International Journal of Women in Technical Education and Employment
ISSN: 2811-1567. Volume 3 – Issue 1. July 2022
https://fpiwitedjournal.federalpolyilaro.edu.ng

An application of Hamilton Model of Switching with Autoregressive Dynamics on Exchange Rate Movement in Nigeria

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ARTICLE HISTORY

Received: March 28, 2022
Revised: May 12, 2022
Accepted: May 17,2022

Abstract

This study looks at the relationships existing between the exchange rate and two important determinants such as crude oil price and foreign external reserves in Nigeria between 2006 and 2022. The exchange rate movement has been a major economic growth driver in many countries. In Nigeria, various governments through the Central Bank of Nigeria, in a bid to sustain a stable exchange rate regime came up with various policies. Our objective is to develop a powerful predictive model using the Time-invariant Hamilton model of switching with autoregressive dynamic techniques to generate probabilities of transiting from one state to another. Specifically, a two-state Time-invariant Markov-Switching model was estimated under the assumption that the errors are serially correlated with order four. In our specification, the economy is in regimes 1 and 2 whenever the exchange rate drops and rises respectively and the transition between the two regimes is modelled as the solution of the first-order Markov process. If it is in the "rise" state, this signifies depreciation in Naira but if in the "drop" state, then the currency is said to appreciate against the USDollar. The regime-specific, common and transition parameters were estimated by maximizing iteratively the normal mixture log-likelihood function. The analysis of the constant transition between regimes suggests that the exchange rate is more likely to transit from a "drop" state to a "rise" state. The likelihood that the exchange rate remains in either a "rise" or "drop" regime of origin is high. Generally, the exchange rate is expected to spend more time in the "rise" state than in the "drop" state.

Keywords: Crude oil price, Exchange rate, Foreign external reserve, Markov-switching.

Citation
Alabi, N. O. & Bada, O. (2022). An application of Hamilton Model of Switching with Autoregressive Dynamics on Exchange Rate Movement in Nigeria. *International Journal of Women in Technical Education and Employment* 3(1), 66-76

Introduction

The exchange rate movement has been a major economic growth driver in any country. In Nigeria, the story is not different. According to the Central Bank of Nigeria (CBN), "the main objectives of exchange rate policy are to preserve the value of the domestic currency, maintain a favourable external reserves position and ensure external balance without compromising the need for internal balance and the overall goal of macroeconomic stability". However, historically, it has passed through several interesting faces over time. Various governments through the CBN, in a bid to sustain a stable exchange rate regime, came up with various policies. These policies started with the Second-Tier Foreign Exchange Market (SFEM) which introduced the Bureau de change (BDC) into the economy in 1986. Its introduction which was meant to facilitate industrialization through a weaker currency saw the exchange rate persistently depreciating without achieving the set objective. In 1995, another policy change brought in the Autonomous Foreign Exchange Market (AFEM) which allowed the apex bank to sell foreign exchange to users at the market price fixed. However, pressure from the global crude oil market made this unsustainable. Crude oil was selling for $US20 per barrel in the global market which created a short fall in the supply of forex. This brought about for the first-time round-tripping by commercial banks in connivance with the BDCs.

In early 2000, a new exchange rate regime was introduced through the creation of the Interbank Foreign Exchange Market (IFEM). The goal of the IFEM was to bridge the rapidly growing gap between the official rate and the black markets by raising the CBN rate while keeping the BDC rate at a constant level. The justification for this sudden increment was that the previous government depleted the foreign reserve to sustain the official rate for so long which was seen to be no longer viable.

