PROFIT MAXIMIZATION USING LINEAR PROGRAMMING AND INTEGER LINEAR PROGRAMMING MODELS: A FOCUS ON NIGERIAN BOTTLING COMPANY OTA PLANT, OGUN STATE.

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ABSTRACT

Most companies are established to make financial profit. With regard to this, the aim of such establishments is to maximize profit. This paper focuses on some veritable tools in achieving such optimum profit. The study is designed to apply linear programming technique (LP) and Integer linear programming technique (ILP) to optimize profit for Nigerian Bottling Company, Coca-Cola industry, Ota plant, Ogun State. Linear Programming model of the production operations of the company was formulated. The study considered five (5) products of the company namely, Fanta Orange 35cl, coke 35cl,Fanta lemon 35cl, Sprite 35cl, Schweppes 35cl. The work exploits two methods to analyze the formulated models using Simplex method and Branch & Bound method. The result indicates that two particular products, Coke 35cl and Fanta Orange 35cl should be produced to maximize profit. Comparing results of the two models, it was noted that ILP model has an advantage over round- off error encountered in LP model.
KEYWORDS: Optimization, Integer linear programming (ILP), Linear programming (LP), Simplex method, Branch & Bound method.

INTRODUCTION

Company managers are often faced with decisions relating to the use of limited resources. These resources may include men, materials and money. In other sector, there are insufficient resources available to do as many things as management would wish. The problem is based on how to decide on which resources would be allocated to obtain the best result, which may relate to profit or cost or both. Linear Programming is heavily used in Micro-Economics and Company Management such as Planning, Production, Transportation, Technology and other issues. Although the modern management issues are error changing, most companies would like to maximize profits or minimize cost with limited resources. Therefore, many business problems can be characterized as Linear Programming Problems (Sivarethinamohan, 2008).

A linear programming model can be formulated and solutions derived to determine the best course of action within the constraint that exists. The model consists of the objective function and certain constraints. For example, the objective of Nigerian Bottling Company (Coca-Cola) is to maximize profit with high quality products needed by its customers, subject to limited resources, which will still conform with the standard of National Agency for Food and Drug Administration Control (NAFDAC) and Standard Organization of Nigeria (SON). The problem then is on how to utilize limited resources to the best advantage, to maximize profit and at the same time selecting the products to be produced out of the number of products considered for production that will maximize profit.

The research focus is to decide how limited resources, (raw materials) of Nigerian Bottling Company (Coca-cola), Ota plant, would be allocated to obtain the maximum contribution to profit and to determine the products that contribute to such profit.

The Simplex method, also called Simple technique or Simplex Algorithm, was invented by George Dantzig, an American Mathematician, in 1947. It is the basic workhorse for solving Linear Programming Problems up till today. There have been many refinements to the method, especially to take advantage of computer implementations (Chinneck, 2000).The essential elements are still the same as they were, when the method was introduced (Gupta and Hira, 2006).The Simplex method is a Pivot Algorithm that transverses through Feasible Basic Solutions while Objective Function is improving. The Simplex method is, in practice, one of the most efficient algorithms but it is theoretically a finite algorithm only for non-degenerate problems.

Linear programming and its many extension have come into wide use. In academic circles, industries, military, business and others.

Integer linear programming which is an extension of LP can be categorized into two. For convenience we define a pure integer problem as linear programs in which all the variables are integer. Otherwise the problem is a mixed integer problem.

More so, if all the variables in the optimal solution are allowed to take 0 or 1, such is referred to as 0-1 or standard discrete programming problem (Kalavathy 2002). Meanwhile, our approach in this study is a pure integer linear programming techniques.

The significances of ILP are numerous, several occurring situations in business and industry that extend to planning models involve integer valued variables. In manufacturing, production is frequently scheduled in terms of batches, lots or runs. In allocation of goods, shipment must involve discrete number of trucks, personnel management where numbers of staff should strictly assume positive integers and in particular where number of bottles of soft drink produced must not be a fraction.

Aim and Objectives.

- To effectively identify and estimate the products mix that must be produced out of the considered products, in other to maximize profit.

