A Simultaneous Technique Estimation of Keynesian Economic Growth Model

Adeboye, Nureni Olawale (PhD)[1] and **Agunbiade, Dawud Adebayo (PhD)**[2]
[1]Department of Mathematics & Statistics, Federal Polytechnic, Ilaro, Nigeria. P.M.B 50
[2]Department of Mathematical Sciences, Olabisi Onabanjo University, Ago-Iwoye, Nigeria
nureni.adeboye@federalpolyilaro.edu.ng and bayoagunbiade@gmail.com

Abstract
This research compared different Simultaneous techniques in the estimation of Keynesian model specified to estimate the effects of International trade on the Economic Growth of Nigeria within the periods of 1991 to 2018. The study reveals that truly Keynesian theory of consumption can promote production because consumption responds perfectly to the output in the previous periods. Consumption responds to 72% output while investment responds to 38 % output in the previous year and a unit increase in time with first lag of consumption and investment fixed, bringing about 14% and 0.03% increase in consumption and investment respectively. The t-statistic shows that the first lag of consumption and investment are significant each in their respective equations while F- statistic revealed that all parameters are simultaneously significant in their respective equations. The estimators (2SLS, 3SLS, FIML and LIML) were compared based on Root Mean Square Error, it shows that Full Information Maximum Likelihood (FIML) is the best in estimating the system of Simultaneous Equation because of its least RMSE value.

Keywords: Full Information Maximum Likelihood, Economic Growth, Keynesian Model, Simultaneous Equation.

1. Introduction

GDP growth is one of the most important criteria to evaluate the performance of an economy. Economic growth is a rise in per capital income (Jelilov *et al.*, 2015). That is, an increase in the total output of an economy per person, all things being equal. A large number of studies have been conducted.to identify the main drivers of economic growth and the potential sources of growth (See Jordan and Eita, 2007; Ullah *et al*., 2009; Pazim, 2009; Shahbaz *et al*., 2011; Shahbaz *et al*., 2018). The studies indicate different drivers of growth including Foreign Direct Investment, Domestic Investment, Financial Development and Export. Adapting the foreign trade-led-growth model of Giles and Williams (2000), this study however aims to study economic growth via the long run relationship between International Trade and Gross Domestic Product of Nigeria by employing Keynesian Model.

According to Awe (2013), the relevance of foreign trades in boosting economic growth and prosperity is captured in the theoretical justification for international trade. It has a great impact on the economy of developing countries most especially for those who basically rely on it to increase their GDP. The theory of comparative cost advantage states that global output will reach its optimum level if every country specializes in the production of the commodity (or commodities) in which it has comparative cost advantage over others; this is seen as the basis for profitable trade (Ozughalu and Ajayi, 2004). Meanwhile, it is pertinent to emphasize that Nigerian economy is fast deteriorating as a result of much dependency on the imports aspect of international trade, due to its greater negative impact on the country's exchange rate.

The Keynesian Multiplier Theory emphasized the impact of government spending on the National income such that government spending formed an integral part of the model under consideration. Thus, the research model specified for this study identified Import (IM_t), Export (Ex_t), Consumption (C_t,), Investment (I_t) and Government Expenditure (G_t) as explanatory variables while GDP (Y_t) is tagged as the response variable. In developing countries, government expenditure policy not only accelerates economic growth & promotes employment opportunities, but also plays a useful role in reducing poverty and inequalities in income distribution (Jelilov and Onder, 2016).

The theoretical gist of the model's design is based on the Keynesian National Income Accounting Identity that employed the above listed phenomena. However, using only a single equation model to explore the relationship between Foreign Trade and Economic Growth is not enough considering the probable two-way causal link existing in the variables. As a result, establishing Simultaneous Multi-equations Model is a more appropriate choice.
A few other authors who have written extensively on the theoretical framework of foreign trade-led-growth model but conjectured differently from the theoretical gist of The Keynesian Multiplier Theory are Leontief (1990); McCombie and Thirwall (1997); Shan and Sun (1998); Rodriguez and Rodrik (2001); Alimi and Atanda (2011); Sarbapriya (2011); Alimi and Atanda (2011); Omoju and Adesanya (2012); Omoju (2012); Mishra (2012); Chen (2013); Ajmia *et al.* (2013).

2. Materials and Methods

The variables included in this model are based on data collected from CBN Statistical Bulletin for periods of 27 years (1991-2018).

