SMOOTHING PARAMETER AND THE PERFORMANCE OF EXPONENTIALLY WEIGHTED MOVING AVERAGE AND VARIANCE CONTROL SCHEME

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ABSTRACT: *Exponentially weighted moving average and variance (EWMA&V) control charts are regarded as one of the most convenient tools used in monitoring process shifts. Although EWMA&V control charts have been extensively used to monitor quality characteristics, there are few studies concentrating on the effect of different values of smoothing parameter on the control scheme. In this paper, we look at how different values of the smoothing parameter influence the exponentially weighted moving average and variance (EWMA&V) control chart. Exponentially weighted moving average and variance are applied on simulated process with different values of smoothing parameter and the obtained results show that the EWMA&V control chart are very sensitive in detecting shift in production process when the smoothing parameter is small and every shift in the process mean is always preceded by shift in the process variability.*

Keywords; statistical process control, control chart, Exponentially Weighted Moving Average & Variance (EWMA&V), smoothing parameter.

INTRODUCTION

Data smoothing is way of creating an approximating function that attempts to capture important pattern in the data, while leaving out rapid phenomena. In smoothing, the data points of a signal are modified so individual points (presumably because of noise) are reduced, and points that are lower than the adjacent points are increased leading to a smoother signal.

An exponentially weighted moving average is a moving average of past data where each data point is assigned a weight. These weights decrease in an exponentially decaying fashion from present into the remote past, thus the moving average tends to be a reflection of the more recent process performance, because most of the weight is allocated to the most recently collected data. The amount of decrease of the weights is an exponential function of the smoothing parameter, (λ) which can assume values between 0 and 1. Exponentially weighted moving variance is a moving variance of past data that can equally be smoothed.

Exponentially weighted moving averages will gradually depending on the smoothing parameter, move to the new mean of the process if a shift in the mean occurs, while the exponentially weighted moving variances will remain unchanged. If there is a shift in the process variability, the exponentially weighted moving variances will gradually move to the new level while the exponentially weighted moving averages still vary about the process mean.

EWMA &V charts are becoming much better popular among the practitioners because of their superior ability to detect small process shift (Narinderjit Singh 2006). The EWMA and EWMV procedures will usually give tighter process control than the classical quality control schemes such as Shewhart schemes, because EWMA and EWMV procedures give an early indication of process changes, this is consistent with a management philosophy that encourages doing it right the first time(Adekeye2009).

The Shewhart control schemes for mean and range provide information on variability of the quality characteristics, consistency of performance and the mean of the quality characteristics, however Shewhart control scheme are only efficient in detecting large shift value in process mean and process variability. The (EWMA) control schemes can be designed to quickly detect small shifts in the mean of process, (Macgregor and Harris 1990), and exponentially weighted moving variance (EWMV) control scheme is very efficient in detecting small shift in process variability. An exponentially weighted moving average and variance gives improved properties when shift in both process mean and process variability are to be detected.

This work focus on studying the effect of the various values the smoothing parameter can take, ($0 \le \lambda \le 1$) on the performance of Exponentially Weighted Moving Average (EWMA&V) control scheme.

2.0 Methodology: Formulation of the EWMA and the EWMV charts.

For a sequentially recorded observations, which can either be individually observed values (X_t) from the process or sample averages ($\bar{X}_t$), the formulation of the EWMA and the EWMV charts are given below.

The EWMA chart:

$$Z_t = \lambda Q_t + (1-\lambda)Z_{t-1},\ 0 < \lambda \le 1,\ t = 1,2,\ldots,n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1.0)$$

$$CL = \bar{X}$$

$$LCL = \bar{X} - 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \quad (3.0)$$

$$UCL = \bar{X} + 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \quad (4.0)$$

Where: Z_t is the value plotted on the control chart and is a weighted average of all previous plotted values

Z_0 is the estimated process mean and a starting value for the EWMA

λ is a smoothing parameter

Q_t is the sequentially recorded observations (which can either be individually observed values (X_t) from the process or sample averages ($\bar{X}_t$) obtained from a designated sampling plan).

