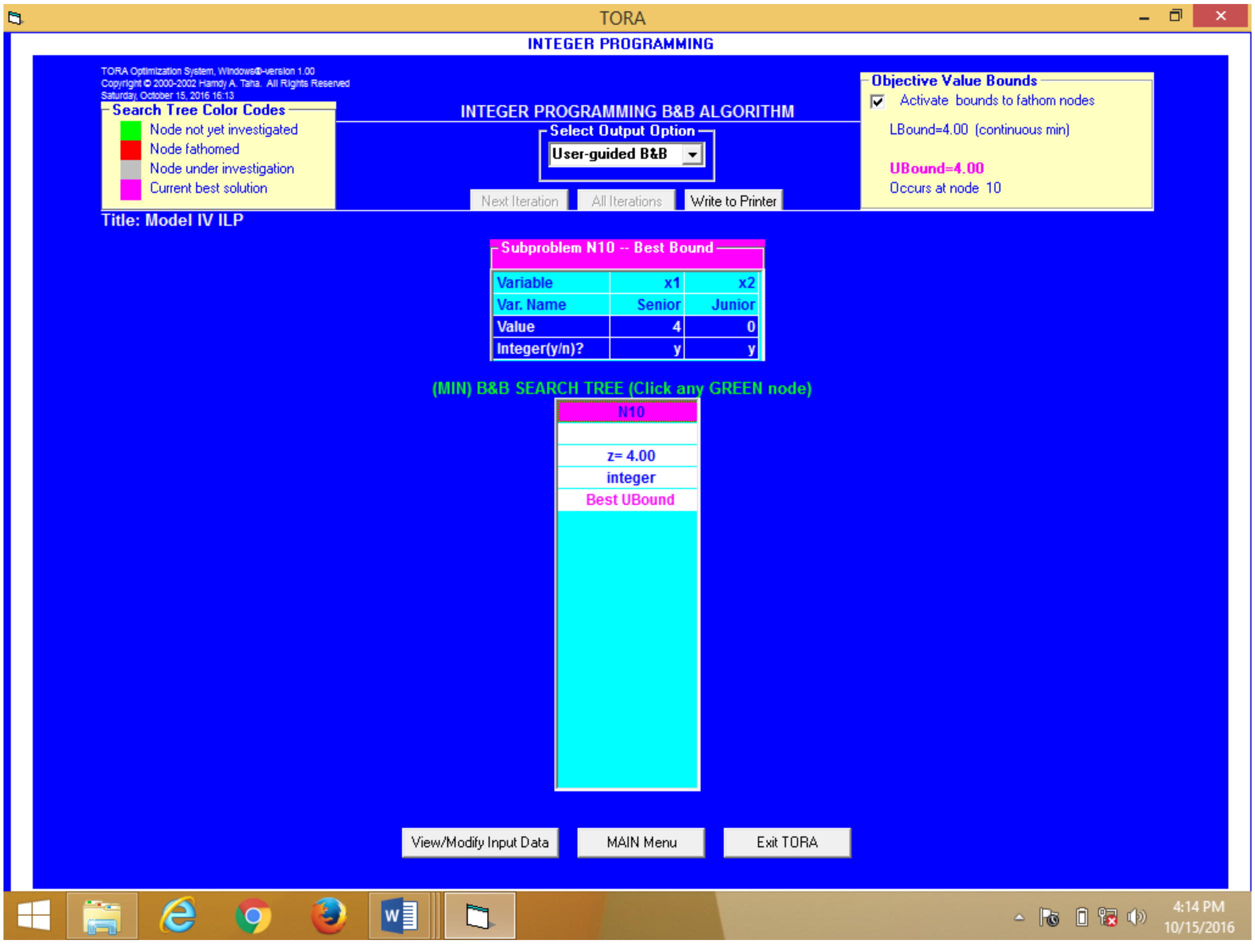

TORA
INTEGER PROGRAMMING
TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Saturday, October 15, 2016 15:19
Search Tree Color Codes
Node not yet investigated
Node fathomed
Node under investigation
Current best solution
INTEGER PROGRAMMING B&B ALGORITHM
Select Output Option
User-guided B&B
Next Iteration
All Iterations
Write to Printer
Objective Value Bounds
Activate bounds to fathom nodes
LBound=1.23 (continuous min)
UBound=2.00
Occurs at node 31
Title: Model I ILP
Subproblem N31 -- Best Bound
Variable | x1 | x2
Var. Name | Senior | Junior
Value | 1 | 1
Integer(y/n)? | y | y
(MIN) B&B SEARCH TREE (Click any GREEN node)
N21
x1>=1
z= 1.33
x2=0.33
<N30, N31>
N31
x2>=1
z= 2.00
integer
Best UBound
View/Modify Input Data
MAIN Menu
Exit TORA
3:21 PM
10/15/2016