Objectives of the study

The main objectives of the study are:

i. To develop a realistic LP maximization model.

ii. To consider the models as LP model and solve by simplex method.
iii. To consider the models as ILP model and solve by Branch & Bound method.
iv. To compare the efficacy of ILP and LP model.

LITERATURE REVIEW

Application of LP began in 1947, (in connection with the planning activities of the military) by George B. Dantzig, shortly after world war II and has been keeping the pace ever since with the extraordinary growth of computing power. Dantzig was fascinated by the work of Wassily Leontief who proposed in 1932 a large but simple matrix structure which he called the interindustry input-output model of the American Economy (Dantzig, 2002).

Linear programming technique is a very resourceful method in various fields. (Snezana, 2009) presents a method for modeling and optimizing an industrial steam condensing system by linear programming techniques. LP is used to minimizing the total cost for energy net costs in steaming condensing systems.

(Waheed, et al 2012), demonstrates the use of linear programming methods as applicable in the manufacturing industry where KASMO industry limited, Osogbo, Nigeria was taken as a case study.

(Kourosh, et al 2013), solve transportation problems using linear programming in services company. The paper reveals that an evaluation of 500 largest companies in the world showed that 85% of them have used linear programming.

(Akinyele, 2007), applies LP model based on integer programming to the determination of effective size of manpower to be engaged. His study also incorporate global constraints such as production capacity/demand rate and allowable time of operation into the model to reflect the relative activities in production organization in developing countries.

(Agarana, et al 2014), this paper applies LP to the management of loan portfolio of banks, where an answer is provided to the question of how to avoid possible occurrence of non-performing loans, bad and doubtful debts in banks.

(Mina, et al, 2013) exploit LP to establish the optimal combination of production and the optimal allocation of human resources in a beverage company.

Integer linear programming began in 1958 by R.E. Gomory, unlike the earlier work on the travelling salesman problem (TSP) by P.R. Fulkerson (1954)

Land & Doig (1960), introduced another method called Branch & Bound (B&B) which has turned out to be one of the most successful ways to solve practical ILP (Kurtz, 1992).

(Fulkerson and Johnson, 1954), on travelling salesman problem (TSP) provided a remarkable sources of idea for solving by hand combinational optimization problems including cutting, B&B and Lagrangian duality. Dantzing, Fulkerson and Johnson pioneered the idea of employing LP relaxation and valid inequalities to solve integer programs by solving (including a proof of optimality) a 49- city TSP in USA (Richard 1991). It is amazing that the three authors were able to find an optimal solution of such large TSP at instant and prove its optimality by manual computation.

(Biniyam & Atench 2013), worked on personnel scheduling using ILP model in which Avantis Blue-Nile Hotels, in Ethiopia serves as a case study. They used ILP to determine an optimal weekly shift schedule for the Hotel's engineering department personnel.

(Rajan et al, 2010), in their paper propose a vendor selection model using ILP model for multi-product, multi-vendor environment. As such, their model is validated with a case study by implementing the model for Agricultural equipment whole sale company.

(Christodoulos and Xianxia, 2005), review the advances of mixed-integer linear programming (MILP) for the scheduling of chemical processing systems.

Ezema and Amakom (2012) worked on the optimizing profit with the linear programming model: A focus on Golden plastic industry limited, Enugu, 2012. The result they had showed that only 2 sizes of the total 8 ‘PVC’ pipes should be produced.

Khan et al. (2011) in their work, optimal production levels for the different product manufactured at ICL, a multinational Company in Pakistan has a result that showed that the amount was raised by changing production patterns within the first, second, third and fourth digit respectively.

Adamu (2013) carried out a research on application of parametric linear programming in Coca-Cola company using a developed algorithm. In his paper, he critically examined parametric linear programming problem with interval in the coefficients of the objective function and the composition of Coca-Cola, Fanta, Sprite and Schweppes soft drink. The findings of soft drinks production in coca cola company in Kaduna, Nigeria were studied and the problem formulated. The formulated problem was tested using the developed computer program by varying the parameter at regular interval and obtaining the corresponding values of the objective function. The final result of the research refilled that an increase in parameter values resulted in the increased of the values of the objective function and the developed computer program generate these results faster, and accurate than the other methods.

