HEAT AND MASS TRANSFER OF MAGNETO-MICROPOLAR FLUID PAST A NONLINEAR STRETCHING SHEET IN A POROUS MEDIUM WITH CHEMICAL REACTION

BY
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INTRODUCTION I

Abstract to the Study

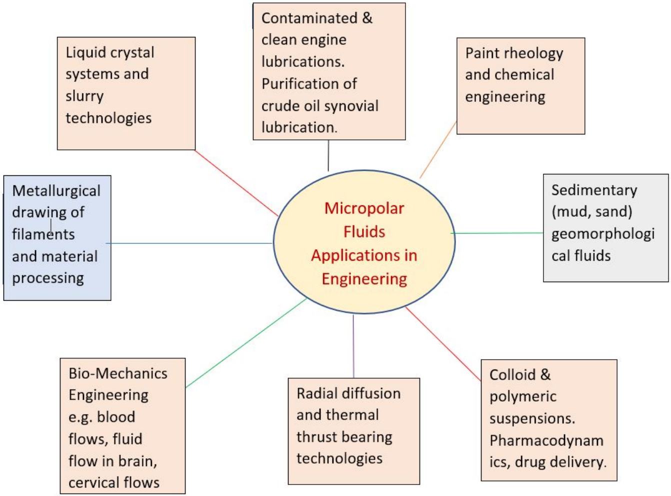
The objective of this study is to analyze the flow, heat and mass transfer characteristics of magneto-micropolar chemically reactive fluid over a nonlinear stretching sheet in a saturated non-Darcy porous medium. The impact of viscous dissipation and non-uniform heat source/sink are checked under the influence of prescribed surface temperature and concentration boundary conditions. The model equations governing the problem are reduced from partial to ordinary differential equations using a suitable similarity conversion analysis and then solved via shooting technique with Runge-Kutta algorithms. The numerical outcomes of the simulation are illustrated graphically while a comparison of the present numerical analysis with previous reported data in literature for some limiting situations shows an excellent agreement. The results indicate that the velocity and microrotation profiles advance as the material parameter increases with the motion of micropolar fluid higher than that of Newtonian fluid and that the momentum and thermal boundary layer thicknesses become thin with an increase in the nonlinear stretching parameter.

INTRODUCTION I

Background to the Study

- The study of non-Newtonian fluids has gain prominence due to its practical industrial and diverse applications in engineering processes.
- ***Why Micropolar Fluid?***
- Newtonian fluids cannot effectively describe the complex mechanical behaviour that fluid exhibit at micro level.
- Micropolar fluid is prominent among others because it offers a good mathematical model for simulating the flow characteristics of polymeric suspensions, colloidal fluids, liquid crystals, animal blood etc.

Micropolar Fluids Applications in Engineering I



Various Applications of this Study I

The study of heat and mass transfer over stretching surfaces has gained prominence due to its applications in industrial and engineering processes, e.g.

- extrusion of plastic sheet and metal extrusion, hot rolling.
- paper and textile production.

Convective Transport phenomena in Porous media is Applicable in:

- crude oil extraction; ground water hydrology, underground disposal of waste.
- biomedical engineering; spreading of chemical pollutants in saturated soil.

Heat and Mass Transfer with Chemical Reaction and Radiation is Applicable in:

- the design of chemical processing equipment & combustion processes.
- temperature& moisture distribution over agricultural fields, food processing.
- nuclear power plant, steel rolling, electric power generation etc.

Review of Literature I

Authors who have worked on stretching sheet on different geometries:

- Ahmad *et al.*, (2014) worked on MHD micropolar fluid flow on nonlinearly impermeable stretching sheet with constant properties.
- Salem (2013) studied on MHD micropolar nonlinearly stretching sheet with variable viscosity.
- Waqas *et al.*, (2016) studied MHD micropolar liquid flow on nonlinearly stretching sheet with constant properties.
- Fatunmbi & Fenuga (2018) MHD micropolar nonlinearly stretching sheet with variable viscosity.
- Tripathy *et al.* (2016) investigated numerical MHD flow of micropolar fluid on stretching fluid with constant properties.
- Cortell (2007, 2008); Shamshuddin & Thunma (2019) etc.
- Present work MHD micropolar fluid flow on a nonlinear stretching sheet with prescribed surface temperature and concentration.