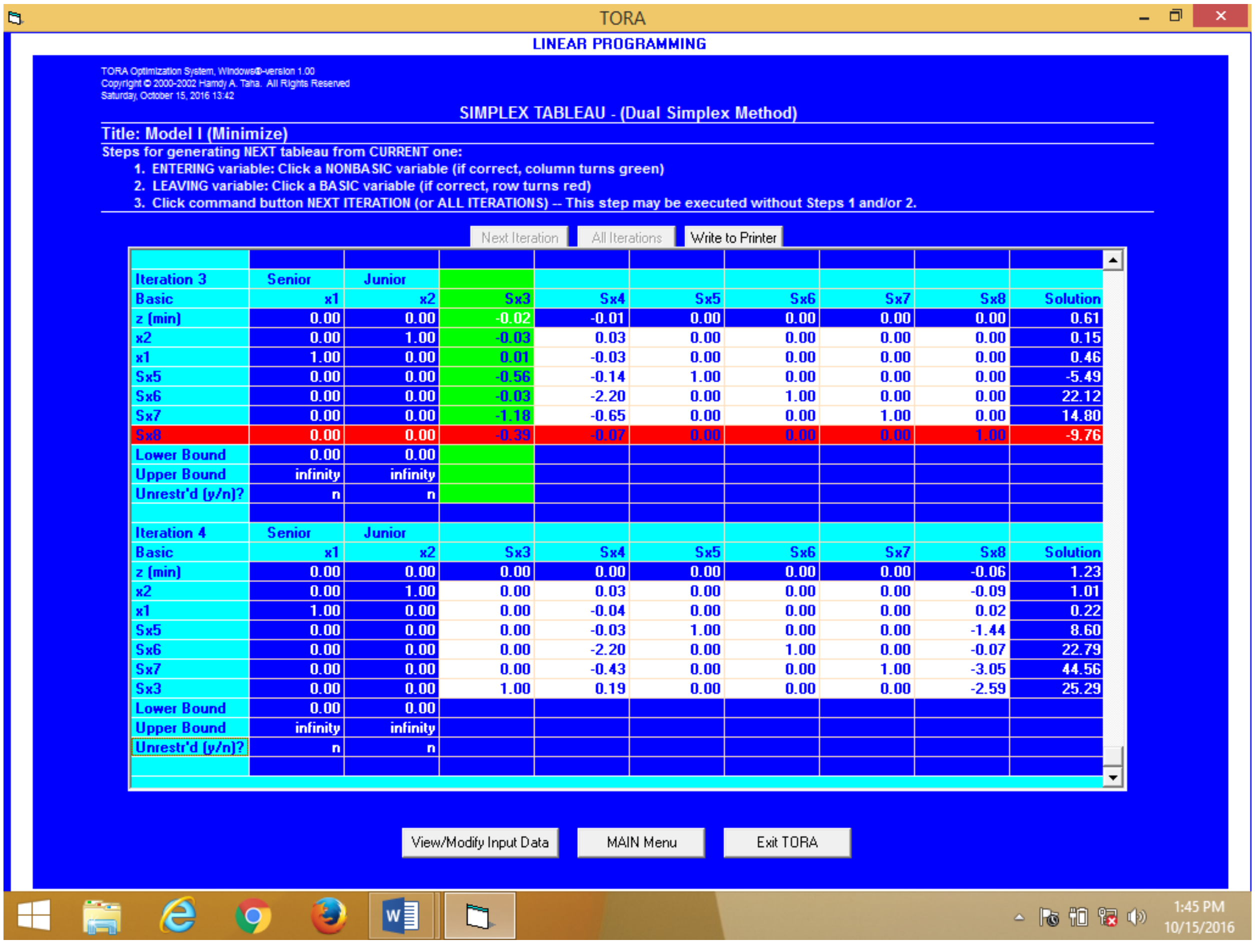


TORA
INTEGER PROGRAMMING
TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Saturday, October 15, 2016 16:00
Search Tree Color Codes
Node not yet investigated
Node fathomed
Node under investigation
Current best solution
INTEGER PROGRAMMING B&B ALGORITHM
Select Output Option
User-guided B&B
Next Iteration
All Iterations
Write to Printer
Objective Value Bounds
Activate bounds to fathom nodes
LBound=5.14 (continuous min)
UBound=6.00
Occurs at node 31
Title: Model II ILP
Subproblem N31 -- Best Bound
Variable
x1
x2
Var. Name
Senior
Junior
Value
2
4
Integer(y/n)?
y
y
(MIN) B&B SEARCH TREE (Click any GREEN node)
N21
x1>=2
z= 5.14
x2=3.14
<N30, N31>
N31
x2>=4
z= 6.00
integer
Best UBound
View/Modify Input Data
MAIN Menu
Exit TORA
4:01 PM
10/15/2016


