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Estimating regression parameters in the presence of extreme influential observations: A case of Nigeria Exchange Rate

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Influential observations in data (both multivariate and univariate) can alter the estimates of the regression coefficients subjectively such that the underlying statistical relationships particularly in a least square estimation are rendered meaningless. Our objective is to examine the effects of extreme influential observations on model coefficients by comparing the classical OLS estimators with the more robust least square of maximum likelihood-like plus scale estimators such as MM-estimators. In order to achieve this set objective, the authors postulated an exchange rate regression model in the presence of extreme influential observations. Four major macroeconomic variables were included in the study as regressors. These regressors comprise foreign external reserve, foreign direct investment inflow, crude oil price and credit to private sector. The analysis showed that the standard errors and p-values of the model coefficients for the robust least square are smaller than the classical OLS method largely due to detection and handling of the influential observations. Furthermore, the forecast values from the robust MM-estimation indicate that the model slightly provide more precise estimates than the OLS.

Keywords: Influential observations, OLS estimators, *MM*-estimators.

JEL Classifications: C510, E400, E520

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1.0 INTRODUCTION

Exchange rate movement has been a major economic growth driver in any country. Over the years, in most developed and developing countries around the world, the stability of exchange rate has been hinged on favourable economic climate. Its role in international trade cannot be overemphasized. This is particularly the case because it depicts the value of a country's currency in terms of convertibility relative to the currencies of trading partners. In Nigeria and indeed many other countries, researchers have established strong nexus between some major macroeconomic variables and exchange rate. Such macroeconomic variables include foreign external reserve, gross domestic products, industrial production, credit to private sector, crude oil price, inflation rate, interest rates, public debts, terms or balance of trade and current account deficit. Others include government intervention, speculation and political stability. According to the Central Bank of Nigeria (CBN), "the main objectives of exchange rate policy are to preserve the value of the domestic currency, maintain a favourable external reserve position and ensure external balance without compromising the need for internal balance and the overall goal of macroeconomic stability". Theoretically, a drop in the market price of a currency is called depreciation of the currency while a rise is known as appreciation. By implication, a rise in the market value of the local currency i.e naira against a major currency like US Dollar, means appreciation and a drop indicates depreciation of the currency. These two regimes describe exchange rate policy and are greatly determined by government's intervention through the Central Bank of Nigeria (CBN). Alabi and Bada (2022), Alabi and Ajibode (2022) postulated powerful Markov-switching with autoregressive dynamics models on exchange rate movement in Nigeria. Their works revealed that by modeling exchange rate using crude oil price (i.e. bonny light *"sweet"* price which is one of eleven crude oils making up the new OPEC Reference Basket benchmark), foreign external reserve and inflation rate, exchange rate is more likely to transit from the appreciation regime to the depreciation regime. Also, the analysis shows that the exchange rate is expected to spend considerable time in the depreciation state than in the appreciation state in Nigeria. Furthermore, highly significant negative relationship in only the appreciation regime was established between exchange rate and the price of Nigeria's main mineral resource i.e. crude oil (Alabi and Bada, 2022). Similarly, foreign external reserve and the measure on purchasing power of the naira have significant positive influence in these two regimes. Lawal and Aweda (2015) opined that rise in crude oil price lead to appreciation of the naira to the US Dollar by estimating a level relationship between exchange rate, crude oil price and inflation rate in Nigeria. Their conclusion was reached through an autoregressive distributed lag (ARDL) bound testing procedure proposed by Pesaran and Yongcheol (2001). A similar study by Frankel (2007), based on logit and multinomial regression models concluded that the South African rand value is strongly dependent on the index of real prices of minerals commodities alongside interest rates, inflation rate and default risk/country risk.

In another research by Dada and Oyeranti (2012), real exchange rate does not have a direct relationship with economic growth. The authors concluded that government spending and monetary policy variables such as interest rates and inflation rate, crude oil price and exports were perceived to have more direct connection with economic growth in Nigeria. Ngerebo-a and Ibe (2013) through Granger causality tests revealed unidirectional causal relation from exchange rate to balance of payment, external reserve and gross domestic products in Nigeria. The study shows that no causality existed in all directions between exchange rate, external debt and inflation rate. Kuijs (1998), through cointegration analysis, investigated the reason for secular depreciation of the Nigeria real exchange rate experienced since the implementation of Structural Adjustment Programme (SAP) in 1986 and revealed that in a short-run dynamic setting, real exchange rate responds to shocks on balance of payments but in the long run, it is determined by the real demand and supply of foreign exchange in the economy. Suthar (2008) studied the major determinants of exchange rate in India. The research employed the classical least square (OLS) and resolved that the bank rate by RBI, short-term and long-term domestic interest differentials, interest yield differentials, and changes in the foreign external reserve all have significant impact on the average monthly exchange rate in India. Hypothetically, long-run determinants like trade openness, export share, and income per capital are partial determinants of exchange rate in six Central American countries (Papaioannou, 2003). This implies that though these factors are adequate in the determination of exchange rate regimes in these six countries, they are not strong predictors. Mpofu (2016) in a working paper on the volatility of exchange rate in South Africa particularly on testing the hypothesis that trade openness devalues the rand, estimated GARCH models and found that switching to floating exchange rate regime, increases the value of rand against the US Dollar.