Mathematical Formulation of the Problem I

Basic Assumptions

- The flow is two-dimensional (x, y), incompressible and steady.
- The sheet is permeable and stretching nonlinearly with velocity $u_w = cx^r$.
- The corresponding velocity components are (u, v).
- x axis is taken along the direction of flow with y axis normal to it.
- The fluid is electrically conducting Micropolar fluid.
- Applied magnetic field is normal to the flow direction.
- The induced magnetic field is sufficiently small & negligible and no electric field.
- The surface mass flux is assumed to be a function of x.
- Non-uniform heat source/sink is applied with viscous dissipation effect.
- A chemical species diffuses into the ambient fluid initiating an irreversible homogeneous first order chemical reaction.

Mathematical Formulation of the Problem II

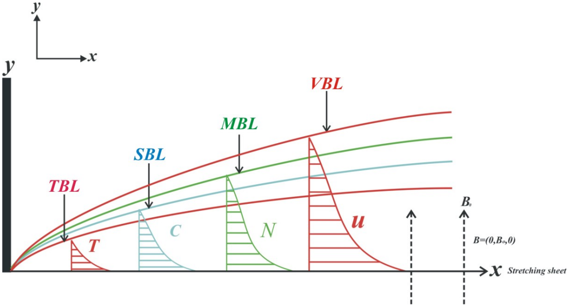


Figure 1. *Flow Configuration and the Coordinate System.*

The Governing Equations I

Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

Momentum Equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{(\mu+\mu_r)}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{\mu_r}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma B^2(x)}{\rho}u-\frac{\nu}{K_p}u-\frac{F}{K_p}u^2, \tag{2}$$

Microrotation Equation

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{\mu_r}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right), \tag{3}$$

Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{(\mu+\mu_r)}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B^2(x)}{\rho C_p}u^2+\frac{q'''}{\rho C_p}, \tag{4}$$

The Governing Equations II

Concentration Equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=Dm\frac{\partial^2 C}{\partial y^2}-k_c\left(C-C_\infty\right). \tag{5}$$

The Boundary Conditions

$$u=u_w=cx^r, v=v_w, N=-h\frac{\partial u}{\partial y}, T=T_w=\left(Ax^n+T_\infty\right), C=C_w=\left(Bx^m+C_\infty\right.$$

$$u\longrightarrow 0, N\longrightarrow 0, T\longrightarrow T_\infty, C\longrightarrow C_\infty \quad \text{as} \quad y\to\infty. \tag{6}$$

The Non-uniform Heat Source/Sink

$$q'''=\frac{\kappa u_w}{x^r\nu}\left[A^\star\left(T_w-T_\infty\right)f'+B^\star\left(T-T_\infty\right)\right] \tag{7}$$

The Suction Velocity & the Applied Magnetic Field varies in strength such that

$$V_w=V_0x^{(r-1)/2}, B(x)=B_0x^{(r-1)/2} \tag{8}$$

where V_0 and B_0 are also a constants.

The Governing Equations III

The Transformation Variables, (see Hayat *et al.*, 2008; Salem, 2013)

$$\eta = y\left[\frac{c(r+1)x^r}{2x\nu}\right]^{1/2},\ N = x^{(3r-1)/2}\left[\frac{c^3(r+1)}{2\nu}\right]^{1/2} g(\eta),\ u = cx^r f',$$
$$v = -\left[\frac{c\nu(r+1)}{2}\right]^{1/2} x^{(r-1)/2}\left(f + \frac{(r-1)}{(r+1)}\eta f'\right),\ \gamma = \left(\mu + \frac{\mu_r}{2}\right) j, \tag{9}$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}.$$

