AN APPLICATION OF MULTIVARIATE GAUSSIAN DISCRIMINANT CLASSIFIERS TO THE DIRECTION OF INFLATION RATE IN NIGERIA

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ABSTRACT: *In this research work, we have considered two different statistical learning techniques for classification such as Linear Discriminant Analysis (LDA), and Quadratic Discriminant Analysis (QDA) in modeling and forecasting the direction of inflation rate in Nigeria between 2004 and 2016 using two macroeconomic variables such as the nominal exchange rate and crude oil price. The direction of inflation rate was classified into k = 3 levels i.e. "Down", "Unchanged" and "Up". Analysis revealed that during the period under review, inflation rate rose in 86 months, decline in 66 months and remained unchanged for 4 months out of the 156 months with prior probabilities of 0.4231, 0.0256 and 0.5513 respectively. LDA on the direction of inflation rate was based on the assumption that the two predictors are multivariate normally distributed with common covariance matrix. LDA classifier was based on two linear discriminant functions. The between- and within-class standard deviation ratios for the two linear discriminant variables generated are 2.2299 and 0.0826 respectively. The LDA output shows that whilst about 53.20 per cent of the training observations were correctly classified into "Down", "Unchanged" and "Up" categories, 46.80 per cent were misclassified. Hence the accuracy of the LDA classifier is 53.20 per cent. In addition to the assumption of the vector of expanatory variables (i.e crude oil price, exchange rate and interest rate) coming from a multivariate Gaussian or normal distribution, QDA estimates a separate covariance matrix for each class. The QDA output shows that 40.38 per cent of the training observations were incorrectly classified while 59.62 per cent of the observations are correctly classified. Based on a specific economic scenario whereby a test dataset of observations between 2015 and 2016 was used, the performances of the classifiers revealed a test error rate of 95.83 per cent for the LDA and 8.33 per cent test error rate for QDA.* ***R*** *programming language packages were employed throughout the analysis.*

Keywords: LDA, QDA, Direction of inflation rate, crude oil price, exchange rate and test error rate.

1.0 Introduction

Understanding the direction of inflation in an economy is an important challenge to most governments all over the world. In Nigeria, researchers and policy makers have made several attempts to know and understand the direction of inflation rate and its dynamics for economic and financial policy decision-making. Inflation is characterized by a fall in the value of a country's currency and a rise in her exchange rate with other nation's currencies.

Since mid-1960s, inflation has become so serious and contentious problems in Nigeria. Several policies were introduced to control inflation in the economy and despite these policies, inflationary trends continue to increase. Government needs to control high levels of unpredictable inflation since it can severally disrupt the economy. The tools governments normally use include monetary policy (i.e. increase or decrease in the money supply and interest rate), fiscal policy (changes in the amount of taxes and government spending) and various controls on prices, tariffs, etc. Many nations choose monetary policy as their primary tool since it has proven to be effective, less disruptive to the market operations and easier and quicker to implement since adjusting the money supply does not require legislative approval as would be for instance changing the tax structure. Though inflation rate is not new in the

Nigerian economic history, the recent rates of inflation have been a cause of great concern to many. In Nigeria, inflation was curtailed effectively in the late 1990s when the country recorded single digit of 8.5% and 6.6% in 1997 and 1999 respectively. Inflation entered the two-digit range between 2001 and 2004 when 18.9%, 13.2%, 14% and 15% were recorded in 2001, 2002, 2003 and 2004 respectively.

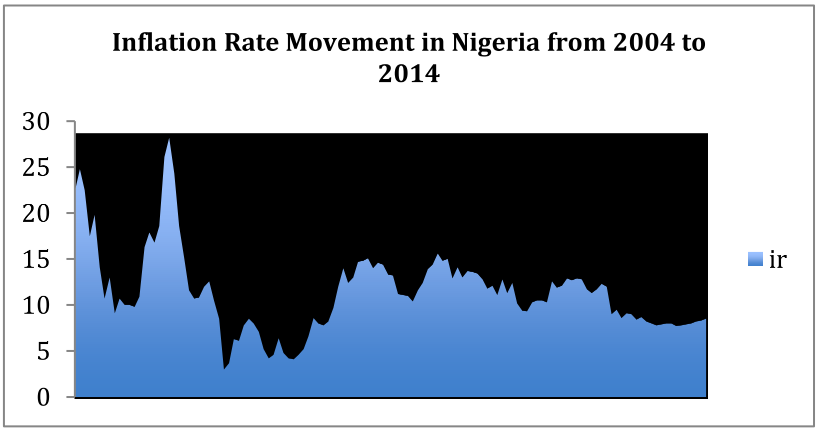


Figure 1 *Plot showing the direction of inflation rate in Nigeria from 2004 to 2014.*

Consumer prices in Nigeria jumped to 11.4 per cent in February 2016, following a 9.6 percent increase in the previous month and way above market expectations of a 10 per cent rise, it is the highest reading since December 2012, as a falling naira keeps pushing import prices up, mainly food. Consumer prices increased to 12.8 per cent in March 2016, it was the highest figure since July 2012, mainly boosted by food prices. Consumer prices in Nigeria surged 13.7 per cent in April 2016, compared to a 12.8 per cent jump in March. The inflation rate accelerated for the third straight month, reaching the highest since August 2010. Consumer prices in Nigeria jumped 15.6 per cent in May 2016. It was the highest reading in more than 6 years, as cost of food, housing, utilities and transport surged mostly due to 67 per cent increase in gasoline prices. Nigeria struggles with rising import cost due to a weak naira after lower oil prices led to a decline in foreign reserves and as oil production reached a historical low in May after Niger Delta Avengers attacked Chevron oil facilities. On a monthly basis, consumer prices went up 2.75 per cent. Consumer prices in Nigeria surged by 16.5 per cent in June 2016, following a 15.6 percent jump in May. Figures came above market expectations of 16.3 per cent increase, mostly driven by sharper rise in cost of food, housing and utilities. On a monthly basis, consumer prices went up 1.7 per cent compared to a 2.75 per cent rise in the preceding month. Inflation Rate in Nigeria averaged 12.15 per cent from 1996 until 2016, reaching an all-time high of 47.56 per cent in January 1996 and a record low of -2.49 per cent in January 2000. The National Bureau of Statistics, Nigeria, reports inflation rate in Nigeria on monthly basis. Currently, the monthly computation of inflation rate by the government agency centers around prices of 740 commodities and services which include consumer price indices from major markets around the country. The commodities and relative prices for each item are divided by the total weight of the items in that category multiplied by 100 to give the corresponding CPI. CPI is classified into four major indexes according to the urban or rural classification sector for each of the 36 states and the Federal Capital Territory.