2.0 EFFECTS OF EXTREME INFLUENTIAL OBSERVATIONS IN REGRESSION ANALYSIS

Influential observations in data (both multivariate and univariate) can alter the estimates of the regression coefficients subjectively such that the underlying statistical relationships particularly in a least square estimation are rendered meaningless. In literature, several techniques have been proposed in detecting these influential observations. Obafemi and Alabi (2019) opined that it is rather more difficult to detect influential observations in multivariate dataset than univariate dataset. However, in most econometric literature, detection of multivariate influential observations are done via diagnostic tests when checking for model stability. According to Obafemi and Alabi (2019), influential observations occur in dataset with changes in location in random directions, clusters of influential observations in particular direction or different directions. Influential observations could also occur from shift in some elements of the location vector (Rocke and Woodruff, 1996; Obafemi and Alabi, 2019). In the identification of an influential observation in a dataset, a detection specification is normally set at $\bar{x} \pm 2\sigma_{\bar{x}}$ (Obafemi and Alabi, 2019).

The detection and handling of these influential observations are crucial in econometric analyses because ordinary least squares (OLS) estimation technique is extremely sensitive to them. In fact, the OLS estimation breakdown completely asymptotically at 1/n point. In order to overcome influential observations and associated problems, several techniques of estimating model parameters, broadly classified into two; those based on diagnostic statistics and the robust procedures have been proposed.

The diagnostic approach such as leverage plots are particularly useful graphical techniques for diagnosing any deviations of the underlying assumptions of regression models such as linearity. Belsley, Kuh and Welsch (2004) proposed an effective technique in the identification of influential observations by plotting leverage for the k^{th} coefficient when multivariate regression are converted into a set of univariate regressions. Their work showed that the result of auxiliary regression of the residuals of original regression on the residuals of the regression of k^{th} regressor on the vector of regressors is identical to the k^{th} coefficient from the original regression. Irrespective of the dataset (univariate or multivariate), these plots are useful in detecting misspecification of the model in addition to influential observations. Alternatively, influence statistics such as the studentized residual (RStudent), scaled difference in fitted values (DFFITS), dropped residual (DR-Resid), ratio of covariance matrix (COVRATIO), Hat Matrix and the scaled difference in the estimated β's (DFBETAS) detect outliers or influential observations by measuring the impact of each observation on the regression output.

The robust approach is the class of least squares referred to as the robust least squares (RLS) which are very efficient on multidimensional dataset (Khan *et al.,* 2021). The diagnostic approaches are particularly useful when the number of influential observations are less than two observations. In spite of the differences in the two approaches, they are less sensitive to influential observations when compared to the OLS. According to (Khan *et al.,* 2021), the main objective of any robust estimation is to determine reliable estimates and inferences for regression model parameters in the presence of influential observations by replacing the error sum of squares in OLS with another objective function depending on the methods. These methods include Least Absolute Deviation method (LAD), M-estimation, S-estimation, and MM-estimation. Others are the Least Absolute Deviation (LAD) which replaces the error sum of squares with the least absolute residuals during minimization, least median square (LMS), least trimmed square (LTS) and the variance-covariance estimator (VCE) of the best unit from the dataset that satisfies some optimality criteria proposed by Obafemi and Alabi in 2019. Like the least square estimation, LAD breaks down whenever there are high leverage points in the dataset. M-estimation (Huber, 1973) is more efficient than the LAD since it deals with influential observations in dependent variable in a regression model with large residuals. It achieves this by minimizing the symmetrical objective function of residuals rather than the squared errors. This claim was confirmed by the work of Ayinde, Lukman and Arowolo (2015) which compared the performances of some robust least squares such as M-estimation, MM-estimation, least trimmed square (LTS) and S-estimation against the classical OLS and concluded that the estimators are mostly efficient with $M-$ and $MM-$ robust estimators, S-estimation (Rousseeuw and Yohai, 1984) on the other hand handles influential observations in the regressors in which high leverages exist. MM-estimation combines S-estimation and M-estimation. Its iteration starts with S-estimation, then uses the results to determine M-estimates.

Other methods of detecting outliers in multivariate dataset include those proposed by Rocke and Woodruff (1996), Barnet and Lewies (1994), Rousseeuw and Van Zomeren (1990), Maronna (1976), Campbell (1980), Stahel (1981), Donoho (1982), Rousseeuw (1985), Jackson and Chen (2004).

In this current work, we intend to study the effect of influential observations on model parameters by fitting two regression models (OLS and MM-estimation) of exchange rate movement on four macroeconomic variables such as the foreign external reserve, foreign direct investment inflow, crude oil price, and credit to private sector in Nigeria between 2006 and 2021.