The Transformed Equations I

Substituting (9) into the governing equations (2-6) gives

$$(1+K)\,f''' + ff'' + Kg' - 2\left(\frac{r+Fs}{r+1}\right)f'^2 - \left(\frac{2}{r+1}\right)(M+Da)f' = 0, \tag{10}$$

$$(1+K/2)\,g'' + fg' - \left(\frac{3r-1}{r+1}\right)f'g - K\,(2g+f'')\left(\frac{2}{r+1}\right) = 0, \tag{11}$$

$$\theta'' + Pr\left(f\theta' - \frac{4n}{r+1}f'\theta\right) + (1+K)\,PrEcf''^2 + \left(\frac{2}{r+1}\right)PrMEcf'^2 + \left(\frac{2}{r+1}\right)Pr\,(\alpha f' + \beta\theta) = 0. \tag{12}$$

$$\phi'' + Scf\phi' - \left(\frac{2m}{r+1}\right)Sc\phi f' - \left(\frac{2}{r+1}\right)Sc\zeta\phi = 0, \tag{13}$$

The boundary conditions are:

$$\begin{aligned} &\eta = 0 : f' = 1,\ f = fw,\ g = -hf'',\ \theta = 1, \phi = 1 \\ &\eta \longrightarrow \infty : f' = 0,\ g \longrightarrow 0,\ \theta \longrightarrow 0, \phi \longrightarrow 0. \end{aligned} \tag{14}$$

The Quantities of Engineering Interest I

$$C_{fx} = \frac{\tau_w}{\rho u_w^2}, \ Nu_x = \frac{xq_w}{\kappa\left(T_w - T_\infty\right)}, \ Sh_x = \frac{xq_m}{Dm\left(C_w - C_\infty\right)}, \tag{15}$$

with

$$\begin{aligned} \tau_w &= \left[\left(\mu + \mu_r\right)\frac{\partial u}{\partial y} + \mu_r N\right]_{y=0}, \ q_w = -\left(\kappa\frac{\partial T}{\partial y}\right)_{y=0}, \\ q_m &= -\left(Dm\frac{\partial C}{\partial y}\right)_{y=0} \end{aligned} \tag{16}$$

The dimensionless skin friction coefficient is

$$C_{fx} = \left(\frac{r+1}{2}\right)^{1/2}\left[1 + (1-h)\,K\right] Re_x^{-1/2} f''(0), \tag{17}$$

while the Nusselt and Sherwood numbers respectively simplify to

$$Nu_x = -\left(\frac{r+1}{2}\right)^{1/2} Re_x^{1/2}\theta'(0), \ Sh_x = -\left(\frac{r+1}{2}\right)^{1/2} Re_x^{1/2}\phi'(0). \tag{18}$$

Comparison of The P.R. with Existing Work I

Table 1: Comparison of values of the skin friction coefficient C_{fx} with existing results

K	Kumar (2009)	Present	r	Hayat et al. (2008)	Present
0.0	1.000008	1.00000837	00	0.627555	0.627555
1.0	1.367996	1.36799627	0.2	0.766837	0.766837
2.0	1.621575	1.62157505	0.5	0.889544	0.889544
3.0	1.827392	1.82738216	1.0	1.000000	1.000008
4.0	2.005420	2.00542027	1.5	1.061601	1.061601
			3.0	1.148593	1.148593
			7.0	1.216850	1.216850
			10.0	1.234875	1.234875
			20.0	1.257424	1.257424
			100.0	1.276774	1.276774

Comparison of The P.R. with Existing Work II

Table 2: Comparison of values of heat transfer rates Nu_x for changes in Pr when $K = \lambda = Ec = M = G_2 = \alpha = \beta = fw = 0$ and $r = 1$

Pr	Grubka & Bobba (1985)	Chen (1998)	Present
0.01	0.0294	0.02942	0.12036573
0.72	1.0885	1.08853	1.08862246
1.0	1.3333	1.33334	1.33333334
3.0	2.5097	2.50972	2.50972158
10.0	4.7969	4.79686	4.79687061
100.0	15.712	15.7118	15.71196466