Many scholars from different perspective in terms of its supposed causes have defined inflation. It is a measure used in determining the relative changes in the prices of commodities and services over a period of time and it is an economic indicator that has direct effect on the state of the economy. Furthermore, it can be said to be a sustained upward trend in general price level and not the price of only one or two goods. It is not the high prices but the price level that constitute inflation. There are several types of inflation as developed by many experts according to their different categories but this study will only be limiting itself to the direction of inflation. Some of the major questions researchers ask "Does Inflation rate have direct or indirect relationship with government fiscal policy and how does it impact government budget?" Inflation rate and government fiscal policy have direct relationship. Fiscal policy is based on the theories of British economist John Maynard Keynes. Also known as Keynesian economics, this theory basically states that governments can influence macroeconomic productivity levels by increasing or decreasing tax levels and public spending. This influence in turn, curbs inflation, increases employment and maintains a healthy value of money. Fiscal policy is the spending of money by government, and if there are scarcities in labor and capital (low unemployment), such spending can cause inflation by driving up prices for goods which cannot be produced in greater quantities due to unavailable labor or capital. The primary objective of fiscal policy is to maintain the price stability, economic growth and employment empowerment. Hence an appropriate fiscal policy can help in combating rising inflation rates. Inflation in Nigeria has been accelerating since 1960s and has become a major concern to the government. Understanding and forecasting the behavior and direction of inflation plays a central role in the conduct of economy activities for decision or policy makers. It is thus important to accurately predict the factors that affect the economy in other to know the future dynamics and trends of the inflation. To conduct this forecast, most researchers take a number of economic variables into consideration also known as macro-economic variables and different statistical methods are applied. Several macroeconomic variables influence the direction of inflation rates in any economy. These include interest rates, exchange rates, unemployment rates, crude oil price, government spending interms of capital and recurrent expenditures, stock market liquidity, depth of financial development and credit to the private sector amongst other variables. However, this research features prominently the crude oil price, interest rate and exchange rate. The selection of these three variables is to avoid the problem usually associated with over-fitting when the number of predictors is large. We propose two alternative methods to traditional econometric methods; the linear discriminant analysis and quadratic discriminant analysis. These methods were employed to achieve the research set objectives, which include

- ➢ Modeling the direction of inflation rate in Nigeria using some selected macroeconomic variables.
- ➢ Comparing the performances of the linear and quadratic discriminant analysis models in order to select the best for forecasting the direction of inflation rate in Nigeria.
- ➢ Forecasting the direction of inflation rate using the preferred model.

1.0 Related Works

The maintenance of price stability is one of the major macroeconomic challenges facing the Nigerian government. This elusive factor is also referred to as inflation, which is defined by economists as a continuous, rise in general level of prices. Furthermore, inflation is a persistent and appreciable rise in the general level of prices (Jhingan, 2002). Not every rise in the price level is termed inflation. Therefore, for a rise in the general price level to be considered inflation, such a rise must be constant. Some attempts have been made to study the character of inflation in Nigeria.

Prior to this research, many works have looked into and developed models for predicting inflation rates using macroeconomic variables. This research work prefers to look at the relationship using the information on the direction of inflation rates in Nigeria. This is important because, these past works though came up with models using appropriate statistical methods have not predicted precisely the inflation rate due to a problems associated with model bias and large variance of the error terms and excessive flexibility. Some of these research works include the works of Buhari (1989) who concluded that, inflation in an economy, can be disadvantageous to those people on fixed incomes such as pensioners and students, debtors gains at the detriment of the lenders, deterioration of standard of living due to fall in real income, savings and capital formation can be discourage, inflationary situation tends to discourage efficiency, it may lead to increase in investment as a result of rise in prices, inflation may lead to industrial unrest as trade unions mount pressure for higher wages in order to sustain the existing standard of living, inflation may worsen the balance of payments of a country. Adamson (2000) opined that inflation retards economic growth and can only negate the development efforts of the country. Then the frustration experienced by the class becomes a breeding ground for chaos. Inflation is a situation of a rising general price level of broad spectrum of goods and services over a long period of time Balami (2006). It is measured as the rate of increase in the general price level over a specific period of time. According to Asogu (1991) undertook an empirical investigation based on ten different specifications that covered monetary, structural and open economy aspects of inflation. Variables used in the regressions include money supply and its lagged value, real GDP and its lagged value, aggregate domestic credit to the economy and its lagged value, government expenditure and its lagged value. The study therefore confirms the importance of the structural character of the economy, open economy and monetary aspects of inflationary trend in Nigeria.

Several macroeconomic variables influence the direction of inflation rates in any economy. These include interest rates, exchange rates, unemployment rates, crude oil price, government spending interms of capital and recurrent expenditures, stock market liquidity, depth of financial development, credit to the private sector amongst other variables. However, this research features prominently the crude oil price, interest rate, exchange rate and unemployment rate. The selection of these variables is to avoid the problem usually associated with over-fitting when the number of predictors is large.