By the end of 2008, the effect of the global financial crisis which started in July of 2007 was felt in Nigeria when the price of crude oil started to drop reaching $US50 per barrel. Despite a huge foreign reserve of $US62 billion of the country at this time, currency devaluation was put in place (Figure 1*c*). The exchange rate rose by ₦20 per $US1 (Figure 1*a*), representing a more than a 15 per cent increase between 2003 and 2009. This upward trend has been recorded since then. Despite different policies aimed at alleviating the pressure on the exchange rate, it continues to rise all through 2014. By mid-2016, another global crude oil price crisis resulted in a significant drop in the commodity to as low as $US15 per barrel (Figure 1*b*). This further put pressure on the already depleted country's foreign reserve. Between 2016 and 2021, the apex bank introduced a different policy regime referred to as the managed floating exchange rate (MFER) to attract more investment into the country's financial system. This new policy allows the market forces of demand and supply to determine the value of the Naira. Its implementation was enhanced through the introduction of Special Secondary Market Intervention Sales (SMIS), Invisible Windows, Small and Medium Enterprises Window, Investors' and Exporters' Foreign Exchange Window, Naira-settled OTC FX futures etc. In 2018, stability in the exchange rate was achieved through the sale of dollars. This move was also to ensure that the inflation rate was kept down. However, the sale of dollars was at the expense of forex savings or depletion of the reserve. In 2020, CBN devalued the Naira by selling dollars to banks in the Investors and Exporter Window at ₦380/US$ (Figure 1*a*). The drop in the value of the Naira was associated with the drop in crude oil price which traded at $30 per barrel. In the parallel market, the currency continues to drop in 2020 amidst COVID19 pressures. In late 2021, the apex bank turned its attention to marketers involved in illegal trading. This policy led to the closure of several BDCs. However, the country's currency has continued to experience depreciation.

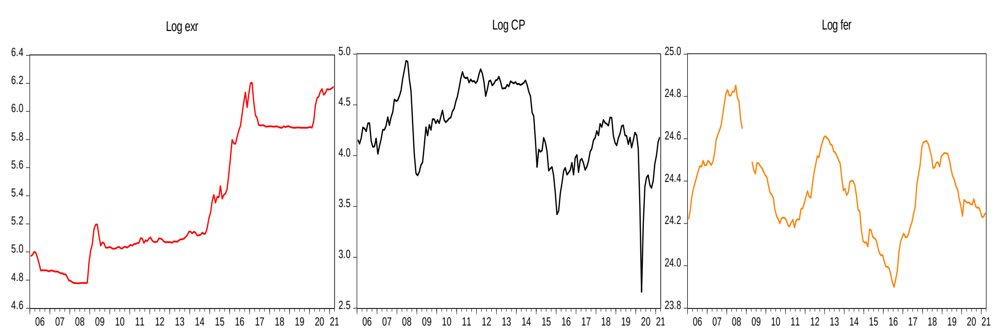

Figure 1 Time Plots on Nigeria Exchange Rate, Foreign External Reserve, and Crude Oil Price

Throughout these periods, the country's currency experienced depreciation and appreciation at various points. Hence, this current work intends to develop a Markov-switching (MS, hereon) model that can help in the prediction of the exchange rate in these two regimes using crude oil price and foreign external reserve as its determinants. Several works relating exchange rates with macroeconomic variables exist in the literature. These works include but not limited to (Onavrote & Eriemo., 2012); Sibanda, Mlambo (2014); Hassan & Mohammed (2011); Ardian, Lanier and Hudson (2009); Selien and Peersman (2012); Rickne (2009); Al-Mulali & Normee (2009); Sebastian (2006) and Lawal & Aweda (2015). We do not dwell extensively on this previous research.

Methodology

MS models are powerful in characterizing time series belonging to different and recurring states or regimes. They are also very useful in computing the probability that a series transit from one regime to a different regime at any given period. In MS models, nonlinearities due largely to discrete changes in state or regime are incorporated into a linear model. According to Hamilton (1990), these models are different from regression models with structural breaks in that the model parameters are invariant to a temporary, unobserved but finite number of recurring regimes. Furthermore, the cyclical components of time series are easily modelled. Under this model, the assumption is that the log first differences of the macroeconomic time series follow a nonlinear rather than a linear stationary process (Hamilton, 1989). Earlier works on MS are credited to Goldfeld and Quandt (1973); Maddala (1986); Hamilton (1989), (1994), (1990), (1996); Fruhwirth-Schnatter (2006). In this current study, we let exchange rate (*exr*), $\boldsymbol{y}_t$, be the endogenous variable which follows a data generating process (*d.g.p*, hereon) relying on values of unobserved discrete regime/state variable, r_t. Also, we let the number of regimes be *G*, so that the series is in state *g* in ~~time~~ period *t* given that $r_t = g$, for $g = 1, 2, 3 \ldots G$. A major assumption in regime-switching is that each regime has a distinct and unique regression.