σ is the standard deviation of the observations

The EWMV chart:

$$V_t^2 = \lambda(Q_t - Z_t)^2 + (1-\lambda)V^2_{t-1},\ 0 \le \lambda \le 1, \quad t = 1,2,\ldots,n \qquad (5.0)$$

$$CL = \sigma^2 \qquad (6.0)$$

$$LCL = \sigma^2 - 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \qquad (7.0)$$

$$UCL = \sigma^2 + 3\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \qquad (8.0)$$

Where: V^2_t is exponentially weighted moving variance

V^2_0 is the variance of the individually observed values or the sample averages.

Q_t is the sequentially recorded observations (which can either be individually observed values (X_t) from the process or sample averages ($\bar{X}_t$) obtained from a designated sampling plan).

Z_t is the corresponding EWMA value

λ is EWMV weighting parameter

An algorithm is developed to compute EWMA&V statistics from the observed values or the sample averages.

The algorithm is as follows;

1 Initialize variable

1.1 Set counter = 0

1.2 Set sum of X = 0

1.3 Set mean of X = 0

2 Open “data.txt”

3 Loop while end of file “data.txt” = false

3.1 Set counter = counter + 1

3.2 Read X from file “data.txt”

3.3 Array Data (counter) = X

4 Close file (data.txt)

5 Set counter = 0

6 Loop while counter ≤ upper bound of ArrayData

6.1 Set counter = counter +1

6.2 Set sum of X = sum of X + ArrayData(counter)

7 Mean of X = sum of X/counter

8 Set counter = 0

9 Set sum of X = 0

10 Loop while counter ≤ upper bound of ArrayData

10.1 Set counter = counter + 1

10.2 Set Sum of X = Sum of X + (ArrayData(counter) – Mean of X)**2

11 End loop

12 Set V_0 = Sum of X/(counter -1)

13 Set Z_0 = Mean of X

14 Open “ComputedData.txt”

15 Set Counter = 0

16 Loop while counter ≤ upper-bound of ArrayData

16.1 counter = counter + 1

16.2 EWMA = lamda*ArrayData(counter) + (1- lamda)*Z_0

16.3 EWMV = lamda*(ArrayData(counter)-EWMA)^2 + (1-lamda)*V_0

16.4 Write counter, ArrayData(counter), EWMA, EWMV into file “ComputedData.txt”

16.5 Set Z_0 = EWMA

16.6 Set V_0 = EWMV

17 End loop

18 Close file “ComputedData.txt”

19 Stop.

Choosing The Value Of The Smoothing Parameter (λ)

Various approaches have been proposed for choosing the value of λ. Lucas and Saccucci (1990), states that the choice "can be left to the judgment of the quality control analyst" and points out that the smaller the value of λ "the greater the influence of the historical data". Furthermore, smaller values of λ should be used if early recognition of smaller shifts is desired.

If λ = 1, the EWMA control chart degrades to the usual Shewhart $\overline{X}$ chart. Thus the larger the value of λ the more the weight assigned to the recent data and the shallow the memory of the EWMA&V. The smaller the value of λ, the more the weights given to older data and the deeper the memory of the EWMA&V. In order to see the effect of the smoothing parameter **(λ)** on the control charts, different value λ was applied on the simulated process in this study.

Simulation Process

Simulation is the use of mathematical model to recreate a situation, often repeatedly, so that the likelihood of various outcomes can be more accurately estimated. Simulation has also been defined as the use of a system model that has the designed characteristics of reality in order to produce the essence of actual operation by Gupta and Hira(2012).