1.1 Inflation rate and crude oil price

The oil industry is very important to the Nigerian economy, as it is evident that export of petroleum earns valuable foreign exchange in Nigeria. Nigeria is highly vulnerable to fluctuations in the international oil market despite being the 6th largest producer of oil in the world. This is given the fragile nature of the Nigerian macro economy and the heavy dependence on crude oil proceeds. Evidently, it is believed that the dominance of petroleum in Nigerian economy resulted in significant instability in the economy, which as a result makes price instability of oil products to be more prevalent in Nigeria than other countries. Runl (2010) asserted that people say Nigeria is dominated by oil and they are right because Nigeria seems to be exporting nothing but oil. The government revenues are so dependent on oil, which has been managed quite protectively. Arinze (2011) in his work "the impact of oil price on the Nigerian economy," he asserted that there is a direct relationship between fuel price increase and inflation rate in Nigeria and also recommended that more resources should be tapped to diversify the economy.

1.2 Inflation rate and exchange rate

Exchange rate movement and its impact on the Nigerian economy have received some attention over the years. This macroeconomic variable plays a major role in the stability of any economy. Increased exchange rate directly affects the prices of imported commodities and an increase in the price of imported goods and services contributes directly to increase in inflation (CBN, 2008). Consequently assessing the relationship among monetary policy, exchange rate and inflation rate is relating because an understanding of the connection between these variables is prerequisite for the successful conducting and adoption of inflation targeting. Inflation is a monetary phenomenon which is usually measured by changes in Consumer Price Index (CPI). When a persistent increase occurs in the level of prices that lowers the purchasing power of money. Productivity of a country is enhanced through stable currency by keeping costs and prices of goods and services as low as possible.

Monetary policy has always been seen as a fundamental instrument over the years for the attainment of macroeconomic stability, often viewed as prerequisite to achieving sustainable output growth. The legal backing for Monetary Policy by the bank derives from the various statues of the bank such as CBN Act of 1958 as amended in CBN Decree No 24 of 1991, CBN Decree Amendment 1993, No 3 of 1997, No 4 of 1997, No 37 of 1998,1999 and CBN Act 2007. Section 12 sub-sections (1) To (5) CBN Act of 2007 (Amended) "In order to facilitate the attainment of price stability and to support the economic policy of the Federal Government". Thus, in the pursuit of macroeconomic stability, the managers of monetary policy have often set targets on intermediate variables, which include the short term interest rate, growth of money supply and exchange rate. Among these intermediate variables of monetary, the exchange rate is argued to have a greater influence on the economy through its effect on the value of domestic currency, domestic inflation, the external sector, macroeconomic credibility, capital flows and financial stability. Increased exchange rate directly affects the prices of imported commodities and an increase in the price of imported goods and services contributes directly to increase in inflation (CBN, 2008). Chimobi and Uche (2010) examined the relationship between money, inflation and output in Nigeria covering the period of 1970 to 2005. They suggests that monetary stability can contribute towards price stability in the Nigerian economy since the variation in price level is mainly caused by money supply, the study concluded that inflation in Nigeria is to a much extent a monetary phenomenon. The research work conducted by Folawewo and Oshinubi (2006) examined the efficacy of monetary policy in controlling inflation and exchange rate instability in Nigeria covering the period of 1980 to 2000 and employing the rational expectation framework and time series analysis. The study observed that the effort of monetary policy at influencing the finance of government fiscal deficit through the determination of the inflation-tax rate affects both the rate of inflation and the real exchange rate, thereby causing volatility in their rates. The study found that inflation affects volatility of its own rate as well as the rate of real exchange and the study concluded that monetary policy should be set in such a way that the objective it is to achieve is well defined.

Calvo, Guillermo, Reinhart and Veigh (1994) identified correlations between the temporary components of inflation and real exchange rate in Brazil, Chile and Colombia. Odedokun (1996), Canneti and Greene (1991), Egwaighide, Chete and Falokun (1994) and London (1989) reached similar conclusion for some selected African countries. It was explicitly concluded that exchange rate devaluation is a major factor for the upsurge of inflation. The study by Okhira and Saliu (2008) which evaluated the impact of exchange rate on major macroeconomic variables such as inflation rate, government expenditure, money supply exchange rate, oil revenue and inflation in Nigeria concluded by recommending that the policy maker should try to cushion the effect of inflation on the economy when the need arises so that rise in exchange rate will not lead to inflationary pressure in the short run even though inflation and exchange rate have no long term relationship, short term relationship seems to exist.

2.0 Methods, Estimation and Results

Selecting the appropriate statistical technique for empirical analysis involves understanding the type of data and data collection. Here, secondary data was collected from major government agencies and other reliable sources. We adopted and compare the results of two classification techniques such as Linear Discriminant Analysis (LDA) and Quadratic Discriminant Analysis (QDA). The choice of these two statistical methods is due largely to the fact that our response variable, direction of inflation rate is a qualitative/categorical data with $k \geq 2$ classes and each of the predictors in each k classes is normally distributed. The predictor variables, crude oil price and nominal exchange rate are both quantitative data and continuous in nature. However, these variables can assume categorical form just like the response variable. Using these statistical learning methods of analysis, we intend to model and forecast the direction of inflation rate in Nigerian. As earlier mentioned we define direction of Inflation rates (Direction) as our response variable Y while Crude oil price (*cp*) and Exchange Rate (*er*) are predictor variables represented by the vector X. We assume that the X = (crude oil price, exchange rate) is drawn from a multivariate Gaussian (or multivariate normal) distribution with class specific mean vector and a common covariance matrix.

Discriminant analysis is a powerfully less direct approach classification tool in which $k \geq 2$ classes or clusters are known *a priori* and one or more new observations are classified into one of the known k classes based on the measured characteristics. By applying the Bayes' theorem for classification, we model the conditional distribution of the response $\boldsymbol{Y}$ (Direction of inflation rate), given the predictor(s) $\boldsymbol{X}$ (crude oil price ***cp*** and exchange rate, ***er***) employing two alternatives and less direct approaches to estimating these probabilities such as discriminant analyses inform of linear and quadratic discriminant analyses. In these alternative approaches, we model the distribution of the predictors $\boldsymbol{X}$ separately in each of the response classes and then use Bayes' theorem of (1) to flip these around into estimates for $\Pr(\mathbf{Y}=k|X=\boldsymbol{x})$. The Baye's rule is a powerful tool for calculating the posterior probabilities. It includes a mechanism for weighting the strength of evidence in the likelihood part of equation (1). Therefore, Bayesian flip requires the knowledge of or estimation of prior probabilities. For simplicity g = 2 predictors and k = 3 levels. We use the "***lda***" and "***qda***" functions found in the **MASS** library of the ***R*** Studio throughout the research work.