Basic switching model (BSM)

Prior to postulating the time-invariant Markov-Switching Autoregressive (MSAR) models, we started with a basic switching model in which crude oil price (*cp*) and foreign external reserve (*fer*) are two regressors in the matrix of regressors, X_t. Also, we defined $\boldsymbol{\Gamma}_t$ as the conditional mean of y_t in a regime *g* which is assumed to be a linear specification given as

$$\mu_t(g) = X_t^{'}\theta_g + \Gamma_t^{'}\delta \qquad (1)$$

Where θ_g and δ are k_x and k_z vectors of coefficients. δ are coefficients of $\boldsymbol{\Gamma}_t$ which are regime indifferent. The model errors are assumed to be distributed normally with variances dependent on the regime, *g*. Let

$$y_t = \mu_t(g) + \sigma(g)\varepsilon_t \qquad (2)$$

When the state $\boldsymbol{r}_t = g$, $\boldsymbol{\varepsilon}_t$ is independently and identically distributed standard normally random errors i.e. *iid* normally distributed random errors. We let $\sigma(g) = \sigma_g$. The estimation of the model parameters is based on likelihood contribution for an observation obtained by weighting the *p.d.f.* in each of the regimes, $g = 1, 2, \ldots, G$ by one-step ahead probability of being in *g* as follows:

$$L_t(\theta,\delta,\sigma,\gamma) = \sum_{g=1}^{G} \frac{1}{\sigma_g}\phi\left(\frac{y_t - \mu_t(g)}{\sigma(g)}\right) P\left(r_t = g \mid \varsigma_{t-1},\gamma\right) \qquad (3)$$

Where θ and σ are vectors of *G* coefficients, γ are parameters which determine the regime probabilities, $\phi(.)$ is the standard normal density function and ς_{t-1} is the information set in period *t*-1. We obtained the estimators of the model parameters by maximizing the log-likelihood function in equation (4) w.r.t θ, δ, σ and γ.

$$l(\theta,\delta,\sigma,\gamma) = \sum_{t=1}^{T} \log\left\{ \sum_{g=1}^{G} \frac{1}{\sigma_g}\phi\left(\frac{y_t - \mu_t(g)}{\sigma(g)}\right) P\left(r_t = g \mid \varsigma_{t-1},\gamma\right) \right\} \qquad (4)$$

The switching can either be a simple switching or the MS depending on whether the errors in equation (2) are serially correlated or not. If the errors are

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ISSN: 2811-1567. Volume 3 – Issue 1. July 2022
https://fpiwitedjournal.federalpolyilaro.edu.ng

uncorrelated, the regime probabilities $P(r_t = g \mid \varsigma_{t-1}, \gamma)$ are varying probabilities by assuming that they are a function of vectors of exogenous variables U_{t-1} and coefficient γ which are parameterized using multinomial logit expression such as

$$P(r_t = g \mid \varsigma_{t-1}, \gamma) = p_g(U_{t-1}, \gamma) = \frac{\exp(U_{t-1}'\gamma_g)}{\sum_{j=1}^{G} \exp(U_{t-1}'\gamma_j)} \quad (5)$$

Equations (4) and (5) results in a normal mixture log likelihood function (NMLLF) expressed as

$$l(\theta,\delta,\sigma,\gamma) = \sum_{t=1}^{T} \log\left\{ \sum_{g=1}^{G} \frac{1}{\sigma_g} \phi\left(\frac{y_t - \mu_t(g)}{\sigma(g)} \right) p_g(U_{t-1,}\gamma) \right\} \quad (6)$$

The NMLLF in equation (6) is maximized iteratively using a method such as Broyden, Fletcher, Goldfarb and Shanno (BFGS) alongside Marquardt steps. The coefficients covariances were computed via inverse negative Hessian. Equation (6) is dependent on one-step-ahead probabilities of being in a particular regime g = 1, 2, 3… G. Alternatively, these regime probabilities may also depend on *filtering* and *smoothing*. The former is a process of contemporaneously updating the regime probability estimates from additional information obtained when the values of the dependent variable can be observed in a given time period. Introducing Bayes' theorem, we have

$$P(r_t = g \mid \varsigma_t) = P(r_t = g \mid y_y, \varsigma_{t-1}) = \frac{f(y_t \mid r_t, \varsigma_{t-1}) P(r_t = g \mid \varsigma_{t-1})}{f(y_t \mid \varsigma_{t-1})} \quad (7)$$

$$P(r_t = g \mid \varsigma_t) = \frac{\frac{1}{\sigma_g} \phi\left(\frac{y_t - \mu_t(g)}{\sigma(g)} \right) p_g(U_{t-1}, \gamma)}{\sum_{j=1}^{G} \frac{1}{\sigma_j} \phi\left(\frac{y_t - \mu_t(j)}{\sigma(g)} \right) p_j(U_{t-1}, \gamma)} \quad (8)$$