Fagoyinbo, et al. (2013) carried out a research on maximization of profit in manufacturing industries using linear programming techniques in GEEPEE Nigeria limited. The research work applied the concept of revised simplex method; an aspect of linear programming to solving industrial problem with the aim of maximizing profit. The industry GEEPEE Nigeria Limited specializes in production of tanks of various types. Four different types of tank were sampled for the study, which are the Combo, Atlas, Rambo and Jumbo tanks of various sizes. Based on the analysis of the data collected, it was observed that, given the amount of materials available, polyethylene (Rubber) and Oxy-acetylene (Gas) used in the production of the different sizes of the product, Combo tanks assumed more objective value contribution and gave maximum profit at a given level of production capacity.

From the available literature, the work notes that little has been done to exploit the resourceful power of ILP models in term of profit maximization. Hence, this study is to justify the advantages of ILP models.

METHODOLOGY

A linear programming is the problem of maximizing (or minimizing) a linear function subject to a finite number of linear constraints.

Maximize $\sum_{j=1}^{n} c_j x_j$

Subject to $\sum_{j=1}^{n} c_j x_j \geq b_i (i = 1,2, \dots m)$

with $x_j \geq 0$ (j = 1,2, … n).

if $x_j \geq 0$ *and* x_j *are integers then LP model becomes an ILP model*

METHOD OF DATA COLLECTION

The data for this study was obtained mainly from secondary source. The data were collected on quantity of raw materials available in the stock per annum, the cost and selling prices with profit made on each create of the selected product, such as coke35cl, Fanta orange 35cl, Fanta lemon

35cl, Schweppes, sprite 50cl. With respect to four raw material such concentrates, sugar, water (H_2O), and carbon (IV) oxide.

Mathematical formulation of the model

Instead of obtaining information from the whole products produced by Coca-Cola industry (bottling company).A sample of five (5) common products are selected with four raw materials, for the effectiveness of this research study.

Mathematical formulation of the model

(i) Decision variables:

Let x_1 represent each crate of coke 35cl.

Let x_2 represent each crate of Fanta orange 35cl.

Let x_3 represent each crate of Fanta lemon 35cl.

Let x_4 represent each crate of Schweppes 35cl.

Let x_5 represent each crate of Sprite 35cl.

(ii) Objective function

Maximize (z) = $c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5$

Where c_1 ... ... c_5 are the cost coefficients of Coke, Fanta orange, Fanta lemon, Schweppes, and Sprite respectively?

(iii) Constraints:

The constraints for this work are basically the raw materials for the production of the selected drinks.

Therefore the general model governing the work is given as:

$$max \sum_{j=1}^{5} c_j x_j$$

Subject to:

$$\sum_{j=1}^{5} a_j x_j \quad \leq bi \ \ (i = 1,2, \dots n)$$

With $x_j \geq 0$, *in case of ILP* $x_j \in \mathbb{Z}^+$

(iv) Non negativity condition:

Case I

$x_1, x_2, x_3, x_4, x_5 \geq 0$

Case II

$x_1, x_2, x_3, x_4, x_5 \in \mathbb{Z}^+$

Case II renders the system as pure ILP models

Valuation of the model

The secondary data collected are implemented to validate the models. The models were validated with the production unit of Nigerian bottling company (Coca-Cola Plc. Ota Plant). The data were collected on quantity of raw materials available in the stock, the cost and selling prices with profit made on each create of the selected product such as coke35cl, Fanta orange 35cl, Fanta lemon 35cl, Schweppes, sprite 50cl. With respect to raw material such concentrates, sugar, water (H_2O), and carbon (IV) oxide ($C0_2$).