The monthly data on crude oil price, nominal exchange rate, inflation rate and the direction of inflation rate in Nigeria from 2004 to 2016 were obtained from Central Bank of Nigeria database. The data on crude oil price is in US Dollars per barrel, nominal exchange rate is in ₦/$US Dollar, inflation rate is a year on year inflation rate. We transformed the inflation rate into a categorical data with three levels (i.e. "***Up***", "***Unchanged***", and "***Down***"), which are considered to be the responses in our discriminant models.

2.1 Linear Discriminant Analysis (LDA) classifier

We classified an observation of $\boldsymbol{X}$ into one of k = 3 classes (i.e. "***Down***", "***Unchanged***" and "***Up***"). In other words, the qualitative response variable Direction can take on only three possible distinct and unordered values. We let π_k represent the overall or prior probability that a randomly selected observation comes from the k^{th} class (***Down***, ***Unchanged*** and ***Up***) which is also the probability that a given observation is associated with the k^{th} category of the response variable $\boldsymbol{Y}$ (i.e Direction of Inflation rate). Also, we let $f_k(\boldsymbol{X}) \equiv \Pr(X = \boldsymbol{x}|\mathbf{Y} = k)$ denote the density function of $\boldsymbol{X}$ for an observation that comes from the k^{th} class. In other words, $f_k(\boldsymbol{X})$ is relatively large if there is a high probability that an observation in the k^{th} class has $\boldsymbol{X} \approx x$, and $f_k(\boldsymbol{X})$ is small if it is very unlikely that an observation in the k^{th} class has $\boldsymbol{X} \approx x$. In order to implement linear discriminant analysis effectively, we estimated $\pi_{k,}$ μ_k and common covariance matrix Σ. Then Bayes' theorem states that

$$\Pr(Y = k / X = x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{K} \pi_l f_l(x)} \quad (1)$$

Firstly, we estimated π_k by simply computing the fraction of the random sample of n = 156 ***Y's*** from the population that belong to the k^{th} class and $f_k(\boldsymbol{X})$ by assuming some simple forms for the densities. We refer to $Pr(\mathrm{Y=k/X}=\boldsymbol{x})$ or simply $p_k(\boldsymbol{x})$ in (1) as the posterior probability that an observation $\boldsymbol{X} = x$ belongs to the k^{th} class. That is, it is the probability that the observation belongs to the k^{th} class, given the predictor value for that observation. The multivariate Gaussian density is defined as

$$f(x) = \frac{1}{(2\pi)^{p/2}|\Sigma|^{1/2}} \exp(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)) \qquad (2)$$

By inserting the density function for the k^{th} class, $f_k(X = x)$ into (1), reveals that Bayes classifier assigns an observation $X = x$ to the class for which the linear function

$$\delta_k(x) = x^T \Sigma^{-1}\mu_k - \frac{1}{2}\mu_k{}^T \Sigma^{-1}\mu_k + \log \pi_k \qquad (3)$$

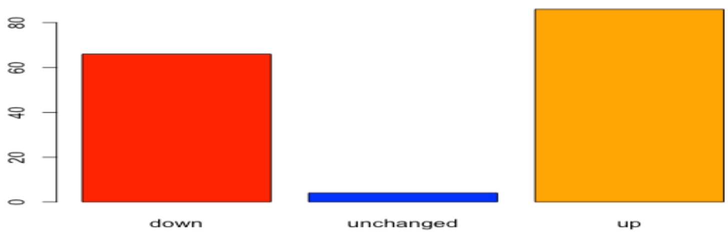
is largest. (3) is a linear function of x which implies that the LDA decision rule depends on x only through a linear combination of its elements. In order to assign a new observation X = x, the LDA plugs these estimates into (3) and classifies to the class for which estimate of (3) is largest. Table 1 is the table of prior probabilities for the 3 classes of $\boldsymbol{Y}$ = Direction.

Table 1: Prior Probabilities (π_k) of Classes

"***Down***", k=1, n_1= 66	"***Unchanged***", k=2, n_2= 4	"***Up***", k=3, n_3= 86
0.4231	0.0256	0.5513

Source*: Personal Computation using* ***R*** *Studio*

Figure 1: Bar plot on Direction of Inflation rate in Nigeria (2004 to 2016)



Source*: Personal Computation using* ***R*** *Studio*

In order to calculate the posterior probabilities $p_k(\boldsymbol{x})$ employing (1), assumptions were made about the probability densities f_k i.e. they follow the multivariate Gaussian distribution with mean μ_k and common covariance $\boldsymbol{\Sigma}$ that is, there is a shared variance term across all three classes. We calculated the discriminant functions d_k which are linear functions of $\boldsymbol{x}$ using estimates of the population mean μ_k and population covariance $\boldsymbol{\Sigma}$.

Table 2: Estimated group mean of classes μ_k for LDA

	Independent variables (x)	
Class	Crude oil price (***cp***)	Exchange rate (***er***)
Down	81.0994	144.2058
Unchanged	78.0650	160.7075
Up	75.8406	162.5884

Source*: Personal Computation using* ***R*** *Studio*

We estimated these two parameters using the formulas:

$$\hat{\mu}_k = \frac{1}{n_k}\sum_{i: y_i = k} x_i$$

$$\Sigma_k = \frac{1}{n-k}\sum_{k=1}^{K} \sum_{i:y_i=k}(x_i-\hat{\mu}_k)^2$$

where $n = 156$ is the total number of training observations, and n_k is the number of training observations in the k^{th} class. The estimate for μ is simply the average of all the training observations from the k^{th} class, while $\boldsymbol{\Sigma}$ can be seen as a weighted average of the sample variances for each of the k classes. Since we do not have knowledge of the class membership probabilities π_1, π_2 and π_3, linear discriminant analysis (LDA) estimates each prior probability π_1, π_2 and π_3 using the proportion of the observations that belong to the k^{th} class.