We simulate a process that is normally distributed, the model is given as;

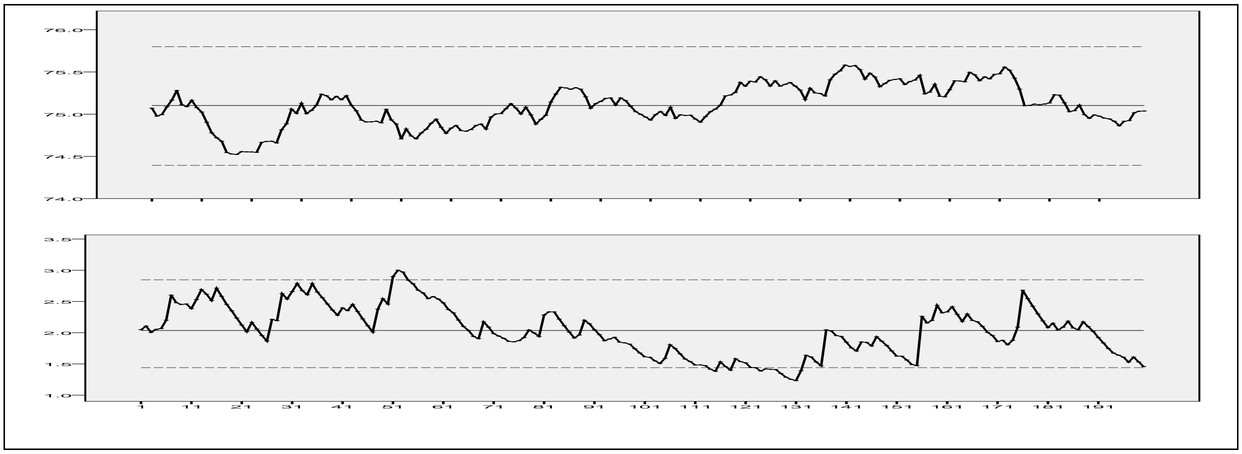
$$f_{(x)} = f_{(x:\mu,\sigma^2)} = \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}, -\infty \le x \le \infty, \mu \ge 0, \sigma \ge 0 \quad (9.0)$$

$\sigma^2 = 2.144$ and $\mu = 75$ are set to simulate 200 cases.

3.0 Results

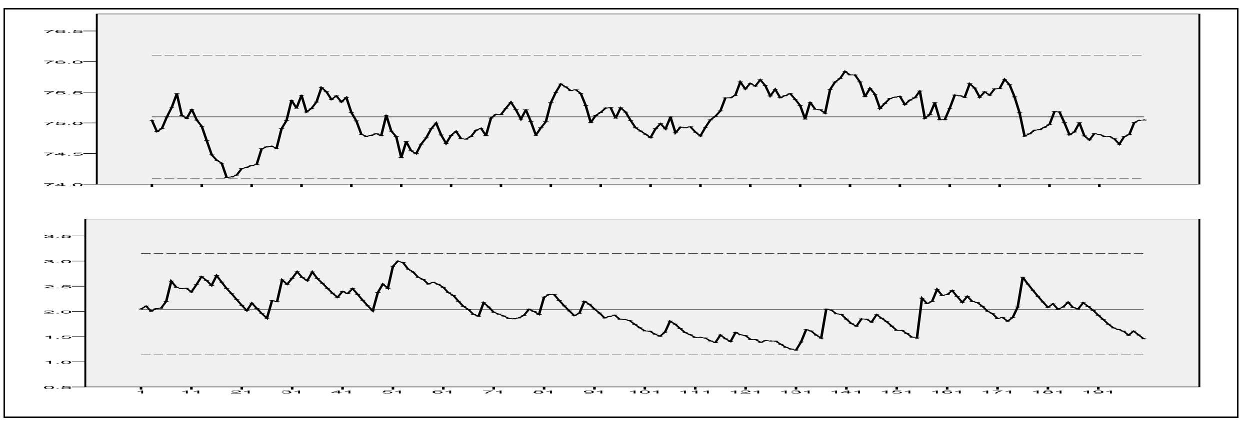
The EWMA&V control scheme is implemented on the simulated process with variance equal 2.144 using six different values for the smoothing parameter (λ), the control charts are presented below in Figure 1 to Figure 6.