Markov Switching

Extending the simple regime framework (discussed so far) by specifying the first-order Markov Process results in an MS (MS). The regime probabilities under this framework assume a first-order Markov process requiring that the probabilities that we are in a regime g are dependent on the previous regime such that:

$$P(r_t = j \mid r_{t-1} = i) = P_{ij}(t) \quad (9)$$

$P_{ij}(t)$ are assumed to be the same irrespective of t i.e. $P_{ij}(t) = P_{ij}$ for all t. the first-order Markov model has a transition matrix expressed as

$$P^1(t) = \begin{bmatrix} p_{11}(t) & . & . & . & p_{1g}(t) \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ p_{g1}(t) & . & . & . & p_{gg}(t) \end{bmatrix}$$

where ij^{th} element denotes the probability of moving to regime j in period t from regime i in period t-1. Equation (5) can be written as

$$P_{ij}(U_{t-1}, \gamma_i) = \frac{\exp(U_{t-1}'\gamma_{ij})}{\sum_{s=1}^{G} \exp(U_{t-1}'\gamma_{is})} \quad (10)$$

For all j = 1, 2… G, i = 1, 2… G, $\gamma_{iG} = 0$.

Filtering of one-step-ahead regime probabilities

One property of MS $p_{ij}(t)$ is that the full log-likelihood in equation (4) is evaluated recursively. The recursion process involves filtering probabilities, $P(r_{t-1} = g \mid \varsigma_{t-1})$ and is carried out in three steps:

a. One-step ahead prediction of regime probabilities are formed from laws of probability and the transition matrix given as

b. $$P(r_t = g \mid \varsigma_{t-1}) = \sum_{j=1}^{G} P_{jg}(U_{t-1}, \gamma_j) P(r_{t-1} = j \mid \varsigma_{t-1}) \quad (11)$$

c. The predictions in the earlier step above are used to derive the one-step-ahead joint density predictions and regimes in period *t*.

$$f(y_t, r_t = g \mid \varsigma_{t-1}) = \frac{1}{\sigma_g}\phi\left(\frac{y_t - \mu_t(g)}{\sigma(g)}\right)P(r_t = g \mid \varsigma_{t-1}) \quad (12)$$

d. A sum of the joint probabilities across unobserved regimes to obtain the likelihood contribution for time period *t*. These are the marginal distribution of the observed data

$$L_t(\theta,\delta,\sigma,\gamma) = f(y_t \mid \varsigma_{t-1}) = \sum f(y_t, r_t = j \mid \varsigma_{t-1}) \quad (13)$$

e. Filtering is then carried out to obtain updated one-step-ahead predictions of the probabilities

$$P(r_t = g \mid \varsigma_t) = \frac{f(y_t, r_t = g \mid \varsigma_{t-1})}{\sum_{j-1}^{G} f(y_t, r_t = j \mid \varsigma_{t-1})} \quad (14)$$

The process is repeated for $t = 1, 2, \ldots\ldots, T$ time periods. Filtering requires initial one-step ahead regime probabilities $P(r_t = g \mid \varsigma_0)$ which are equivalent to initial filtered probabilities $P(r_0 = g \mid \varsigma_0)$ set to steady-state (ergodic) values imposed by the Markov transition matrix.

Smoothing of one-step-ahead regime probabilities

Unlike the filtered estimates which are based on contemporaneous information ς_t , the smoothed estimates for the regime probabilities at time *t* use data in the closing time ς_T. according to Kim (2004) and Kim and Nelson (1999), smoothing can be done efficiently through an efficient smoothing algorithms which involve simple backward recursions through the data by specifying a joint probability as follows

$$P(r_t = i, r_{t+1} = j \mid \varsigma_T) = \frac{P(r_t = i, r_{t+1} = j \mid \varsigma_t)}{p(r_{t-1} = j \mid \varsigma_t)} P(r_{t+1} = j \mid \varsigma_T) \quad (15)$$

The smoothed probabilities at time *t* is obtain by marginalizing the joint probability w.r.t r_{t+1} such that

$$P(r_t = i \mid \varsigma_T) = \sum P(r_t = i, r_{t+1} = j \mid \varsigma_T) \quad (16)$$