Table 3: Coefficients of Linear Discriminants

Linear Discriminant	**LD1**	**LD2**
Crude oil price (***cp***)	-0.0059	-0.0353
Exchange rate (***er***)	0.0263	-0.00999
Between-class standard deviation	2.2299	
Within-class standard deviation	0.0826	

***Source**: Personal Computation using **R** Studio*

The LDA classifier plug the estimates of the population mean μ_k and covariance $\boldsymbol{\Sigma}$ (obtained through maximization of the Rayleigh quotient) into the discriminant function of (3) using the Fisher's approach $LD = a^T X$ where the between-class covariance of LD is $a^T B_a$ and the within-class covariance of LD is $a^T W_a$. Therefore, we $\max(\frac{a^T B_a}{a^T W_a})$ by sphering the data such that the predictors have the identity as their within-class covariance matrix. This implies that we only need to maximize $a'\boldsymbol{B}_a$ subject to $||\boldsymbol{a}|| = 1$ which we solved by taking $\boldsymbol{a}$ to be the eigenvector of $\boldsymbol{B}$ corresponding to the largest eigenvalue. Hence, $\boldsymbol{a}$ is unique up to a change of sign unless there are multiple eigenvalues.

$$B = \frac{(IM - 1\bar{x})^T (IM - 1\bar{x})}{k-1}$$

$$W = \frac{(x - IM)^T (x - IM)}{n-k}$$

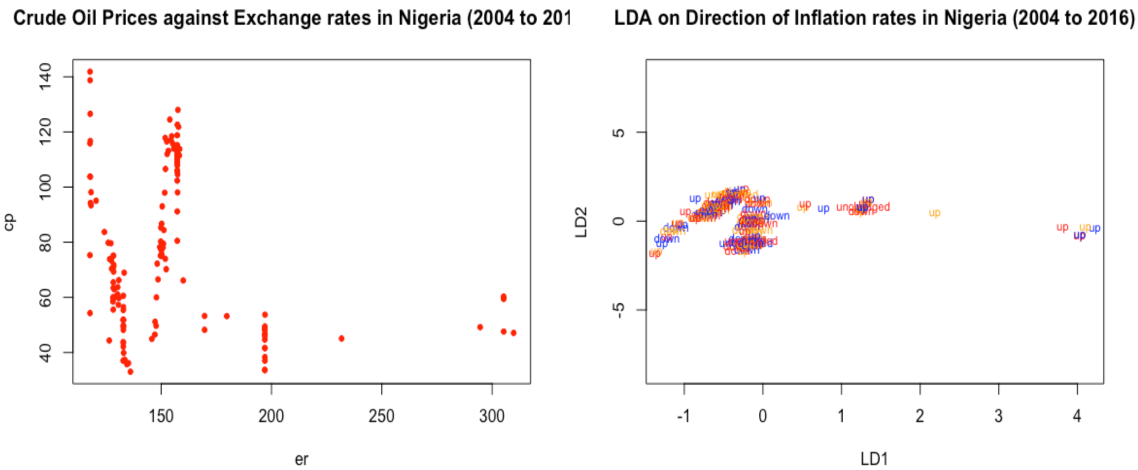
I is the n x k matrix of class indicator variables so that $I_{ij} = 1$ if and only if observation i is classified into class j. M is the k x g matrix of class means, $1x$ is the n x g matrix where each row is formed by x and k is the total number of classes. LDA classifier aims to maximize the between-class covariance while at the same time minimizing the within-class covariance.

An observation $X = \boldsymbol{x}$ ws assigned to the class for which the linear discriminant scores given by

$$LD1 = -0.0059cp + 0.0263er \quad \text{and} \quad LD2 = -0.0353cp - 0.00999er$$

is largest. If these values are large the LDA will predict an "***Up***" in the direction of inflation rate and "***Down***" otherwise.

Figure 2: Classification based on the LDA Classifier



Source: *Personal input using* ***R*** *Studio*

Plot of the linear discriminants by computing $LD1 = -0.0059cp + 0.0263er$ and $LD2 = -0.0353cp - 0.00999er$ for each of the training observations in our Inflation Rates data. The plot shows that the classes are not well separated especially for negative values of linear discriminant scores of *LD1* and *LD2*. Furthermore, discriminant scores between 1 and 2 of *LD2* contain overlapping classes indicating high within-class variance being retained in our data.

2.2 Estimating Error of Misclassification for the LDA classifier

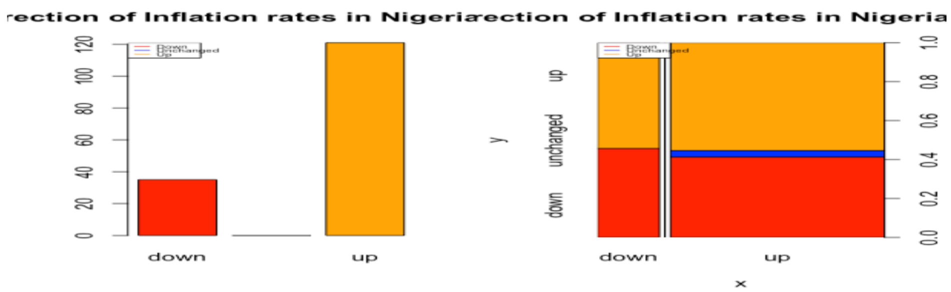
The confusion matrix, which is a confusion table, describes how the discriminant function will classify each observation in the data set. The confusion table for LDA classifier on our data takes the form.