This research work is divided into six sections: section one covers the introduction. In section two, we discussed briefly the effect of extreme influential observations in multivariate analysis. Section three covers the methodology which is broken into sources of data, regression analysis and robust regression through MM-estimation. Section four include the model specification. Section five and six comprises empirical results and discussion, and conclusion respectively.

3.0 METHODS

In this section, we describe the data collected on the macroeconomic variables, the postulated models in this current study, and methods of analysis conducted to achieve the set objectives.

3.1 *Sources of data*

Secondary data were collected on five macroeconomic variables including exchange rate, foreign external reserve, foreign direct investment inflow, crude oil price and credit to private sector. These data were sourced from the database of the Central Bank of Nigeria (CBN) between 2006 and 2021 on monthly basis, a total of N = 178 data points. Naira per US Dollar exchange rate (nominal) was used as the response variable, while foreign direct investment inflow, foreign external reserve, crude oil price and credit to private sector were the regressors. Since the variables involved are based on different units, we converted all data to natural logarithm. This also helps to make interpretation of model coefficients easier.

3.2 *Regression Analysis*

Regression analysis is a powerful statistical tool that utilizes the relation between two or more quantitative variables so that one variable can be predicted from the other or others. This methodology is widely used in social and behavioral sciences such as business, and many other disciplines (Bada and Alabi, 2022). The primary purpose of all regression analyses is to use data to estimate the form of this relation. A functional relation may be defined as

$$Y \approx X\beta + \varepsilon \tag{1}$$

Where Y is an $Nx1$ column vector of the response variable, X is an Nxp (where $p = q+1$) designed matrix of regressors, β is an $(1+p)x1$ vector of regression coefficients and the ε is the $Nx1$ vector of residuals. The estimators $\hat{\beta}$ are usually estimated by the minimizing the residual sum of squares otherwise called the method of least squares according to some underlying assumptions such as the normality of residuals i.e. $\varepsilon \sim N(0, \sigma^2)$.

$$\hat{\beta}_{LS} = \text{argmin} \sum_{i=1}^{N} r_i(\beta)^2 \tag{2}$$

The least square method attempts to find the value of the parameter (s) that makes the sum of squares residual to be zero. The function $r(\beta)^2 = (Y - X\hat{\beta})^2$ is very sensitive to influential variables especially on high leverage cases. At this point, the least squares breaks down and all the model's assumptions are no longer satisfied. As mentioned earlier, the estimation breaks down completely asymptotically at $1/n$ point.

3.2 *Robust Regression through MM-estimation*

The MM-estimation is one of the most powerful statistical techniques used in detecting influential observations in both the response and the regressors. This is achieved through a procedure that combines the S-estimation and the M-estimation. It begins with the S-estimates of the regression coefficients and σ used as scale. It uses the estimate of the standard error, σ as fixed value in iteration in order to solve the following M-estimation k nonlinear first-order equations:

$$\sum_{i=1}^{N} \psi_c \left(\frac{r_i(\beta)}{\sigma w_i} \right) \frac{x_{ij}}{w_i} = 0 \qquad j = 1, 2, \ldots, k \tag{3}$$

Where $\psi_c(\cdot) = \rho_c^{'}(\cdot)$, the first derivative of $\rho_c(.)$ in the maximum likelihood estimator-like of the model parameters expressed as

$$\hat{\beta}_M = \mathrm{argmin}_\beta \sum \rho_c \left(\frac{r_i(\beta)}{\sigma w_i} \right) \tag{4}$$

The residuals are scaled with $\sigma, c > 0$ a tuning constant corresponding to the function ρ and w_i are down-weighted using Hat matrix and expressed as $w_i = \sqrt{1 - X, (X'X)^{-1} X_i'}$. The X_i represent observations in the regressors with high leverages. In the computation of $\rho_c(.)$, there are several choices such as Andrews, Bisquare, Cauchy, Fair, Huber-Bisquare, Logistic, Median, Talworth, Welsch. However, MM-estimation procedure requires the Bisquare given as

$$\rho_c(X) \;=\; \begin{cases} \dfrac{c^2}{6} \left(1 - \left(1 - \left(\dfrac{X}{c} \right)^2 \right)^3 \right) & if \ |X| \ \leq c \\ \\ \dfrac{c^2}{6} & \text{elsewhere} \end{cases} \tag{5}$$

The tuning constant $c = c_M$ is chosen in order achieve a desired breakdown value. S-estimators are coefficients of the model parameters (β), $\hat{\beta}$ with smallest estimate of the scale S in equation (6) which is estimated using the second stage Median Absolute Deviation, zero centered (MADZER0) approach.