Figure 1: EWMA&V control charts for simulated data with Var = 2.144 & λ = 0.05



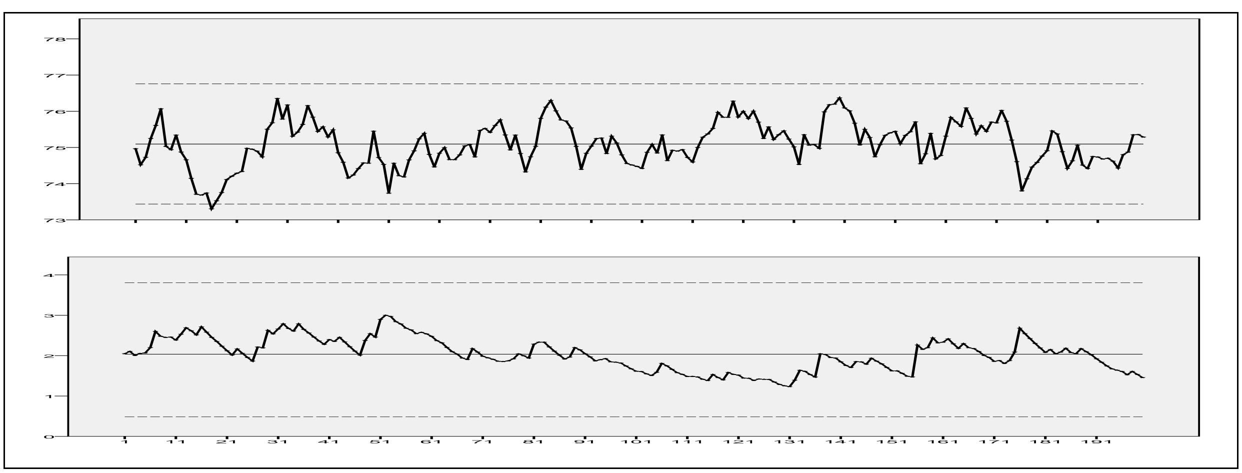
SAMPLING INTERVAL

Figure 2: EWMA&V control charts for simulated data with Var = 2.144 & λ = 0.1



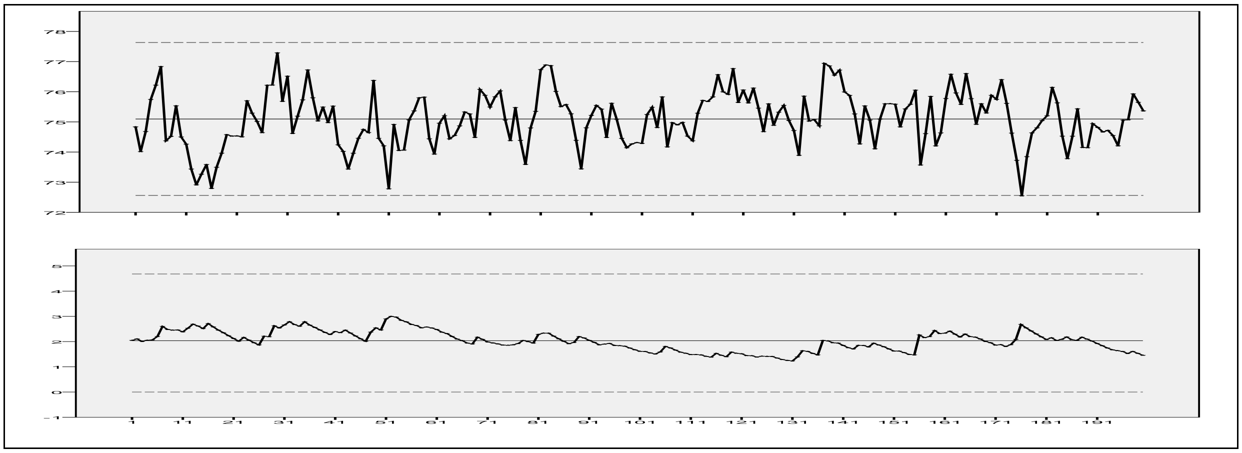
SAMPLING INTERVAL

Figure 3: EWMA&V control charts for simulated data with Var = 2.144 & λ = 0.25



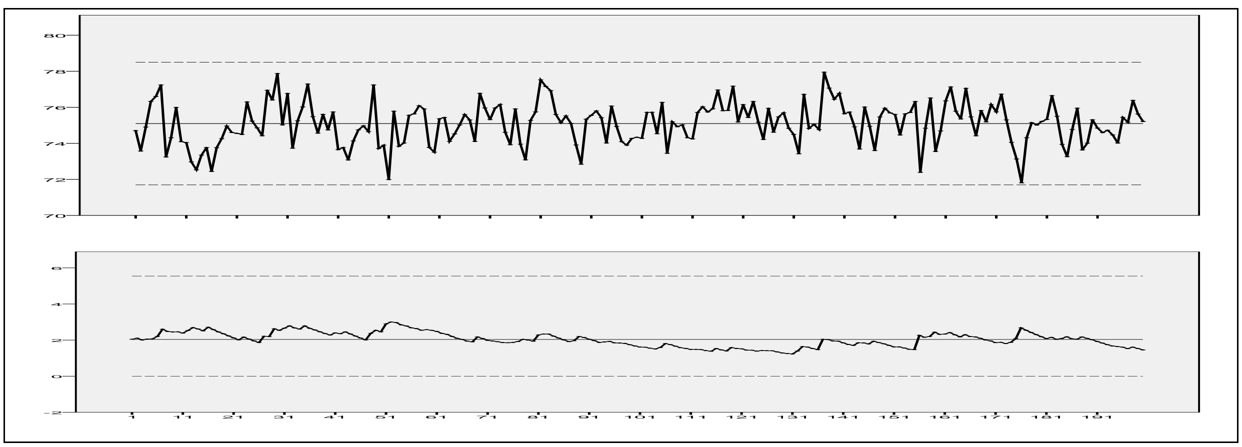
SAMPLING INTERVAL

Figure 4: EWMA&V control charts for simulated data with Var = 2.144 & λ = 0.5



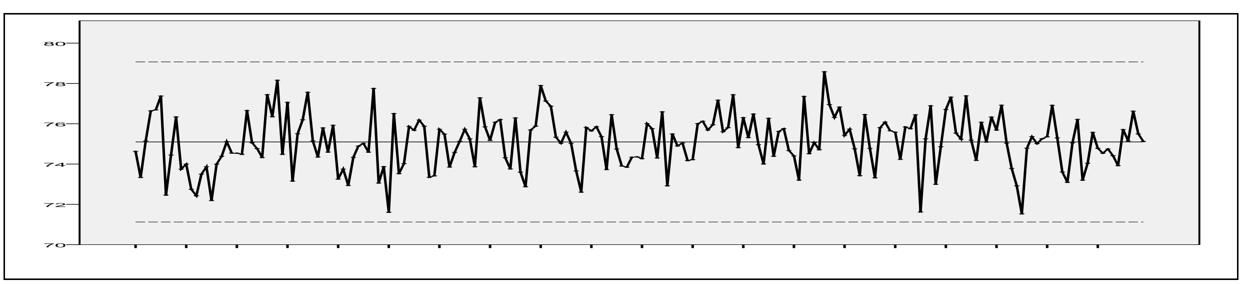
SAMPLING INTERVAL

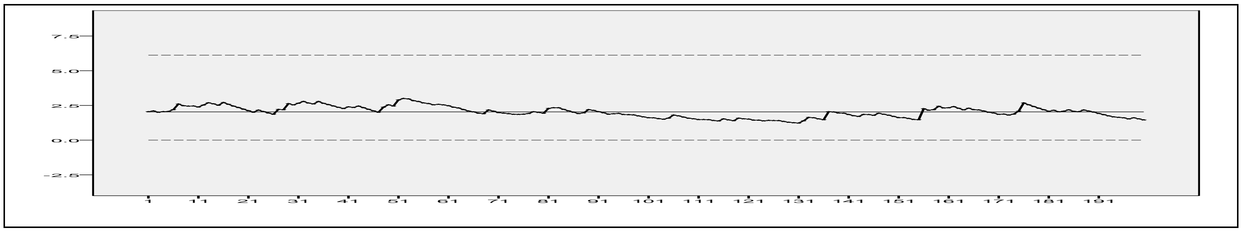
Figure 5: EWMA&V control charts for simulated data with Var = 2.144 & λ = 0.75



SAMPLING INTERVAL

Figure 6: EWMA&V control charts for simulated data with Var = 2.144 & λ = 0.9





SAMPLING INTERVAL

4.0 Discussion of Results

The results of the application of the EWMA&V control scheme on the simulated process using small value of smoothing parameter showed a control chart that is highly sensitive to shift as evident in Figure. 1 and 2, where the smoothing parameter had the values 0.05 and 0.1 respectively. The process variability was clearly out of control around the 40th and 120th sample when the smoothing parameter is 0.05, however