Hamilton model of switching with autoregressive dynamics

MS discussed so far can be modified to include lagged endogenous variables and serially correlated errors. In section 2.2, we mentioned that by extending the simple regime framework which involves the introduction of the first-order Markov process, the transition probability is dependent on the previous regime with serially uncorrelated errors. If the errors are serially correlated, Hamilton (1989) proposed an AR specification with a serial correlation of order *p* as follows:

$$y_t = \mu_t(r_t) + \sum_{m=1}^{p} \rho_m(r_t)(y_{t-m} - \mu_{t-m}(r_{t-m})) + \sigma(r_t)\varepsilon_t \quad (17)$$

Equation 17 is generally regarded as the Hamilton model of switching with dynamics. According to Fruhwirth-Schnatter (2006) and Krolzig (1997), this model is also referred to as the MS autoregressive (MSAR) and MS mean (MSM) respectively. Under these models, the mean equation

$$\mu_t(g) = X_t^{'}\theta_g + \Gamma_t^{'}\delta + \sum_{m=1}^{k} \varphi_{mg} y_{t-m} \quad (18)$$

depends on the lagged states. From equation 17, a specification with a serial correlation of order *p* is given as

$$(1 - \sum \rho_r(r_t)L^r)(y_t - \mu_t(r_t)) = \sigma(r_t)\varepsilon_t \quad (19)$$

In order to obtain the likelihood function, we need the probabilities for *p* + 1-dimensional state vectors for the current and *p* previous regimes due largely to the presence of regime-specific lagged mean, $\mu_{t-m}(s_{t-m})$, in equation 17. The normal mixture log-likelihood function (NMLLF) for an AR (*p*) specification is given as

$$l(\theta,\delta,\sigma,\gamma,\rho) = \sum_{t=1}^{T} \log\left\{\sum_{i=1}^{p}\sum_{j=1}^{p} \frac{1}{\sigma(i)}\phi\left(\frac{y_t - \mu_t(i) - \rho_g(i)(y_{t-1} - \mu_{t-1}(i))}{\sigma(i)}\right)p(r_t = i, r_{t-1} = j \mid \varsigma_{t-1})\right\} \quad (20)$$

Equation 20 requires *p* + *p* potential regime outcomes probabilities for the state vector (r_t , r_{t-1}) with $G^* = G^{p+1}$ potential realizations. The basic equation 1 is not valid for a MSAR because of its inability to take care of the filtering and smoothing of G^*.

Filtering and Smoothing of Markov-Switching AR

Efficient operational lagged-state nonlinear filtering procedures for obtaining optimal statistical estimates by maximizing the normal mixture log-likelihood function in equation 20 have been proposed in the literature. Specifically, Hamilton (1989) filter is an extension of the Markov-switching filter to accommodate p + 1-dimensional state space which follows the same principles described in section 2.3 except that the filtered probabilities for lagged values of the states r_{t-1}, r_{t-2}, r_{t-3},…………, r_{t-p} conditional on the data ς_{t-1} depend on the previous iteration of the filter and the one-step-ahead joint probabilities. A modified lag-state smoothing procedure known as the Hamilton smoother was also proposed by Hamilton (1989). According to Hamilton (1990), the filter formalizes the statistical identification of "*turning points*" or cycles of time series. The procedure was improved upon by Kim (1994). The Kim smoother is an extension of the basic smoothing procedure described in section 2.4 which is an efficient smoothing filtering procedure that handles the G^* vector of probabilities using the single backward recursion pass.

The presence of lagged states in the specification of the MSAR alters the estimation process which consists of evaluating $p + 1$ vector of state variable representing the present and lag states. The estimation of the MSAR specification parameters is done by handling the model as a restricted MS model with transition probabilities which are independent of the regime of origin. The transition probabilities in equation 9 are modified such that the rows of the transition matrix are indistinguishable i. e.

$$P(r_t = j \mid r_{t-1} = i) = p_{ij}(t) = p_j(t) \qquad (21)$$

and

$$P(t) = \begin{bmatrix} p_1(t) & . & . & . & p_G(t) \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ p_1(t) & . & . & . & p_G(t) \end{bmatrix}$$

The Hamilton Smoother or the Kim Smoother is then applied to produce the one-step ahead, likelihood, filtered and smoothed values, on the restricted MS Model specification. The initial probabilities of the vector of probabilities associated with the $G^* = G^{p+1}$ dimensional state vector are computed starting with the uncorrelated model of equation 2, setting G initial probabilities in period (- p + 1) using the steady-state values (ergodic solution) and recursively updating the process to produce the joint initial probabilities for the G^* vector of probabilities in period zero.