$$\frac{1}{N-k} \sum_{i=1}^{N} \lambda_{cs} \left(\frac{r_i\,(\beta)}{S} \right) \;=\; b \tag{6}$$

where $\lambda_{cs}(\cdot)$ with non-negative tuning constant c_s,

$$b = E_\phi\left(\lambda_{cs}\right) \tag{7}$$

ϕ is standard normal with a breakdown value of $B = b/max\,(\lambda_{cs})$. The objective function in equation 6 depends on the tuning constant, c_s through b and λ_c chosen to achieve a desired B Rousseeuw and Yohai (1984) defined a function based on the integral of the following biweight function (Bisquare function).

$$\lambda_{cs} = \begin{cases} \left(\frac{X}{c_s}\right)^6 - 3\left(\frac{X}{c_s}\right)^4 + 3\left(\frac{X}{c_s}\right)^2 & if\ |X|\ \le c \\ \\ 1, & \text{elsewhere} \end{cases} \tag{8}$$

Salibian-Barrera and Yohai (2006) developed a Fast-S algorithm in the computation of the S-estimates. MM-estimation robust statistics are derived from the second M-estimation stage. This statistics include the coefficient of determination or measure of goodness of fit, robust Wald test to test the null hypothesis $H_0 : \hat{\beta}_M = 0$, deviance which measures the value of the objective function in equation 4 at the final coefficient estimates and estimates of scale derived from the MADZERO at the first-stage (S-estimation stage), robust $\mathrm{AIC_R}$ and $\mathrm{BIC_B}$ information criteria. These summary statistics are expressed in the following forms:

- $$R_W^2 = \frac{\sum_{i=1}^{N} \rho_{c_m i} \cdot (y_i - \overline{y_W})(y_i - \overline{y_W})}{\sqrt{\left(\sum_{i=1}^{N} \rho_{c_m i} \cdot (y_i - \bar{y}_W)\right)\left(\sum_{i=1}^{N} (\rho_{c_m i} \cdot (y_i - \overline{y_w}))\right)}} \sim \chi^2(k_1) \tag{9}$$

  $$\rho_{c_m i} = \rho_{c_M}\left(r_i\left(\hat{\beta}\right)\right)/\hat{\sigma} w_i, \bar{y}_w = \sum_{i=1}^{N} \rho_{c_m i} \cdot y_i \text{ and } \overline{y_w} = \sum_{i=1}^{N} \rho_{c_m i} \cdot \hat{y}_i$$

  $$\text{Adjusted } R_w^2 = 1 - \left(1 - R_w^2\right)\frac{N-1}{N-k}$$

- $$R_n = \hat{\beta}_1 \hat{\sum}_1^{-1} \hat{\beta}_1 \tag{10}$$

  $\hat{\beta}_1$ are the k_1 non-intercept robust coefficient estimates and $\hat{\sum}$ the estimated covariance matrix using Huber (1981) equations.

- $$\text{Deviance} = 2\sigma^2 \sum_{i=1}^{N} \rho_{c_M}\left(\frac{r_i\left(\hat{\beta}\right)}{\hat{\sigma} w_i}\right) \tag{11}$$

- $$AIC_R = 2\sum_{i=1}^{N} \rho_{C_M}\left(\frac{r_i\left(\hat{\beta}\right)}{\hat{\sigma} w_i}\right) + 2k\left\{\frac{\sum_{i=1}^{N} \psi_{c_M}\left(\frac{r_i(\hat{\beta})}{\sigma w_i}\right)^2}{\sum_{i=1}^{N} \psi'_{c_M}\left(\frac{r_i(\hat{\beta})}{\sigma w_i}\right)}\right\} \tag{12}$$

- $$BIC_R = 2\sum_{i=1}^{N} \rho_{c_M}\left(\frac{r_i\left(\hat{\beta}\right)}{\hat{\sigma} w_i}\right) + 2k \cdot \log(T) \tag{13}$$

4.0 MODEL SPECIFICATION

In order to ensure that the model's linearity assumption holds, the authors postulate a specific logarithm functional form of the relation in the equation (1) between exchange rate movement and the aforementioned four macroeconomic variables in Nigeria (including the first lag of the response variable, exr_{t-1}) as follows:

$$\log Y = \beta_0 + \sum_{i=1}^{5} \beta_i \log(X_{i,t}) + \varepsilon_t \tag{14}$$

Where $\log(Y) = \log(exr_t)$ is the log of exchange rate of Naira per US Dollar at time t, $\log(X_{1,t}) = \log(exr_{t-1})$ is the log of the exchange rate at lag one, $\log(X_{2,t}) = \log(ext_t)$ is the country's foreign external reserve at time t, $\log(X_{3,t}) = \log(fdi_t)$ is the log of the amount of foreign direct investment inflow during period t, $\log(X_{4,t}) = \log(cp_t)$ is the log of crude oil price at time t, and $\log(X_{5,t}) = \log(cps_t)$ is the log of credit to private sector at period t. The $\beta_i s$ are the model coefficients. Our interest is in modeling the relationship between the exchange rate and the selected macroeconomic variables in which the former is the response variable and the latter are the regressors (Figure 1).