Graphs and Discussion I

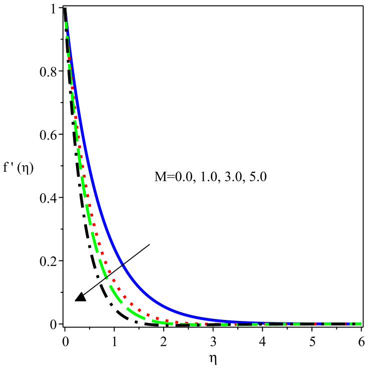


Fig.2 Vel. profiles for M

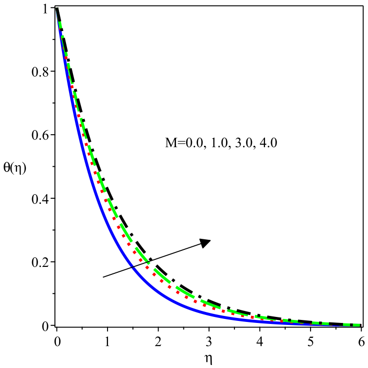


Fig.3 Temp. profiles for M

- Both velocity decreases while temperature increases a rise in M.
- The BLT is thinner in case of Velocity due to Lorentz force.

Graphs and Discussion II

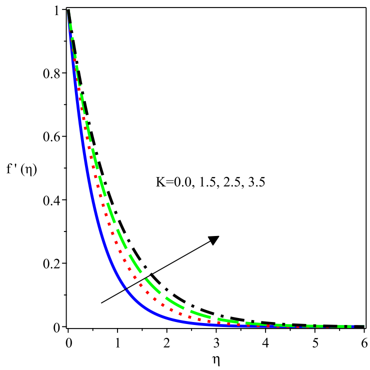


Fig.4 ***Vel. profiles for K***

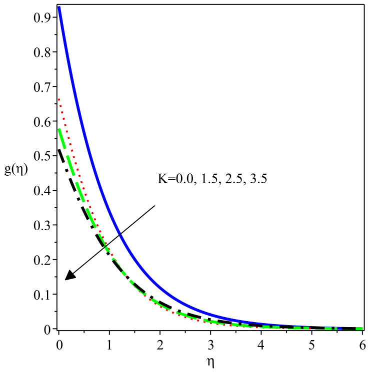


Fig.5 Mic. profiles for K

- The velocity increases while microrotation decrease with a rise in K for.
- The BLT is thinner in case of microrotation profiles.

Graphs and Discussion III

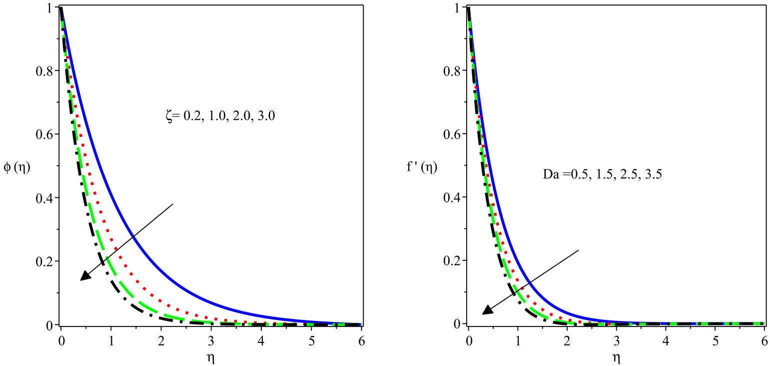


Fig. 6 *Conc. profiles for* ζ *Fig. 7 Vel. profiles for* Da

- Concentration decreases for a rise in chemical reaction parameter ζ.
- Fluid motion decreases with a rise in Da parameter.