Results and Discussion

We collected secondary data on three selected macroeconomic variables (exchange rate (*exr*), crude oil price (*cp*) and foreign external reserve (*fer*)) over 16 years (2006 and 2021) from the database of the Central Bank of Nigeria (CBN). The time-series data were converted to log differences. This was subject to the assumption that the time series (Figure 1) are nonstationary. Exchange rate movement can be categorized either in "*rise*" or "*drop*" phase, with a switch between the two phases controlled by the outcome of a Markov process. Hence, a time-invariant MS Autoregressive model (MSAR) with serial autocorrelation of order $p = 4$ on the Nigeria Exchange rate was postulated. In our specification, the economy is in regime 1 whenever the exchange rate drops and in regime 2 whenever the exchange rate rises and the transition between the two regimes are modelled as the outcome of the first-order Markov process. If it is in the "*rise"* state, this may signify depreciation in the local currency but if it is in the "*drop"* state, then the currency is said to appreciate against the US Dollars. The order of the serial autocorrelation was chosen to ensure the invertibility of the AR specification. The model was estimated using the method of BFGS with Marquardt steps for G = 2 regimes. The covariance and standard errors were computed using the observed inverse negative Hessian method. The optimization involves a random search with varying starting values (using L'Ecuyer random number generator and setting appropriate seeds) and iterations for the model. MS nonlinear iterative filtering was achieved through Ergodic solution of estimating the filtered initial one-step ahead probabilities which also control the evaluation of the likelihood.

Time-invariant MSAR (*p* = 4) on Exchange Rate

Firstly, we estimated the time invariant MSAR with serial autocorrelation of order $p = 4$ with the optimization process reaching convergence after zero iterations. The random search started with 1,000 values with 500 iterations using 1σ. The L'Ecuyer random number generator was employed, and the seed was set at 578,736,543.

Table 1 shows the maximum likelihood estimates for the two regimes obtained by numerically maximizing the conditional log-likelihood function in equation 20. A prior suggestion is that we associate $r_t = 1$ and 2 (i.e., $g = 1 \rightarrow$ *drop* and $g = 2 \rightarrow$ *rise*) with appreciation and depreciation respectively in exchange rate regimes in Nigeria. The coefficients of the conditional mean of the exchange rate at time *t* differ significantly from zero in both regimes with the positive sign which is associated with a high value of the small standard error relative to the value of the estimator. The sample likelihood is maximized by a rise in the exchange rate of more than ten times per month in the two regimes with regime one slightly higher than regime two. We specify log differenced foreign external reserve and crude oil price at time *t*. In each regime, two regimes' specific regressors are estimated. In Regime 1 regression, the two regressors are statistically significant where foreign external reserves have a positive effect on the exchange rate. In Regime 2, whilst foreign external reserves have a positive effect and are significant, Crude oil prices have a statistically insignificant negative effect on the response variable. In this time-invariant MSAR, we assume a common error variance and AR (1), AR(2), AR(3) and AR(4) with values which are highly significant at all levels.

It is revealed that a decline in the exchange rate is associated with higher probabilities of being in a *rise* exchange rate regime. Consequently, the probability of transiting from $g = 2$ is reduced and increasing the transition probability of entering $g = 2$ from $g = 1$. The transition probabilities are shown in **Table** 4. This table is a summary of constant transition probabilities and constantly expected durations spent in each regime

Table 1: Time-invariant MSAR, *p* = 4 model Regime (*g* = 1, 2) coefficients, standard errors, *z* statistics and *p*-values

	$g = 1$				$g = 2$			
Variable	θ	σ_θ	**z-Stat.**	***p*-value**	θ	σ_θ	**z-Stat.**	***p*-value**
c	1129.45	223.3321	5.0572	0.0000*	1080.6820	223.3377	4.8388	0.0000*
$\Delta \log fer_t$	217.8016	34.4018	6.3311	0.0000*	15.7534	9.2025	1.7112	0.0869***
$\Delta \log cp_t$	-0.4439	0.1657	-2.6796	0.0074*	-0.0365	0.0703	-0.5186	0.6041

Note: *,*** indicate coefficient is significant at 1 per cent and 10 per cent levels respectively.