Table 4: Confusion Table on LDA on Direction of Inflation rate in Nigeria

CLASSIFIED AS	**TRUTH**			
	Down	*Unchanged*	*Up*	**Total**
Down	16	0	19	35
Unchanged	0	0	0	0
Up	50	4	67	121
Total	66	4	86	**156**

Source: *Personal computation using* ***R*** *Studio*

Figure 3: LDA Classification using the class priors



Source: *Personal input using* ***R*** *Studio*

From Table 4 above, the number of training observations correctly classified are 83, while 73 observation were misclassified. Hence the probability of misclassification is 0.4680. Consequently, the training error rate for the LDA classifier is 46.80 per cent.

*2.3 Quadratic Discriminant Analysis (**QDA**) Classifier*

LDA assumes that the observations within each class are drawn from a multivariate normal or Gaussian distribution with a class-specific mean vector and a covariance matrix that is common to all k classes. Similar to LDA, the QDA classifier results from assuming that the observations from each class are drawn from a normal distribution, and substituting estimates for the parameters into Bayes' theorem in (1) in order to perform prediction. However, unlike LDA, QDA assumes that each class has its own covariance matrix. That is, it assumes that an observation from the k^{th} class is of the form $X \sim N(\mu_k, \boldsymbol{\Sigma}_k)$, where $\boldsymbol{\Sigma}_k$ is a covariance matrix for the k^{th} class. Under this assumption, the Bayes classifier assigns an observation X = x to the class for which

$$\begin{aligned}\delta_k(x) &= -\frac{1}{2}(x-\mu_k)^T \Sigma_k^{-1}(x-\mu_k) - \frac{1}{2}\log|\Sigma_k| + \log\pi_k \\ &= -\frac{1}{2}x^T \Sigma_k^{-1} x + x^T \Sigma_k^{-1}\mu_k - \frac{1}{2}\mu_k^T \Sigma_k^{-1}\mu_k - \frac{1}{2}\log|\Sigma_k| + \log\pi_k\end{aligned} \quad (4)$$

is largest. Therefore QDA classifier involves inserting estimates for $\sum_k$, μ_k and π_k into (4) and subsequently assigning an observation $X = x$ to the class for which this quantity is largest. Unlike the linear discriminants in LDA, the quantity x appears as a quadratic function in (4). Hence the term QDA.

Table 5: Estimated group mean of classes μ_k for QDA classifier

	Independent variables (x)	
Class	Crude oil price (***cp***)	Exchange rate (***er***)
"Down"	81.0994	144.2058
"Unchanged"	78.0650	160.7075
"Up"	75.8406	162.5884

***Source**: Personal Computation using **R** Studio*

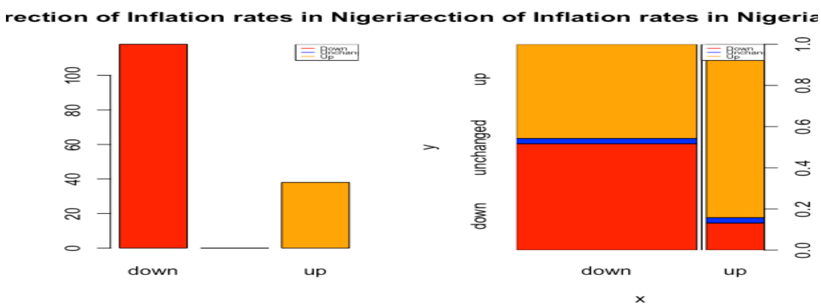
All natural logs are used in this function. The confusion table for the QDA classifier on Direction of Inflation rate in Nigeria is shown in Table 6 below:

Table 6: Confusion Table on QDA classifier on Direction of Inflation rate in Nigeria

CLASSIFIED AS	**TRUTH**			
	"Down"	*"Unchanged"*	*"Up"*	**Total**
"Down"	61	3	54	118
"Unchanged"	0	0	0	0
"Up"	5	1	32	38
Total	66	4	86	**156**

***Source**: Personal computation using **R** Studio*

Figure 3: QDA Classification using the class priors



***Source**: Personal Computation using **R** Studio*

Similar to the confusion table under QDA, **Table** 6 above shows the number of training observations correctly classified is 93, while 63 observations were misclassified. Hence, the probability of misclassification is 0.4038 or 40.38 per cent training error rate for the quadratic discriminant analysis (QDA) classifier. This analysis shows that QDA has a lower error of misclassification when compared with LDA.

2.4 Forecasting Direction of Inflation Rate in Nigeria using LDA and QDA classifiers

One of the primary aims of this research work is to predict the direction of inflation rate using the postulated linear and quadratic discriminant models. We were able to classify each training observations using the prior and posterior probabilities generated by each method to calculate the discriminant scores. The two classifiers were used to predict the direction of inflation rate for the training data set. Another application of the two classifiers is to predict the direction of inflation rate under different economic scenarios as shown below.

2.5 Testing the performance of LDA and QDA classifiers

In order to compare the performances of the two multivariate Gaussian discriminant classifiers, we divided the data into two i.e. training dataset and test dataset and computed the training and test error rates corresponding to each classifier. Specifically we used data between 2004 and 2014 to train the two classifiers and data between 2015 and 2016 to test the classifiers.

The confusion matrix, shown for the Direction of inflation rate data in **Table** 4 and **Table** 6, are conveniently displaying the training error rates. The Table 4 reveals that LDA predicted that a total of 35 "***Down***". Of these, 16 actually are correctly classified as "***Down***" and 19 are misclassified as "***Down***". Similarly, a total of 121 "***Up***" were predicted, out of which 67 are correctly classified, 4 and 50 are misclassified as "***Unchanged***" and "***Down***" respectively. Hence, LDA training error rate was computed as 46.80 per cent. Table 6 reveals that QDA predicted a total of 118 and 38 "***Down***" and "***Up***" respectively. These values represent a training error rate of 40.38 per cent. Comparing these two training error rates indicates that QDA classifier performs better than LDA classifier on our dataset. However, a classifier may perform well on training dataset but perform poorly on test dataset. But in general, our interest is not how well the classifiers perform on the training dataset. Rather, we are interested in the accuracy of the predictions that we obtain when we apply our classifiers to previously unseen test dataset. We use a test dataset corresponding to period between 2015 and 2016, which are left out in training the two classifiers. We then simply compute error rates on the test observations, and select the classifier for which the test error rate is lower as shown in **Table** 7 and **Table** 8 for LDA and QDA classifiers respectively.