MODEL I

Max (z) = 331.49x_1 + 319.09x_2 + 321.33x_3 + 348.15x_4 + 303.96x_5

Subject to;

0.00359x_1 + 0.00419x_2 + 0.0042x_3 +0.00359x_4 + 0.00359x_5 $\leq$ 4332

0.54x_1 + 1.12x_2 + 1.044x_3 + 0.86x_4 + 0.73x_5 $\leq$ 467012

6.882x_1 + 7.552x_2 + 7.671x_3 + 6.539x_4 + 7.602x_5 $\leq$ 16376630

0.0135x_1 + 0.007x_2 + 0.0126x_3 + 0.0125x_4 + 0.0133x_5 $\leq$ 8796

With non- negativity condition x_1, x_2, x_3, x_4, x_5 ≥ 0

MODEL II

Max (z) = 331.49x_1 + 319.09x_2 + 321.33x_3 + 348.15x_4 + 303.96x_5

Subject to;

0.00359x_1 + 0.00419x_2 + 0.0042x_3 +0.00359x_4 + 0.00359x_5 $\leq$ 4332

0.54x_1 + 1.12x_2 + 1.044x_3 + 0.86x_4 + 0.73x_5 $\leq$ 467012

6.882x_1 + 7.552x_2 + 7.671x_3 + 6.539x_4 + 7.602x_5 $\leq$ 16376630

0.0135x_1 + 0.007x_2 + 0.0126x_3 + 0.0125x_4 + 0.0133x_5 $\leq$ 8796

With non- negativity condition $x_1, x_2, x_3, x_4, x_5 \geq 0$ and are integers.

DATA ANALYSIS

The work exploits two techniques namely simplex method and Branch & Bound techniques to obtain solutions to our LP and ILP models respectively via TORA Mathematical package.

Presentation of Results:

Check the Appendices for the worksheets of the calculations

Table I: Optimal results of model I

LP optimal solution
Max (z)= 236,167,536.30 x_1= 580,461.73 crates x_2= 137,109.52 crates

Table II: Optimal results of model II

ILP optimal solution
Max (z)= 236,167,500.00 x_1= 580,461 crates x_2= 137,109 crates

INTERPRETATION OF RESULTS

The optimal solution to the LP model is obtained at the 4th iteration of the tableau.

Max (z) = 236,167,536.30

x_1= 580,461.73

x_2= 137,109.52

Substituting x_1 & x_2 into the LP objective function yield the objective value.

This implies that 580,461 crates and approximately 18 bottles of coke 35cl should be produced and 137,109 crates and approximately 12 bottles of Fanta orange should be produced to maximize profit at ₦ 236,167,536.30.

The optimal solution to the ILP model is obtained at different nodes x_1 is obtained at 21st node while x_2 is obtained at the 41st node with the following output.

Max (z) = 236,167,500

x_1= 580,461

x_2= 137,109

If x_1& x_2 are substituted into the objective function, objective value will be obtained.

This implies that 580,461 crates of coke should be produced and 137,109 crate of Fanta orange should be produced to maximize profit at ₦ 236,167,500.00.

However, the two model suggest that Fanta lemon, Schweppes and Sprite should be produced at zero level.

CONCLUSION

The interest of the study is to see how industries can conveniently maximize profit. The study has given an insight into how best Coca-Cola Company can maximize profit using LP & ILP models.

The model is robust and it can be adapted to production unit of any company. From the results obtained we realized that LP model yields the best optimal value but with fractional decision variables which might be practically difficult to interpret, since the production is done in crates not in bottles. We need to note that fractional part of a bottle cannot be produced. For example result x_1 =580,461.73 the fractional part can be interpreted as 18 bottles, it was 17.52. Meanwhile, the approximated value might not yield the exact objective value.

However, in the case of ILP model the optimal result is slightly less than that of LP model but the decision variables are given as integers which is very easy to interpret without affecting the mixing proportionality of the constituent materials.

Therefore, the study recommends that the two methods can be employed altogether in other to take a precise managerial decision. Though, Coke and Fanta Orange are major soft drinks that can optimize profit for the Company with the objective value contribution of ₦192,417258.35 and ₦43750277.5 respectively. The company still needs to produce Fanta lemon, Schweppes and Sprite at almost zero level despite these products assumed absolute zero from the optimal results. This is necessary for the company to retain their goodwill from some customers that have strong flair in consuming these products.