TORA
INTEGER PROGRAMMING
Search Tree Color Codes
Node not yet investigated
Node fathomed
Node under investigation
Current best solution
Select Output Option
User-guided B&B
Next Iteration
All Iterations
Write to Printer
Objective Value Bounds
Activate bounds to fathom nodes
LBound=10.29 (continuous min)
UBound=11.00
Occurs at node 31
Subproblem N31 -- Best Bound
Variable x1 x2
Var. Name Senior Junior
Value 6 5
Integer(y/n)? y y
(MIN) B&B Search completed. Click any node to reveal its solution
<N30, N31>
N31
x2>=5
z= 11.00
integer
Best UBound
N41
x1>=7
z= 10.75
x2=3.75
<N50, N51>
N51
x2>=4
z= 11.00
integer
View/Modify Input Data
MAIN Menu
Exit TORA
4:08 PM
10/15/2016


TORA
INTEGER PROGRAMMING
TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Saturday, October 15, 2016 16:20
Search Tree Color Codes
Node not yet investigated
Node fathomed
Node under investigation
Current best solution
INTEGER PROGRAMMING B&B ALGORITHM
Select Output Option
User-guided B&B
Next Iteration
All Iterations
Write to Printer
Objective Value Bounds
Activate bounds to fathom nodes
LBound=5.40 (continuous min)
UBound=6.00
Occurs at node 31
Title: Model V ILP
Subproblem N31 -- Best Bound
Variable x1 x2
Var. Name Senior Junior
Value 4 2
Integer(y/n)? y y
(MIN) B&B Search completed. Click any node to reveal its solution
N21
x1>=4
z= 5.50
x2=1.50
<N30, N31>
N31
x2>=2
z= 6.00
integer
Best UBound
N41
x1>=5
View/Modify Input Data
MAIN Menu
Exit TORA
4:21 PM
10/15/2016


TORA
INTEGER PROGRAMMING
TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Saturday, October 15, 2016 16:13
Search Tree Color Codes
Node not yet investigated
Node fathomed
Node under investigation
Current best solution
INTEGER PROGRAMMING B&B ALGORITHM
Select Output Option
User-guided B&B
Next Iteration
All Iterations
Write to Printer
Objective Value Bounds
Activate bounds to fathom nodes
LBound=4.00 (continuous min)
UBound=4.00
Occurs at node 10
Title: Model IV ILP
Subproblem N10 -- Best Bound
Variable | x1 | x2
Var. Name | Senior | Junior
Value | 4 | 0
Integer(y/n)? | y | y
(MIN) B&B SEARCH TREE (Click any GREEN node)
N10
z= 4.00
integer
Best UBound
View/Modify Input Data
MAIN Menu
Exit TORA
4:14 PM
10/15/2016



TORA

LINEAR PROGRAMMING

TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Saturday, October 15, 2016 13:42

SIMPLEX TABLEAU - (Dual Simplex Method)

Title: Model I (Minimize)

Steps for generating NEXT tableau from CURRENT one:

1. ENTERING variable: Click a NONBASIC variable (if correct, column turns green)
2. LEAVING variable: Click a BASIC variable (if correct, row turns red)
3. Click command button NEXT ITERATION (or ALL ITERATIONS) -- This step may be executed without Steps 1 and/or 2.

Next Iteration | All Iterations | Write to Printer

Iteration 3	Senior	Junior							
Basic	x1	x2	Sx3	Sx4	Sx5	Sx6	Sx7	Sx8	Solution
z (min)	0.00	0.00	-0.02	-0.01	0.00	0.00	0.00	0.00	0.61
x2	0.00	1.00	-0.03	0.03	0.00	0.00	0.00	0.00	0.15
x1	1.00	0.00	0.01	-0.03	0.00	0.00	0.00	0.00	0.46
Sx5	0.00	0.00	-0.56	-0.14	1.00	0.00	0.00	0.00	-5.49
Sx6	0.00	0.00	-0.03	-2.20	0.00	1.00	0.00	0.00	22.12
Sx7	0.00	0.00	-1.18	-0.65	0.00	0.00	1.00	0.00	14.80
Sx8	0.00	0.00	-0.39	-0.07	0.00	0.00	0.00	1.00	-9.76
Lower Bound	0.00	0.00							
Upper Bound	infinity	infinity							
Unrestr'd (y/n)?	n	n							

Iteration 4	Senior	Junior							
Basic	x1	x2	Sx3	Sx4	Sx5	Sx6	Sx7	Sx8	Solution
z (min)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06	1.23
x2	0.00	1.00	0.00	0.03	0.00	0.00	0.00	-0.09	1.01
x1	1.00	0.00	0.00	-0.04	0.00	0.00	0.00	0.02	0.22
Sx5	0.00	0.00	0.00	-0.03	1.00	0.00	0.00	-1.44	8.60
Sx6	0.00	0.00	0.00	-2.20	0.00	1.00	0.00	-0.07	22.79
Sx7	0.00	0.00	0.00	-0.43	0.00	0.00	1.00	-3.05	44.56
Sx3	0.00	0.00	1.00	0.19	0.00	0.00	0.00	-2.59	25.29
Lower Bound	0.00	0.00							
Upper Bound	infinity	infinity							
Unrestr'd (y/n)?	n	n							

View/Modify Input Data | MAIN Menu | Exit TORA

1:45 PM
10/15/2016