Table 7: Confusion Table on LDA classifier using test dataset (2015 to 2016)

CLASSIFIED AS	**TRUTH**			
	"Down"	*"Unchanged"*	*"Up"*	**Total**
"Down"	1	1	17	19
"Unchanged"	0	0	5	5
"Up"	0	0	0	0
Total	1	1	22	**24**

Source: *Personal computation using **R** Studio*

Table 8: Confusion Table on QDA classifier using test dataset (2015 to 2016)

CLASSIFIED AS	**TRUTH**			
	"Down"	*"Unchanged"*	*"Up"*	**Total**
"Down"	0	0	0	0
"Unchanged"	0	0	0	0
"Up"	1	1	22	24
Total	1	1	22	**24**

Source: *Personal computation using **R** Studio*

Analysis of **Table** 7 and **Table** 8 reveal that the test error rates are 95.83 per cent and 8.33 per cent for LDA and QDA classifiers respectively. Again, QDA classifier outperforms the LDA classifier.

In summary, we have considered two different classification approaches: LDA, and QDA in modeling and forecasting the direction of inflation rate in Nigeria between 2004 and 2016 using two macroeconomic variables such as the nominal exchange rate and crude oil price. In Chapter four we analysed the data using these two statistical learning methods of classification. Firstly, direction of inflation which was classified into $k = 3$ levels i.e. "***Down***", "***Unchanged***", "***Up***", revealed that during the period under review, inflation rate rose in 86 months, decline in 66 months and remained unchanged for 4 months out of the 156 months with prior probabilities of 0.4231, 0.0256 and 0.5513 respectively. Fluctuation in inflation rate is a consequence of several factors in Nigeria. A major cause of rising inflation rate is the persistent rise in crude oil prices and petroleum products. It has been proven that period of rising crude oil prices result in higher prices of goods and services in an economy. In addition to rising crude oil prices, nominal exchange rate, which is a form of monetary policy instrument, have a direct impact on the inflation rate. Nigeria is an import dependent economy, which implies that any rise in the exchange rate will translate into higher inflation.

Linear Discriminant Analysis (LDA) on the direction of inflation rate was based on the assumption that the two predictors are multivariate normally distributed with common variance-covariance matrix. Since there are $g = 2$ predictors, the variance-covariance matrix requires estimating $g(g+1)/2 = 3$ parameters. Variance-covariance of a classifier plays a crucial role in determining the bias-variance trade-off. LDA classifier compute linear discriminant function which is a linear combination of the predictors that gives the maximum decision boundaries while at the same time minimizing the variation in each class (dimension-reduction property). By assuming that the k classes share a common covariance matrix, the LDA model becomes linear in $\boldsymbol{x}$, which means we require to estimate $k\mathrm{g} = 6$ linear coefficients. However, LDA requires that for $k = 3$ classes, there is at most $k - 1$ linear discriminant functions. Hence no training observation was classified into the "***Unchanged***" class. The ratios of the between- and within-class standard deviations for the two linear discriminant variables are 2.2299 and 0.0826 respectively. These singular values computed for each linear discriminant variable reveal that whilst the *LD1* explained about 99.86 per cent of the between-class variance, *LD2* explained only 0.14 per cent in our data set. Furthermore, **Figure** 2 shows that the classes are not well separated or overlap. We expect the LDA to retain most of between-class variance in our data set. The generated LDA classifier's flexibility is quit low with substantially lower between-class variance when compared to QDA. The LDA output shows that whilst about 53.20 per cent of the training observations were correctly classified into "***Down***", "***Unchanged***" and "***Up***" categories, 46.80 per cent were misclassified. Hence the accuracy of the LDA classifier is 53.20 per cent.

In addition to the assumption of the x coming from a multivariate Gaussian or normal distribution, Quadratic Discriminant Analysis (QDA) estimates a separate covariance matrix for each class, for a total of $k\mathrm{g}(\mathrm{g}+1)/2 = 9$ parameters. The QDA output shows that 40.38 per cent of the training observations were incorrectly classified while 59.62 per cent of the observations are correctly classified. Hence, the QDA classifier is seen to minimize the total probability of misclassification in our data. This is an improvement over the LDA classifier, which can be attributable to the large sample size of 156 observations despite the higher number of parameters in the covariance matrix.

The performances of the classifiers using a test dataset showed that a test error rate of 95.83 per cent for the LDA and 8.33 per cent test error rate for QDA. However, this was based on a specific economic scenario whereby the test dataset of observations between 2015 and 2016 was used.

3.0 Conclusion

This research work favours the QDA because the training observations are large so that the variance of the classifier is not a major concern, and the assumption of a common covariance matrix for the $k = 3$ classes is clearly unsustainable. Furthermore, LDA's assumption that the $k = 3$ classes share a common covariance matrix is badly off which makes LDA suffer from high bias. Theoretically, LDA tends to be a better bet than QDA if there are relatively fewer training observations and so reducing variance is crucial. Crude oil price and exchange rate are two important determinants of direction of inflation rate in Nigeria. The results of linear and quadratic discriminant analysis discussed in this research work indicated that these macroeconomic variables are important in forecasting the direction of inflation rates in Nigeria. We conclude that based on the training error rate, QDA provide a much better forecast of the direction of inflation rate than the LDA given a large sample size and varying covariance matrix of the predictors. As long as inflation rate remains a monetary phenomenon in Nigeria, developing an efficient model to predict or forecast its direction is essential. Central Bank of Nigeria (CBN) relies on the direction of inflation in determining monetary policy (expansionary or contractionary policy) for the economy. For instance, rising inflation rate may lead to the apex bank reviewing the monetary policy rate (MPR) upward and vice versa. Hence, efficiently forecasting the direction of inflation rate in the economy is crucial to economic and financial stability.