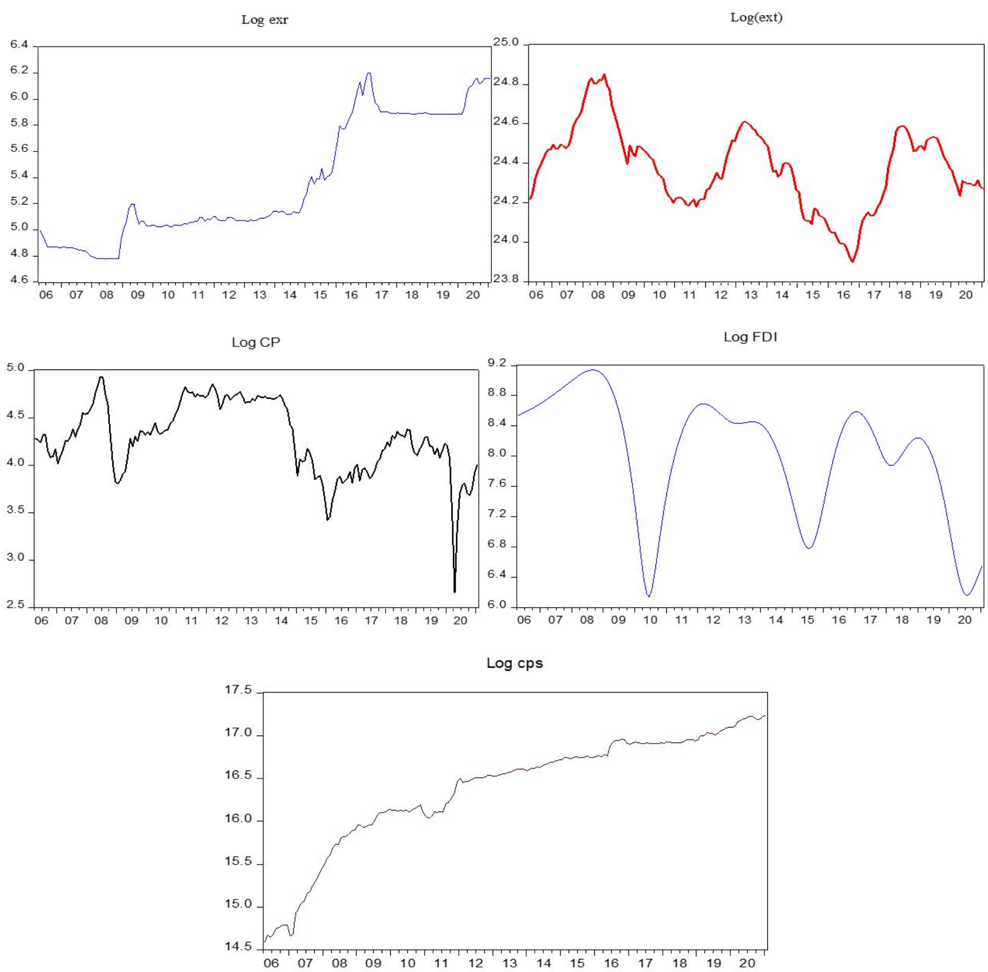

Figure 1: Time plots on the log (exr_t), $\log(cp_t)$, $\log(fdi_t)$ and $\log(cps_t)$ in Nigeria

5.0 EMPIRICAL RESULTS AND DISCUSSIONS

In theoretical and empirical researches, macroeconomic data or time series usually possess some stochastic or random trend as an underlying property. Consequently, most of these time series are non-stationary or follow random walk unless ascertained otherwise. In order to determine the data generating process of these time series, further tests of stationarity are necessary. These are important if the authors are to address the problems of spurious regression usually associated with regressing random walks against each other, producing relationships inform of contemporaneous correlations. Specifically, we conducted Augmented Dickey-Fuller (ADF) test on all variables. ADF procedure is based on DF distribution of the t-ratio statistics. See Lawal and Aweda (2015) for details on ADF test which accommodate ARMA (p, q) models with unknown orders. Table 1 shows the ADF test (intercept only) values and the associated p-values both at levels and first differences.

Table I: Augmented Dickey-Fuller (ADF) Test and p-values

Variable	**@ Level**	**p-values**	**@ 1^{st} diff.**	**p-value**	**remark**
$\log(exr_t)$	-0.0194	0.9548	-9.5949	0.0000*	$I(1)$
$\log(ext_t)$	-2.2954	0.1748	-5.5755	0.0000*	$I(1)$
$\log(fdi_t)$	-2.3142	0.1687	-3.1266	0.0265**	$I(1)$
$\log(cp_t)$	-2.5071	0.1155	-9.0150	0.0000*	$I(1)$
$\log(cps_t)$	0.1279	0.9670	-10.9865	0.0000*	$I(1)$

* ** *indicate that the significance at 1 percent and 5 percent levels of significance respectively*