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APPENDICES

PRESENTATION OF DATA (COCA-COLA INDUSTRY, OTA, OGUN STATE)

Table 1: Quantity of raw materials available in stock.

Raw Materials	Quantity Available
Concentrates	4332(litres)
Sugar	467012(kg)
water(H_20)	16376630(litres)
carbon(iv)oxide	8796(vol. per pressure)

Source: statistical survey in Nigeria bottling company (Coca-Cola industry, Ota Ogun State)

Table II: Quantity of raw materials needed to produce a crate of each product listed.

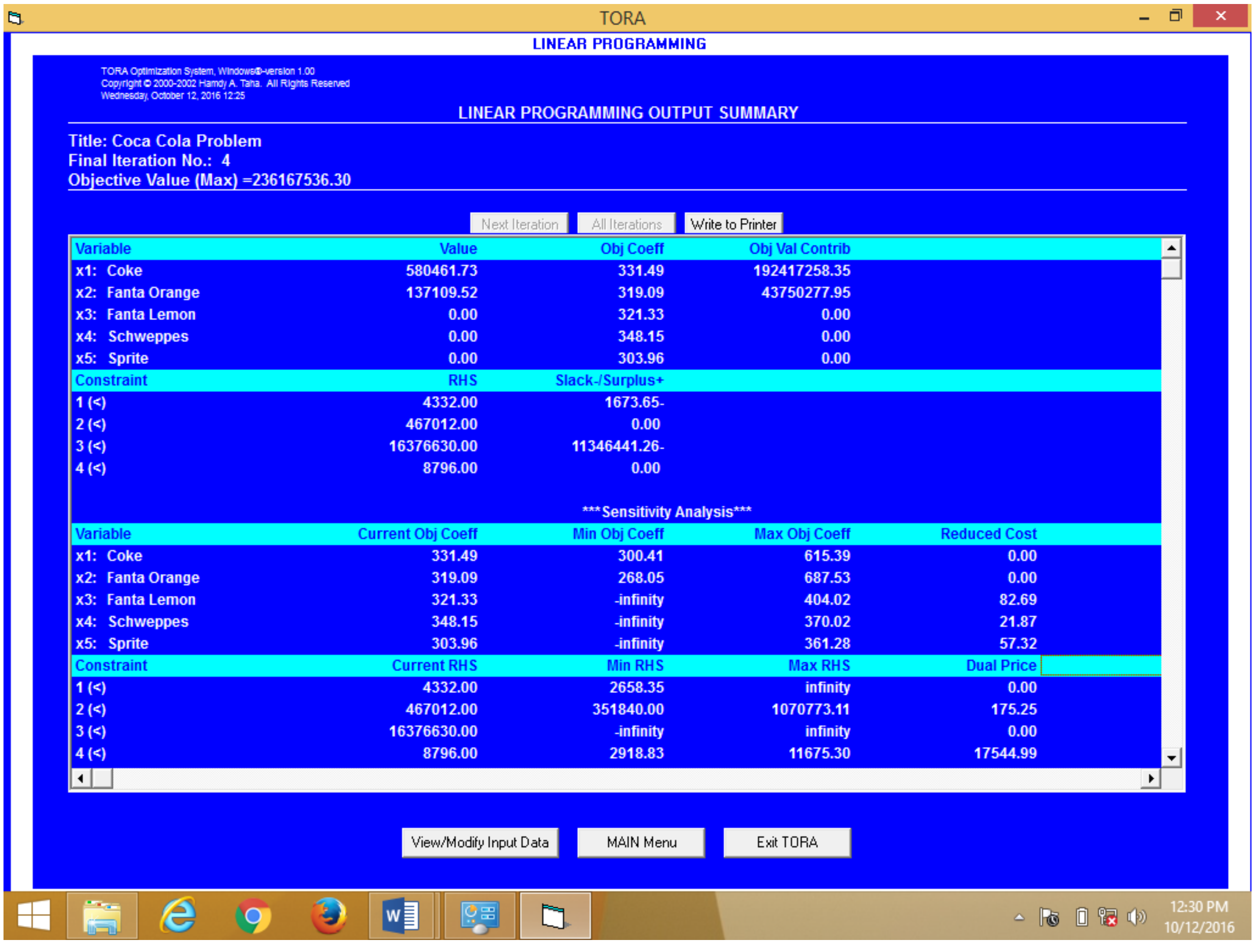
Types of drinks	Concentrate	Sugar	Water	Carbon(iv)oxide
coke 35cl	0.00359	0.54	6.882	0.0135
Fanta orange 35cl	0.00419	1.12	7.552	0.007
Fanta lemon 35cl	0.0042	1.044	7.671	0.0126
Schweppes 35cl	0.00359	0.86	6.539	0.0125
sprite 35cl	0.00359	0.73	7.602	0.0133