the process average was within the control limits, but having several samples above and below the control line is an indication of process shift. The process becomes stable as the value of smoothing parameter increases as evident in Figure. 5 and 6 where the smoothing parameter had values 0.75 and 0.9 respectively. Also the control charts show that shift in the process average is always preceded by a shift in the process variability as evident in Figure 1, where the process variability first shifted away around the sixth sample and the process average started shifting around the twelfth sample, also the process variability shifted around the 95th sample before the process average started shifting around 115th sample. Generally speaking, smaller value of the smoothing parameter seems more sensitive in recognizing shift in simulated process quality characteristics.

4.1 Conclusion

The EWMA&V control scheme has been proposed as alternative means of monitoring process variation, analyzed by varying the smoothing parameter. The results show that the combined EWMA&V control chart is very sensitive to small shift in the process mean and process variability when the value of the smoothing parameter is small.

REFERENCES

Adekeye(2009): *An Introduction to Industrial Statistics(Concepts and Practice).* Revised Ed. Adewumi Printing and Publishing House, Ilorin, Nigeria.

Adekeye(2012): Modified Simple Robust Control Chart Based On Median Absolute Deviation. *Transnational Journal of Science and Technology,May 2012 edition vol.2,No 4*

Gupta, P.K & Hira, D. S(2012); *Operational Research*. S. Chand & Company Ltd. New Delhi.

Lucas, J.M. & Saccucci, M.S. (1990); Exponentially Weighted Moving Average Control Schemes: Properties and enhancement. *Journal of Technometrics.* Vol. 32, No. 1, pp.1-12.

Macgregor, J.F. and Harris T.J.(1990); Discussion of EWMA Control Schemes. *Journal of Technometrics.* Vol. 32, pp.23-26

Montgomery, D.G. and Masterangelo, C.M.(1991); *Some Statistical Quality Control.* John Wiley and Sons, New York, NY

Montgomery, Douglas G. (2001); *Introduction to Statistical Quality control* (4th Edition). John Wiley and Sons Inc., Canada

Roberts, S. W.(1959):"Control Chart Test Based on Geometric Moving Average " *Technometrics.* Vol. 42(1), pp. 97-102.

Shewhart, W.A.(1931): *Economic Control of Quality of Manufactured Product*. D. Van Nostrand, New York, USA.

Stephen, B. V and Max, D. Morris(2011): Statistical Quality Assurance Methods f nds XSor Engineers.