The results on ADF test show that the macroeconomic variables are nonstationary at levels but became stationary after the first difference. All p-values at first difference are highly significant at $\alpha = 0.01$ except the p-value on foreign direct investment inflow which is significant at $\alpha = 0.05$. Hence our postulated model on exchange rate in equation (14) has no problem of spurious regression. Next, we compare the ordinary least squares (OLS) estimators with the robust MM-estimators. We start with OLS estimation of the model in equation (14) by employing the HAC (Newey-West) covariance estimation technique. This is crucial since most time series suffer from the problem of autocorrelation and heteroscedasticity, HAC estimation gives consistent estimators by assuming that the serial autocorrelation dissipate with increasing sample sizes. The method corrects the standard errors of the OLS estimation in the presence of unknown autocorrelation and heteroskasticity. Stability diagnostics were conducted on this fitted OLS. Specifically, influential observations were detected using influence statistics via RStudent, DFFITS and COVRATIO plots (Figure 2).

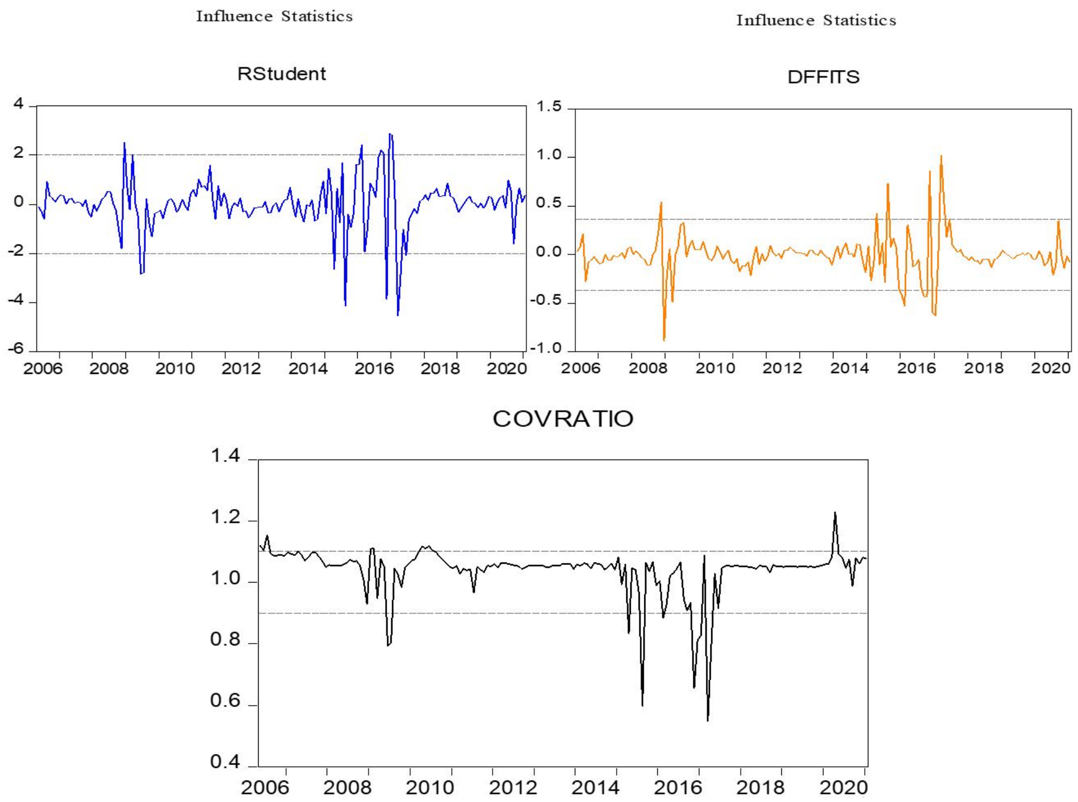

Figure 2: RStudent, DFFITS, and COVRATIO Plots on Exchange Rate Model

We observe a few observations behaving differently from others in these three plots. Influence statistics show that the major influential observations are found between 2014 and 2017 in addition to 2008 and 2009. Individually, these observations makes significant difference to the regression results obtained using the classical OLS estimation technique.

The *MM*-estimation was carried out by using the Fast-S algorithm. This algorithm includes the S-estimation in which the values of the tuning constant c_s were chosen so as to achieve 95% asymptotic efficiency under residual normality. A tuning constant $c_s = 3$ and breakdown value $B = 0.2427$ were computed. The tuning value was taken from Holland and Welsch (1977) which satisfies the 95 percent asymptotic efficiency under $\varepsilon \sim N(0, \sigma^2)$. Five subsamples corresponding to the number of regressors were taken at each trial of 200. Two refinements were carried out and the best five of the 200 trials were compared. The results were then used to implement the second stage involving the M-estimation which has a tuning constant $c_m = 5.787$. Five hundred iterations were carried out using the L'Ecuyer random number generator and the seed was set to 2,138,879,884. The Huber Type III procedure was used to compute the standard errors and covariance matrix. Table II shows the results of the OLS and *MM*-estimation.