Graphs and Discussion IV

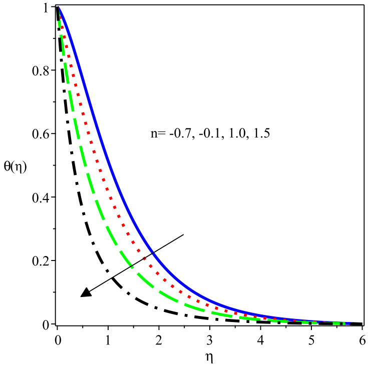


Fig.8 ***Temp.profiles for*** n

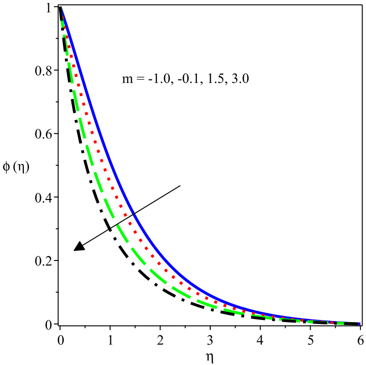


Fig. 9 Conc.profiles for m

Figs. 8 & 9 show the effects n and m on Temp. and Conc. profiles.

- Temp. profiles fall for a rise in n.
- As m increases, concentration profiles drop.

Graphs and Discussion V

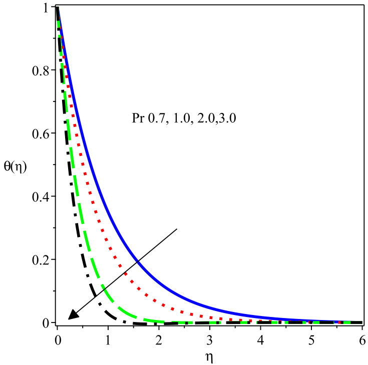


Fig.10 ***Temp.profiles for*** *Pr*

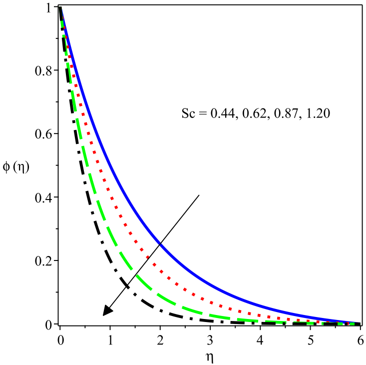


Fig. 11 Conc.profiles for *Sc*

Figs. 10 & 11 show the of effects Pr and Sc on Temp. and Conc. profiles.

- Temperature falls with a rise in Pr.
- Concentration decreases for in Sc

Graphs and Discussion VI

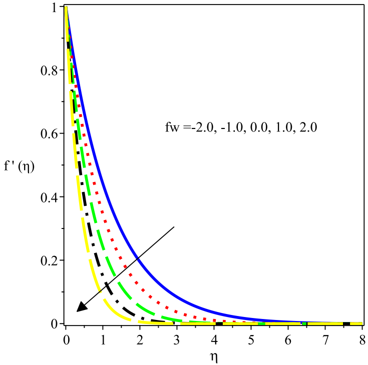


Fig.12 *Temp.profiles for Pr*

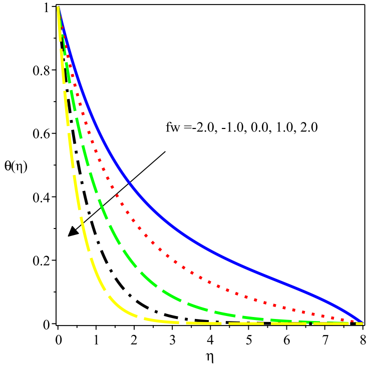


Fig. 13 Conc.profiles for Sc

Figs. 12 & 13 show the effects of fw on velocity & temperature profiles.

- Both MBLT & TBLT fall for an increase in $fw > 0$.
- Velocity & temperature rise with a rise in $fw < 0$

Conclusion I

- The (Cf_x) is found to increase with a rise in r
- Heat transfer rate Nu_x increases for a rise in Pr
- The temperature falls with a rise in the wall surface temperature parameter n.
- The concentration falls with a rise in the wall surface concentration parameter m.
- Injection increases the $f'(\eta)$ while suction lowers it.
- Temperature decreases for an increase in n, Pr and fw.
- Velocity as rises with an increase in K while it falss for M, Da and fw.
- Concentration reduces with a rise in ζ and Sc.
- The BLT is thinner in case of suction than injection in both profiles.

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