Table 3: Transition Matrix Parameters

Variable	Coefficient	Std. Error	z-Statistic	*p*-value
P_{11}-c	1.2975	0.6520	1.9900	0.0466**
P_{21}-c	-3.92478	0.6162	-6.3693	0.0000*

Note: *,** indicate coefficient is significant at 1 per cent and 5 per cent levels respectively.

Table 2: Common Parameters

Variable	Coefficient	Std. Error	z-Statistic	*p*-value
AR(1)	1.8289	0.0729	25.0862	0.000*
AR(2)	-1.4471	0.1419	-10.1969	0.000*
AR(3)	1.0541	0.1409	7.4827	0.000*
AR(4)	-0.4367	0.0772	-6.0801	0.000*
log(σ)	1.9085	0.0569	33.5427	0.000*

Note: * indicate coefficient is significant at 1 per cent level.

Table 4: Transition probabilities and expected durations

$P_{i,j} = P\,(r_t = j \mid r_{t-1} = i)$

Constant transition probabilities

$$P(t)=\begin{bmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{bmatrix}=\begin{bmatrix} 0.7854 & 0.2146 \\ 0.0194 & 0.9806 \end{bmatrix}$$

***Constant expected durations*:**

Regime 1 (*drop*)	Regime 2 (*rise*)
4.6603	51.6402

The transition probability matrix and expected durations show a significant dependence on the state in the transition probabilities in which there exist high probabilities of remaining in the origin, regime 1 (0.7854) and regime 2 (0.9806). The exchange rate is expected to spend about 5 months in the '*drop'* state (regime 1) and 52 months in the '*rise'* state (regime 2). The standard descriptive statistics on this model are AIC = 7.0743, SSR = 18,860.49 and the normal mixture log-likelihood = -581.24.

In Figure 2, the left panels represent the one-step-ahead regime 1 and 2 probabilities. The middle panels are plots on the filtered one-step ahead regime 1 and 2 probabilities. The right panels represent the smoothed regime 1 and 2 probabilities respectively. These plots demonstrate the probability that the Nigerian economy is in increasing and declining exchange rate states at time period *t* using the information available at the time period ς_{t-1}. These plots indicate that as the probabilities of being in regime 1 increase, the probabilities of being in regime 2 decline at a particular time period *t*. Using the plots on the filtered and smoothed probabilities, we were able to identify different dates representing cycles, coinciding with periods of significant change in exchange rate policy by the CBN. These dates are 2015, 2016, 2017 and 2020.

Inverse Roots of the AR **polynomials for the two models**

The stationarity of the model was tested by checking for any root lying outside the unit circle. **Table** 5 and **Figure** 3 both show the result of the check. The roots display the inverse of the Autoregressive (AR) polynomials for the model. The idea is that if the AR specification is stationary, all the AR roots will lie within the unit circle where for each root, the real and imaginary parts are represented by horizontal and

International Journal of Women in Technical Education and Employment
ISSN: 2811-1567. Volume 3 – Issue 1. July 2022
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vertical axis respectively. **Table** 5 shows that the model has both the real value and imaginary part (the sum of squares of the real and imaginary parts).

Table 5: Inverse Roots of AR Polynomials

AR Root(s)	Modulus	Cycle
0.9978	0.9978	
0.7683	0.7683	
0.0314 ± 0.7540*i*	0.7547	4.1089