In Nigeria, the direction of inflation rate is dependent on crude oil price and nominal exchange rate as revealed in this research work, hence stability in these two variables are central to price level control. Therefore, the efficient use of exchange rate as apparatus of monetary policy (inflation-targeting) and crude oil price as major source of government financing is essential to the management of inflation rates in Nigeria.

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#R codes on forecasting the Direction of Inflation rate in Nigeria using multivariate #discriminant classifiers

par(mfrow=c(1,1));summary(cpm)

```
write.csv(summary(cpm),"Direction_Inflation.csv")
plot(Direction, col=c(down="red",unchanged="blue",up="orange"))
plot(x=er,y=cp,pch=20,col="red",main="Crude Oil Prices against Exchange rates in
Nigeria (2004 to 2016)")

#Linear Discriminant classifier
lda.fit=lda(Direction~cp+er,data=cpm);lda.fit
names(lda.fit);lda.fit$svd
plot(lda.fit,col=c(up="red",down="blue",unchanged="orange"),
    main="LDA on Direction of Inflation rates in Nigeria (2004 to 2016)")
lda.pred=predict(lda.fit);lda.pred=predict(lda.fit,data.frame(cp=52.75,ep=305.50))
lda.pred;names(lda.fit);write.csv(lda.pred$x,"Linear_Discriminant.csv")
names(lda.pred);lda.pred$svd;lda.pred$posterior;lda.class=lda.pred$class
table(lda.pred$class,Direction)
mean(lda.class==Direction);mean(lda.class!=Direction)
lda.class;par(mfrow=c(1,2))
plot(lda.class,data=Direction,col=c(down="red",unchanged="blue",up="orange"),
    main="LDA on Direction of Inflation rates in Nigeria (2004 to 2016)")
legend("topleft",legend=c("Down","Unchanged","Up"),
      col=c("red","blue","orange"),lty=1,cex=0.5)
plot(lda.class,Direction,col=c(down="red",unchanged="blue",up="orange"),
    main="LDA on Direction of Inflation rates in Nigeria (2004 to 2016)")
legend("topleft",legend=c("Down","Unchanged","Up"),
      col=c("red","blue","orange"),lty=1,cex=0.5)
lda.pred;names(lda.pred)
write.csv(lda.pred$posterior,"posterior.csv");write.csv(lda.class,"ldaclass.csv")
summary(cpm);cor(cpm[,-5]);var(cpm[,-5])
train=(Year<2015);train;test.2015=cpm[!train,];test.2015
dim(test.2015);direction.2015=Direction[!train];direction.2015;
lda.fit2=lda(Direction~cp+er,data=cpm,subset=train);lda.fit2
lda.pred2=predict(lda.fit2,test.2015);lda.pred2
lda.pred2$class
write.csv(lda.pred2$class,"testdatalda.csv")
lda.pred2$posterior
table(lda.pred2$class,direction.2015)
mean(lda.pred2$class==direction.2015);mean(lda.pred2$class!=direction.2015)
plot(lda.fit2$class,data=Direction,subset=test.2010,col=c(down="red",unchanged="blue",u
p="orange"),
    main="LDA on Direction of Inflation rates in Nigeria (2004 to 2016)")
legend("topleft",legend=c("Down","Unchanged","Up"))

#Quadratic Discriminant Classifier
qda.fit=qda(Direction~cp+er,data=cpm);qda.fit;plot(qda.fit)
names(qda.fit);qda.class=predict(qda.fit)$class
plot(predict(qda.fit)$posterior);plot(predict(qda.fit)$class)
table(qda.class,Direction)
mean(qda.class==Direction);qda.class;qda.pred=predict(qda.fit)
names(qda.pred)
write.csv(qda.pred$posterior,"posteriorqda.csv")
write.csv(qda.class,"qdaclass.csv")
par(mfrow=c(1,2))
plot(qda.class,data=Direction,col=c(down="red",unchanged="blue",up="orange"),
    main="QDA on Direction of Inflation rates in Nigeria (2004 to 2016)")
legend("topright",legend=c("Down","Unchanged","Up"),
      col=c("red","blue","orange"),lty=1,cex=0.5)
plot(qda.class,Direction,col=c(down="red",unchanged="blue",up="orange"),
    main="QDA on Direction of Inflation rates in Nigeria (2004 to 2016)")
legend("topright",legend=c("Down","Unchanged","Up"),
      col=c("red","blue","orange"),lty=1,cex=0.5)
train=(Year<2015);train;test.2015=cpm[!train,];test.2015
```

```
dim(test.2015);direction.2015=Direction[!train]
direction.2015;qda.fit2=qda(Direction~cp+er,data=cpm,subset=train)
qda.fit2;qda.pred2=predict(qda.fit2,test.2015)
qda.pred2;qda.pred2$class
write.csv(qda.pred2$class,"testdataqda.csv")
qda.pred2$posterior;table(qda.pred2$class,direction.2015)
mean(qda)
```

Predicted versus Actual Direction of Inflation rate in Nigeria using test dataset (2015 and 2016)

Year	lda	qda	Actual
2015	down	up	up
2015	down	up	up
2015	down	up	up
2015	down	up	up
2015	down	up	up
2015	down	up	up
2015	down	up	unchanged
2015	down	up	up
2015	down	up	up
2015	down	up	down
2015	down	up	up
2015	down	up	up
2016	down	up	up
2016	down	up	up
2016	down	up	up
2016	down	up	up
2016	down	up	up
2016	down	up	up
2016	down	up	up
2016	unchanged	up	up
2016	unchanged	up	up
2016	unchanged	up	up
2016	unchanged	up	up
2016	unchanged	up	up

***Source**: Personal Computation using **R** Studio*