Source: statistical survey in Nigeria bottling company (Coca-Cola industry, Ota Ogun State)

Table 3: Average cost and selling of a create of each product

Product	Average cost price(N)	Average selling price(N)	Profit
coke 35cl	358.51	690	331.49
Fanta orange 35cl	370.91	690	319.09
Fanta lemon 35cl	368.67	690	321.33
Schweppes 35cl	341.85	690	348.15
sprite 35cl	486.04	790	303.96

Source: statistical survey in Nigeria bottling company (Coca-Cola industry, Ota Ogun State)


TORA

LINEAR PROGRAMMING

TORA Optimization System, Windows®-version 1.00
Copyright © 2000-2002 Hamdy A. Taha. All Rights Reserved
Wednesday, October 12, 2016 12:25

LINEAR PROGRAMMING OUTPUT SUMMARY

Title: Coca Cola Problem
Final Iteration No.: 4
Objective Value (Max) =236167536.30

Next Iteration | All Iterations | Write to Printer

Variable	Value	Obj Coeff	Obj Val Contrib
x1: Coke	580461.73	331.49	192417258.35
x2: Fanta Orange	137109.52	319.09	43750277.95
x3: Fanta Lemon	0.00	321.33	0.00
x4: Schweppes	0.00	348.15	0.00
x5: Sprite	0.00	303.96	0.00

Constraint	RHS	Slack-/Surplus+
1 (<)	4332.00	1673.65-
2 (<)	467012.00	0.00
3 (<)	16376630.00	11346441.26-
4 (<)	8796.00	0.00