Table II: Result of Classical OLS and robust MM-estimations

	OLS estimation				**MM-estimation**			
Variable	$\hat{\beta}$	$S_{\hat{\beta}}$	t**-value**	p**-value**	$\hat{\beta}$	$S_{\hat{\beta}}$	z**-value**	p**-value**
$\log(exr_{t-1})$	0.9241	0.0132	69.8819	0.0000*	0.9366	0.0094	99.4010	0.0000*
$\log(ext_t)$	-0.0015	0.0034	-0.4236	0.6725	-0.0024	0.0022	-1.0953	0.2734
$\log(fdi_t)$	0.0074	0.0045	1.6551	0.0997***	0.0082	0.0031	2.6255	0.0087*
$\log(cp_t)$	-0.0721	0.0109	-6.6185	0.0000*	-0.0701	0.0091	-7.6866	0.0000*
$\log(cps_t)$	0.0425	0.0054	7.9445	0.0000*	0.0391	0.0049	8.0336	0.0000*
	R^2 :	0.9952	*Adj.* R^2	0.9951	R^2	0.8663	*Adj.* R^2	0.8632
	AIC:	-4.0293	*BIC:*	-3.9396	R^2_w	0.998064	*Adj.* R^2_w	0.9981
	$S.E_{res}$:	0.1742			AIC_R :	259.7826	BIC_R :	278.294

* *** *indicate the significance at 1% and 10% levels of significance respectively.* $S_{\hat{\beta}}$ are the standard errors of the model's estimators

The effects of the presence of influential observations on the estimates of the model parameters can be observed in the sizes of the standard errors. The standard errors are smaller in the *MM*-estimation than in the classical OLS producing lower p-values in the robust least squares estimation. The signs of the model coefficients are similar in both the OLS and robust *MM*-estimations. The results of the two models show that the foreign external reserve and crude oil price have negative impact on exchange rate movement in Nigeria. This negative relations implies that growth in these two macroeconomic variables lead to appreciation of the naira during the review period. Analysis show that rise in bonny light *"sweet"* price lead to a minor but significant appreciation of the Naira due largely to growths in the country's forex savings derived from the export of the mineral resource during the corresponding period. This confirms the earlier work of Alabi and Bada (2022) in which highly significant negative relations were established between exchange rate and crude oil price in the appreciation regime of the Markov-switching Autoregressive model. Also, Lawal and Aweda (2015) concluded that crude oil price has negative relations with exchange rate in the short-run in Nigeria by estimating a level relationship between exchange rate, crude oil price and inflation rate via an ARDL (4,4,0) model. This effect on exchange rate indicates the significant importance of the fluctuations in the price of the mineral resource to the country's economy as a main source of foreign exchange earnings and major determinant of government's exchange rate policy over the years. Unfortunately, the Nigerian government does not regulate the fluctuations of bonny light price in the international market making it more difficult for this macroeconomic variable to be used as a stabilizer of exchange rate movement in the long-term.

Nonetheless, upswing in foreign external reserve did not produce any statistically significant relations. Perhaps, a large proportion of foreign exchange earned from sales of crude oil was directly used to offset the ever growing short-term demand for forex in the financial system, rather than reserved externally to stabilize exchange rate in the long-term. In fact, Alabi and Ajibode (2022) stated that growth in foreign external reserve in the company of rising inflation lead to depreciation of the naira both in the appreciation and depreciation regimes of a two-state Markov-Switching model with serially correlated errors. Hence, the insignificance of the Nigeria's external position in the model explains the likelihood that it does not contribute significantly to the preservation of the value of the country's domestic currency. More so, over the years, CBN exchange rate policies have been to deplete the country's foreign external reserve in order to keep prices down and ensure sustainability of the official exchange rate (Alabi and Bada, 2022). Furthermore, Ngerebo-a and Ibe (2013) iterated that a unidirectional granger causality exists from exchange rate to external reserve and not vice versa. A move, too many, that has produced little or no positive impact on the output of an import-dependent economy like Nigeria, where exchange rate is regarded as a weak inflation targeting monetary policy tool. This view is in support of the claim by Lawal and Aweda (2015) that exchange rate is a weak "shock-absorber" during periods of higher prices.