2.1 Keynesian Model Specification

The model is based on Keynesian National Income Accounting Identity and was adapted from the work of Chen (2009) for China Economy as follows:

GDP = Consumption + Investment + Government spending + Exports – Imports (1)
Since a single equation model cannot explain the relationship between economic growth and foreign trade because there is two-way flow of influence among the economic variables, we need to consider multi equations, which leads to set of simultaneous equations, one for each interdependent variable as follows:

$$Y_t = C_t + I_t + + G_t + EX_t - IM_t \quad (2)$$

$$C_t = \beta_0 + \beta_1 IM_t + \beta_2 C_{t-1} + \mu_{1t} \quad (3)$$

$$I_t = \gamma_0 + \gamma_1 Y_t + \gamma_2 I_{t-1} + \mu_{2t} \quad (4)$$

Where β_i and γ_i, (i = 0, 1, 2) are the unknown parameters. Y_t, C_t and I_t are the endogenous variables; G_t $C_{t-1}$and I_{t-1} are the predetermined variables. μ_{1t} and μ_{2t} are the random disturbance terms and it follows a normal distribution. Equations (2) to (4) are structurally identified through their standard forms given as follows:

$$0 - C_t - I_t + Y_t - G_t - EX_t + IM_t + 0C_{t-1} + 0I_{t-1} = 0 \quad (5)$$

$-\beta_0 + C_t + 0I_t + 0Y_t + 0G_t + 0EX_t - \beta_1\ IM_t - \beta_2\ C_{t-1} + 0I_{t-1} = \mu 1_t$ (6)

$-Y_0 + 0C_t + I_t - Y_1 Y_t + 0G_t + 0EX_t + 0IM_t + 0C_{t-1} - Y_2\ I_{t-1} = \mu 2_t$ (7)

All equations (5) to (7) met the exactly and over identification conditions and the LHS can be expressed in matrix form as

$$\begin{Bmatrix} 0 & -1 & -1 & 1 & -1 & -1 & 1 & 0 & 0 \\ -\beta_0 & 1 & 0 & 0 & 0 & 0 & -\beta_1 & -\beta_2 & 0 \\ -Y_0 & 0 & 1 & Y_1 & 0 & 0 & 0 & 0 & -Y_2 \end{Bmatrix}$$

2.2 Estimation Methods

The system of simultaneous equations of this model makes the Ordinary Least Square (OLS) method of estimation inappropriate, hence the following methods of evaluations shall be adopted:

Two Stage Least Square (TSLS): As the name implies, the method involves two successive applications of Ordinary Least Squares (OLS).

Given the linear regression;

Y= $\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$. (8)

where X_1 is an endogenous variable, we regress X_1 on Z_1 , X_1 and X_3 to obtain X_1 as

$$X_1 = Y_0 + Y_1 Z_1 + Y_2 X_2 + Y_3 X_3 + \upsilon \quad (9)$$

Where Z_1 is the instrumental variable. Plug in the fitted values of X_1 derived from equation (9) into equation (8), we have

$$\text{Y}= \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \upsilon \quad (10)$$

Where υ is a composite error term that is uncorrelated with X_1, X_2 and X_3

Limited Information Maximum Likelihood (LIML): LIML has desirable large sample properties and uses instruments to rectify the problem where one or more of the right hand side variables in the regression are correlated with residuals. The linear LIML estimator minimizes:

$$\boldsymbol{\Psi(\beta) = T\frac{(y-X\beta)\prime Z(Z'Z)^{-1}Z\prime(y-X\beta)}{(y-X\beta)\prime(y-X\beta)}} \quad (11)$$

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With respect to $\boldsymbol{\beta}$, where y is the dependent variable, X are explanatory variables, and Z are instrumental variables. Computationally, it is often easier to write this minimization problem in a slightly different form. Let W = (y − X) and β = (1 −β), then the linear LIML objective function can be written as:

$$\boldsymbol{\Psi(\beta) = T\frac{\beta W\prime Z(Z'Z)^{-1}Z\prime W\beta)}{\beta\prime W\beta W)}} \quad (12)$$

λ is the smallest Eigen value of $W'W$ −1 WZ $Z'Z$ −1$Z'W$. The LIML estimator of $\boldsymbol{\beta}$ is the eigenvector corresponding to $\boldsymbol{\lambda}$, with normalization so that the first element of the eigenvector equals 1. The non-linear LIML estimator maximizes the concentrated likelihood function

$$L = -\frac{T}{2}(\log(u'u) + log|X'AX - X'AZ(Z'AZ)^{-1}Z'AX|) \tag{13}$$

Where $\boldsymbol{ut}$= $\boldsymbol{yt}$ − ($\boldsymbol{Xt}$ $\boldsymbol{\beta}$) are the regression residuals and $\boldsymbol{A}$=$\boldsymbol{I}$−($\boldsymbol{u'u}$)−$\boldsymbol{1u'}$.

Three Stage Least Square (3SLS): 3SLS is an appropriate technique when right-hand side variables are correlated with the error terms, and there is both heteroscedasticity and contemporaneous correlation in the residuals. It first computes two stage least square and then applied Seemingly Unrelated Regression (SUR) to the result generated. SUR uses the OLS residuals to obtain a consistent estimate of the cross-equation covariance matrix Σ. This covariance estimator is not however consistent, if any of the right-hand side variables are endogenous. 3SLS uses the 2SLS residuals to obtain a consistent estimate of Σ in the balanced case. The equation as given as

$$\hat{\delta}_{3SLS} = (Z\left(\hat{\Sigma}^{-1} \otimes X(X'X)^{-1}X'\right)Z\left(\hat{\Sigma}^{-1} \otimes X(X'X)^{-1}X'\right)y \tag{14}$$