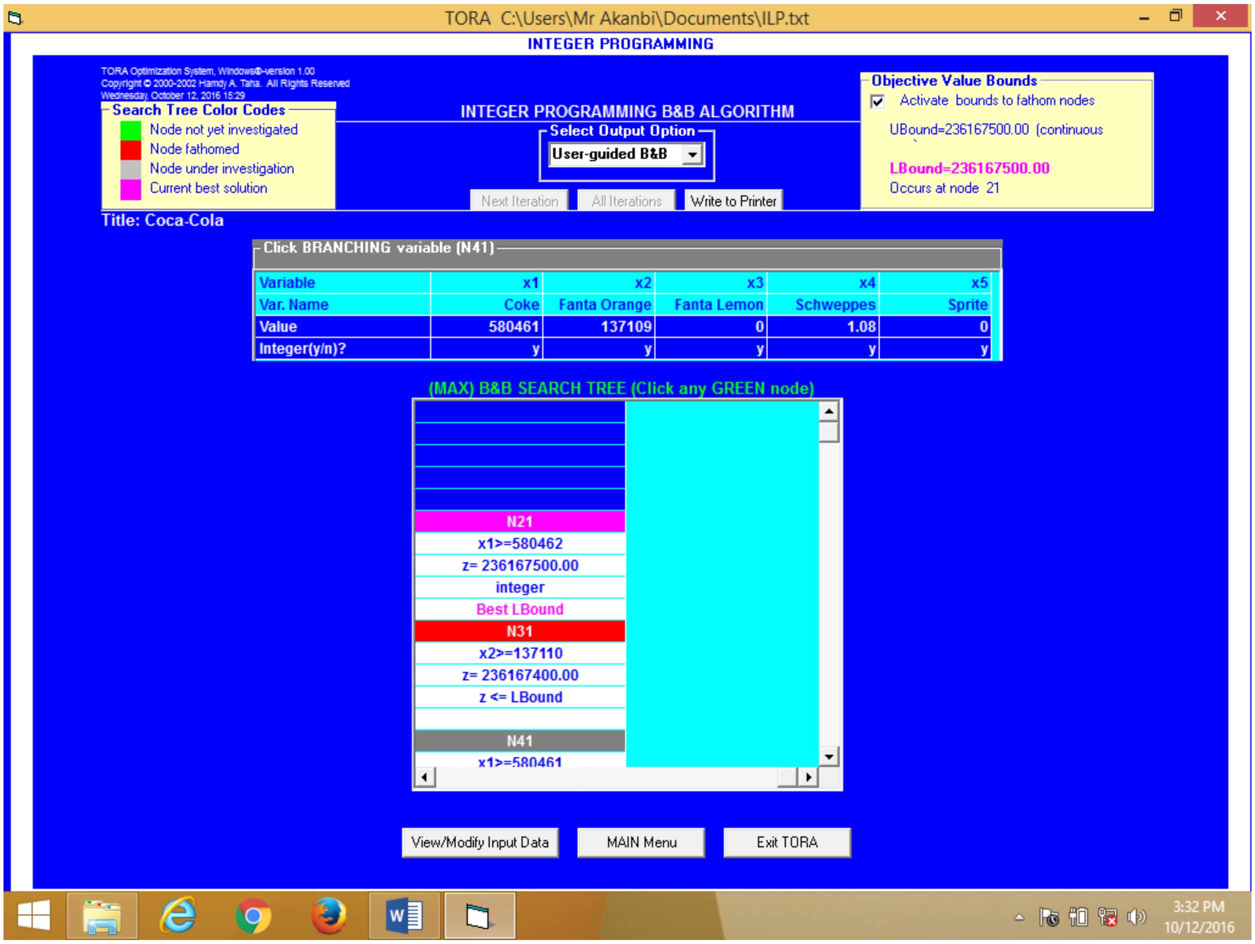
Sensitivity Analysis

Variable	Current Obj Coeff	Min Obj Coeff	Max Obj Coeff	Reduced Cost
x1: Coke	331.49	300.41	615.39	0.00
x2: Fanta Orange	319.09	268.05	687.53	0.00
x3: Fanta Lemon	321.33	-infinity	404.02	82.69
x4: Schweppes	348.15	-infinity	370.02	21.87
x5: Sprite	303.96	-infinity	361.28	57.32

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (<)	4332.00	2658.35	infinity	0.00
2 (<)	467012.00	351840.00	1070773.11	175.25
3 (<)	16376630.00	-infinity	infinity	0.00
4 (<)	8796.00	2918.83	11675.30	17544.99

View/Modify Input Data | MAIN Menu | Exit TORA

12:30 PM 10/12/2016


TORA C:\Users\Mr Akanbi\Documents\ILP.txt
INTEGER PROGRAMMING
Search Tree Color Codes
Node not yet investigated
Node fathomed
Node under investigation
Current best solution
INTEGER PROGRAMMING B&B ALGORITHM
Select Output Option
User-guided B&B
Next Iteration
All Iterations
Write to Printer
Objective Value Bounds
Activate bounds to fathom nodes
UBound=236167500.00 (continuous
LBound=236167500.00
Occurs at node 21
Title: Coca-Cola
Click BRANCHING variable (N41)
Variable x1 x2 x3 x4 x5
Var. Name Coke Fanta Orange Fanta Lemon Schweppes Sprite
Value 580461 137109 0 1.08 0
Integer(y/n)? y y y y y
(MAX) B&B SEARCH TREE (Click any GREEN node)
N21
x1>=580462
z= 236167500.00
integer
Best LBound
N31
x2>=137110
z= 236167400.00
z <= LBound
N41
View/Modify Input Data
MAIN Menu
Exit TORA
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