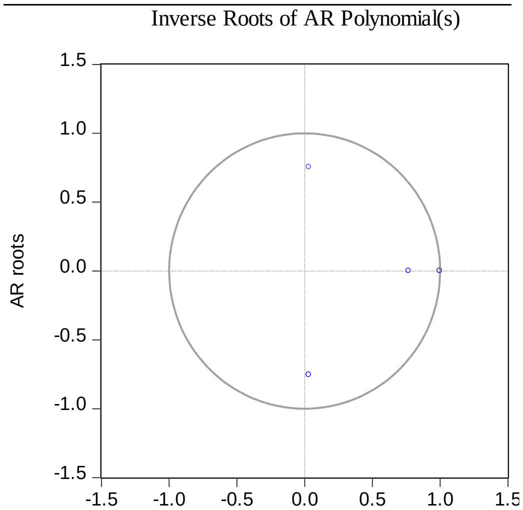


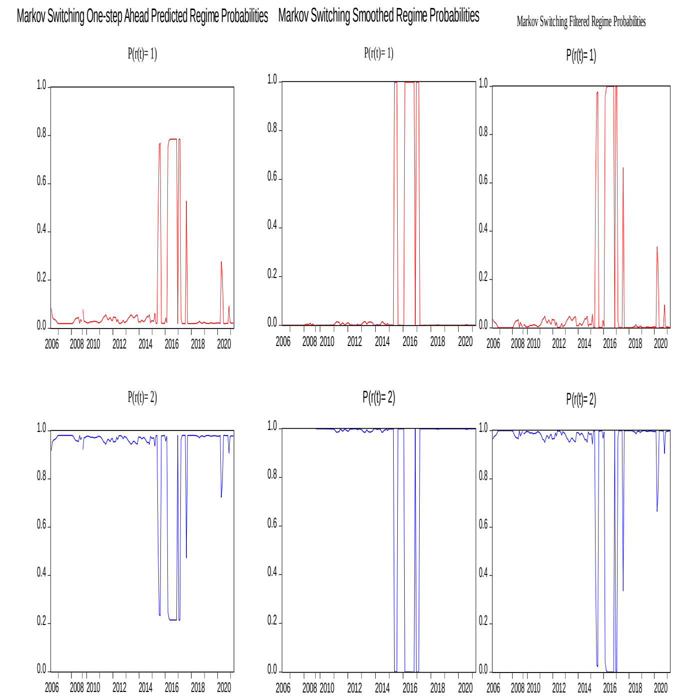
Figure 3: Plots on Inverse Roots of AR Polynomials

We also plotted the cycles corresponding to roots which were calculated using 2π / s, where s = atan(i / h), i and h are the imaginary and real parts of the roots. Since the cycles for the real roots are infinite, we did not include them in the analysis. **Figure** 3 shows that no roots lie outside the unit circles which indicates that our MSAR model is stationary. Hence this model forecast errors diagnosis and performance were carried out as follows.

Forecast error Diagnosis and Performance of the MSAR model

The model was checked for residual stability for purposes of the forecast. The fitted values and residuals are generated by computing the expected values $\hat{y}_t$ from the one-step-ahead regime

Figure 2: Markov-Switching One-step ahead, Filtered, and smoothed regime Probabilities



probabilities and regime-specific values. Before forming the expectations, the regime-specific residuals were scaled by estimating the regime standard errors, to calculate the standardized residuals. The plot is shown in **Figure** 4 below. The plot is on the actual, fitted and standardized residual. A close look at the plot indicates that the actual and the fitted values are close.

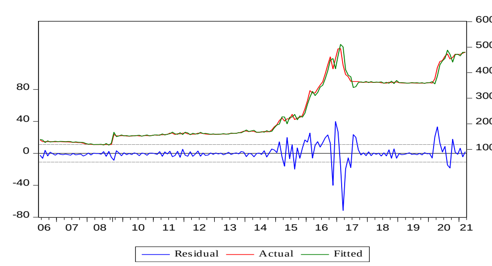


Figure 4: Actual, Fitted and Standardized residual values for the two MSAR models

Summary and Conclusion

In this research work, two important macroeconomic variables influencing the movement of exchange rates such as crude oil price and foreign external reserve were used as regressors in a Hamilton model of switching with autoregressive dynamics or a two-state MS autoregressive setting. This model is a Time-invariant MSAR with fourth-order of autocorrelation. Two unobserved regimes were included in the model. The model produced similar results concerning regime-specific regressor effects on exchange rate. Analysis shows that in the "*drop*" regime, both regressors are statistically significant but only the foreign external reserve is significant in the "*rise*" regime of the exchange rate model. Furthermore, analysis of transition between regimes indicates that the exchange rate is more likely to transit from regime 1 to regime 2. However, the chance that the exchange rate remains in the regime of origin is higher for regime 2 than regime 1, although both are high. Generally, the exchange rate is expected to stay much longer in regime 2 than in regime 1 in the model. In conclusion, the exchange rate movement during the period under consideration is such that it is more likely to transit from a rising to a declining state. In addition, the expected time spent in appreciation is significantly lower than in depreciation. Important dates of policy changes were also captured by the filtered and smoothed probability of the model. These dates coincide with periods of significant policy changes by the apex bank.

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