Furthermore, empirical analysis show that foreign directs investment inflow and credit to private sector have small but significant positive effects on exchange rate in both models. By implication, increase in FDI and CPS lead to depreciation in Naira against the US Dollar. Firstly, previous researches have shown that in a favourable FDI setting, mostly during periods of currency devaluation, managed floating exchange rate policy and so on, the relationship between exchange rate and FDI inflow is always positive. According to Goldberg (2006), growth in FDI is associated with depreciation in a country's currency as a direct consequence of reduced wages and production costs relative to its trading partners. This produces locational advantage for foreign direct investment or productive capacity investments, though experts warn against anticipation of exchange rate movement which can weaken relative wage importance due largely to the effects of rising prices in an economy with negative net exports. Secondly, the positive connection between credit to private sector and exchange rate implies that as credit to the private companies' increases from both local and international lenders, their demand for forex may grow because most of these companies rely greatly on equity and debt financing to cater for their operational costs and cashflows. Hence, higher demand for forex leads to depreciation of the country's currency against the US Dollar. In the OLS and robust *MM*-estimation models, the predictors explain about 99.5 per cent and 99.81 percent of the total variability in exchange rate respectively. These two coefficients indicate that the selected macroeconomic variables are sufficient for the study. Hence, the two models are well fitted.

In the classical OLS model, influential observations were identified during the period under review. Like in the diagnostic approach of the influence statistics, the dates associated with these observations are 2008, 2009 and between year 2014 and 2017. When these influential observations were eliminated from the dataset, FDI becomes highly statistically significant at lower level of significance and the foreign external reserve remains insignificant. This is an indication that the presence of influential observations in datasets alter the regression estimates and results of inference on the model parameters.

The standard descriptive statistics on the two models produced higher R-squared values in the classical OLS than the robust least square. However, Renaud and Victoria-Feser (2010) has shown that the robust least squares' R^2 statistic which is based on the objective function on M-estimation can be highly sensitive irrespective of the coefficients and the

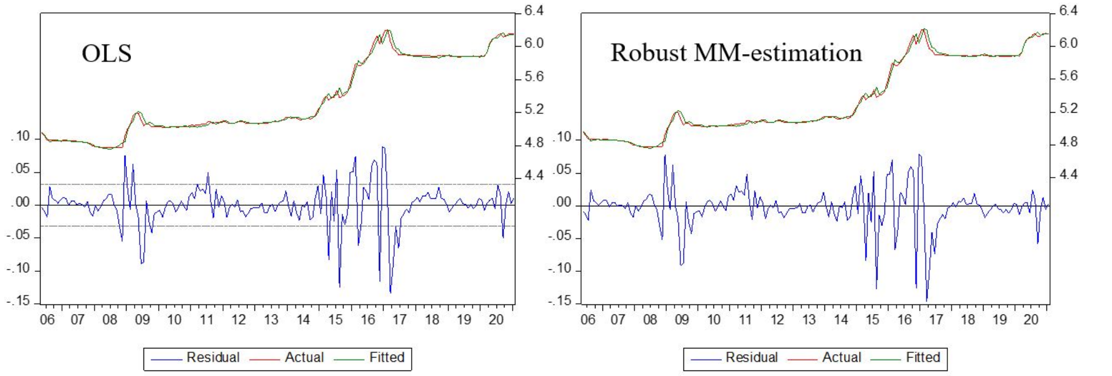

Figure 3: Actual, fitted and residual plots on the two models

standard errors. On the other hand, the alternative R_w^2 statistic provided a slightly higher value (0.9981) when compared to the OLS because the statistic is computed from a function of the residual value (equation 9). The Wald test statistic returned a p-value $= 0.000$ and deviance of 0.1052. The forecast statistics of figure 4 indicate a Mean Absolute Error (MAE), Mean Absolute Percent Error (MAPE) are higher for OLS. Specifically, $\text{MAE}_{OLS} = 5.2707$, $\text{MAPE}_{OLS} = 1.9419$. Similarly, $\text{MAE}_{MM} = 5.2014$, $\text{MAPE}_{MM} = 1.9228$. These results indicate that the robust least square provide slightly better estimates and hence smaller residuals.

6.0 CONCLUSION

Two regression models on exchange rate movement in Nigeria were estimated and compared. These models parameters were estimated using the ordinary least squares (OLS) and "maximum likelihood-like" plus scale estimators also known as the *MM*-estimator. Four macroeconomic variables such as foreign external reserve, foreign direct investment inflow, crude oil price and credit to private sector including the first lag of exchange rate were used as regressors in these two models. Estimation results revealed both significant and insignificant relations. In the two models, foreign external reserve was statistically insignificant. Unsurprisingly, growths recorded in foreign direct investment inflow and credit to private sector lead to depreciation of the naira against the US Dollar. Conversely, rise in crude oil price lead to a small but significant appreciation of the country's currency. Hence, this mineral resource remains a major determinant of CBN's monetary policy decision-making consideration alongside inflation rate in Nigeria. Nevertheless, the government does not determine the price of bonny light in the global market in order to increase foreign exchange earned for stabilizing the exchange rate. Rather, several factors which are out of control of the government are believed to be responsible for crude oil price fluctuations internationally. These factors are financial markets, spot prices, non-OPEC crude oil supply, crude oil balance, oil demand by non-Organization for Economic Cooperation and Development (OECD) members and oil demand by OECD members. Finally, the summary statistics indicate a slightly more precise forecasts were obtained with robust *MM*-estimation.

Conflict of Interest

The authors confirm that this article content has no conflict of interest.

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