Full Information Maximum Likelihood (FIML): FIML estimates the likelihood function under the assumption that the contemporaneous errors have a joint normal distribution. Provided that the likelihood function is correctly specified, FIML is fully efficient.
If we use *X to* represents all the data, then the probability density function (pdf) is given as

$$f(X) = \frac{e^{-\frac{1}{2}(X-\mu)\prime\Sigma^{-1}(X-\mu)}}{\sqrt{(2\pi)^k|\Sigma|}} \tag{15}$$

Thus the likelihood function is given by

$$L(\mu, \Sigma) = \prod_{i=1}^{n} f(X_i; \mu, \Sigma) \tag{16}$$

where X_i is the *i*th row of X. The log-likelihood function is

$$LL(\mu, \Sigma) = \sum_{i=1}^{n} LL_i(\mu, \Sigma) \tag{17}$$

Where

$$LL_i = -\frac{1}{2}(X_i - \mu)'\Sigma^{-1}(X_i - \mu)' - \frac{1}{2}kln(2\pi) - \frac{1}{2}ln|\Sigma|$$

Our goal is to maximize LL, which is the same as minimizing $-2LL$. *Thus,*

$$-2L(\mu, \Sigma) = \sum_{i=1}^{n} -2LL_i(\mu, \Sigma) \tag{18}$$

Where

$$- 2\text{LL} = (X_i - \mu)'\Sigma^{-1}(X_i - \mu)' + kln(2\pi) + ln|\Sigma| \tag{19}$$

The goal is to find the values for Σ and μ which maximize LL, or equivalently which minimize -2LL.

2.3 Validity Check

This aims at estimating the reliability of our estimated parameters. The statistical test employed in this research are test of significances of estimated parameters (F-statistics, Student-test,) and Goodness of Fit (Coefficient of determination), Wald Test, MSE and RMSE.

3. Results and Discussion

The performances of the estimators evaluated by using RMSE are as presented in Table 1 below:

Table 1: Comparison of the Estimators Performance

Estimators	**RMSE**	**Sum of square error**		**Mean square error**	
	Con/Inv	**Consumption**	**Investment**	**Consumption**	**Investment**
2SLS	0.039/0.066	1.07E+14	2.97E+13	635702.4851	913944.264
3SLS	0.042/0.066	1.20E+14	2.97E+13	622140.5175	929895.6298
LIML	0.061/0.067	2.51E+13	2.99E+13	6635702.4821	25205041.51
FIML	0.028/0.053	0.51E+13	1.97E+13	368622.2361	904238.3861

From the table above, Full Information Maximum Likelihood has the minimum RMSE and thus the minimum level of bias, and it is hereby recognized as the best estimator. We therefore provide the summary of FIML estimated results of our Keynesian model as presented in Table 2.

Table 2: Summary of Full Information Maximum Likelihood.

Explanatory Variables	**Consumption**	**Investment**
Y_t	_	-0.001401 (-0.203654)
C_{t-1}	0.723866 (5.367282)	_
I_{t-1}	_	0.424483 (1.956377)
IM_t	0.047530 (0.640973)	_
Constant	304432.1 (0.412861)	566302.7 (0.892701)
R^2	0.628224	0.183364
Number of observations	27	27
Instrumental Variables	G_t ,$C_{t-1,}I_{t-1}$	G_t,$C_{t-1,}I_{t-1}$

From Table 2, the consumption equation is given as

$$C_t = 304432.1 + 0.047530IM_t + 0.723866C_{t-1} \tag{20}$$

where β_{1} = 0.047530 indicates that for a unit increase in import when the first lag of consumption is fixed, will result into about 4% increase in consumption while β_{2} = 0.723866 implies that consumption responds to about 72% of the output in the first lag period. The investment equation is equally given as

$$I_t = 566302.7 - 0.001401Y_t + 0.424483_{t-1} \quad (21)$$

where $\gamma_{1\,=} - 0.001401$ indicates that a unit increase in income when the first lag of investment is fixed will result into about 0.1% decrease in investment while $\gamma_{2\,=} 0.424483$ implies that investment responds to about 42% of the output in the first lag period.

The coefficient of multiple determination (R^2) which measures the overall goodness of fit gives the value $R^2 = 0.628224$ for the consumption equation and $R^2 = 0.183364$ for the investment equation. These imply that approximately 63% and 18.3% of the dependent variable is explained by the independent variables respectively for the two models. The values in parentheses are the results of the t-statistic compute for each of the parameter estimates which measures the significance of the identified parameters.

4. Conclusion

This research was carried out to estimate Keynesian model of economic growth via a simultaneous technique. The study reveals that truly Keynesian theory of consumption can promote production because consumption responds perfectly to the output in the previous period. Consumption responds to 72% output in previous year while investment respond to 42% for the same period. When first lag of consumption and investment are fixed, contribution of about 4% and 0.01% decrease were made in consumption and investment respectively. The implication of this is that government spends more on consumption in previous year than they save for future trade, and to overcome the challenges of growth slowdown as a natural consequence of economic maturity, Government will have to reduce its consumption of imported goods; increase its production so as to participate in international trade which will in-turn earn the country substantive foreign income for the purpose of investments to attract foreign investors and boost the standard of living of her citizenry.

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