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Monte Carlo Estimation of Heterogeneity Effects in a Panel Data Regression Model

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ABSTRACT

Violation of homoscedasticity assumption in a Panel Data Regression Model (PDRM) implies unequal variability of error terms, and this creates heterogeneity problem in estimation. This research thus attempts to investigate this phenomenon by extending the works of Baltagi et al. (2010) and Adeboye and Agunbiade (2017) within the context of fitting an audit fee model via a simulated panel data and its estimation through the derivation of a Conditional Lagrange Multiplier (CLM) test for heteroscedasticity given zero first-order serial correlation via a two-way error component model. Monte Carlo simulations were carried out for 27 different variations, of which its design assumed a uniform distribution under a linear heteroscedasticity function. Each of the variation was iterated 1000 times and the assessments of estimators considered are based on Absolute bias (ABIAS) of parameters estimates. Nine (9) different models at different specified conditions were fitted, and the best-fitted model is that given by a within estimator when heteroscedasticity is severe. This study established that using CLM test; the results provide good size and power at 5% significance level for the specified linear form of heteroscedasticity function with α assigned values 0, 1 and 2 denoting homoscedastic individual specific error, moderate and severe heteroscedastic errors respectively.

Keywords: Audit Fee, Heterogeneity, Lagrange Multiplier Test, Monte-Carlo Scheme, Panel Data Regression Model.

Mathematics Subject Classification: 91G70, 97K80

1. INTRODUCTION

Panel data models examine individual-specific effect, time-effect or both in order to deal with heterogeneity/periodicity of individual effects that may or may not be observed [1-5]. In this paper, focus shall only reflect on the problems which affect the cross sectional aspect of panel data (i.e heterogeneity), which is the problem imposed by heteroscedasticity. This shall be looked into via a Panel Data Regression Model (PDRM) of audit fees estimated from simulated data.

Heteroscedasticity is one of the associated problems with the Pooled Ordinary Least Squares (POLS) [2]. According to [6], Heterogeneity is an important problem faced by the statistician or the econometrician trying to infer the behavior of economic agents from available data sets. Economic decision makers are heterogeneous in their characteristics and they usually operate in heterogeneous (different) environments. As a result, their behavior generates data whose distributions are sometimes difficult to approximate with the traditional single component econometric models. [7] and [8] confirmed that in the presence of heteroscedasticity, POLS estimates are unbiased, but the usual tests of significance are generally inappropriate and their use can lead to incorrect inferences. The pioneering work of [9] has given rise to further researches on the estimation of heteroscedasticity effects in panel data. Prominent among these works are those of [10-22]. However, this work is based on two-way error components model as against the aforementioned that dealt with one-way error component model. [14] suggest a robust version of [23] for no random individual effects, by allowing for adaptive heteroscedasticity of unknown form on the remainder error term. [6] extended the work of [24] to panel data setting by modeling several related mixture models to take care of heterogeneity problems in panel data sets.

This research therefore, intends to extend the works of [25] in compliance with some of the methodology established in [26] and [27] by testing for the effect of heterogeneity in the presence of varying degree of heteroscedasticity via a two-way error components model, where zero serial correlation is assumed. This shall be done within the context of specified audit fees model.
Prominent among authors who have worked on modeling of audit fees are [28-33], but they all conjectured differently from the background knowledge of the audit fees model specified in this research, which is in line with that specified in [25].

2. GENERATION OF THE DATA

The realization of this work was based on simulation process using three sizes of cross-sectional units (i.e N=20, 40 and 60), three time periods (i.e T = 10, 40 and 100), a homoscedastic situation and two degrees of heteroscedasticity (i.e moderate and severe). These variations were used for simulation with each of the combinations iterated 1000 times and the assessment of the three estimators considered based on the Absolute bias (ABIAS).

2.1 *Specification of PDRM for Fixed and Random Effect.*

The two models specified for Fixed and Random effect models are given respectively as

$$AF_{it} = \alpha_1 + \alpha_2 D_{2i} + \alpha_3 D_{3i} + \cdots + \alpha_N D_{16i} + \beta_2 PBT_{it} + \beta_3 TA_{it} + \beta_4 TL_{it} + \beta_5 SHF_{it} + \gamma_1 (D_{2i}\ \mathrm{PBT_{it}}) + \gamma_2 (D_{2i}\ \mathrm{TA_{it}}) + \gamma_3 (D_{2i}\ \mathrm{TL_{it}}) + \gamma_4 (D_{2i}\ \mathrm{SHF_{it}}) + \ldots + \gamma_{4(N-3)} (D_{Ni}\ \mathrm{PBT_{it}}) + \gamma_{4(N-2)} (D_{Ni}\ \mathrm{TA_{it}}) + \gamma_{4(N-1)} (D_{Ni}\ \mathrm{TL_{it}}) + \gamma_{4N} (D_{Ni}\ \mathrm{SHF_{it}}) + \varepsilon_{it} \quad (1)$$

$$AF_{it} = \beta_1 + \beta_2 PBT_{it} + \beta_3 TA_{it} + \beta_4 TL_{it} + \beta_5 SHF_{it} + \omega_{it} \quad (2)$$

Where Profit before Tax (PBT), Total Assets (TA), Total Liability (TL) and Shareholders Fund (SHF) which shall be originated from panel data simulated using a uniform distribution under a linear form of heteroscedasticity.

β_1, β_2, β_3, β_4 and β_5 are estimable parameters and ω_{it} is a composite error term.

In the course of this study, it was demonstrated that the conditional variance of AF_{it} increases as each of $PBT_{it}, TA_{it}, TL_{it}$ and SHF_{it} increases.
Model (1) and (2) were estimated using

Pooled OLS estimator: $\hat{\beta}_{pooled} = (X'X)^{-1} X' y$ (3)

Within estimator: $\hat{\beta} = [X' M_D X]^{-1} [X' M_D \mathrm{y}]$ (4)

GLS estimator: $\hat{\beta}_{\mathrm{RE}} = (\mathrm{X}'\Omega^{-1}\mathrm{X})^{-1} X'\Omega^{-1} Y_{it}^{*}$ (5)

2.2 Model Testing

Here, we shall employ a two-way error component model as earlier emphasized, to test for the violation of homoscedasticity assumption in our researched model.

Considering a two-way error component model stated as:

$$y_{it} = x_{it}\beta + u_{it}\ ,;\ \ i = 1,2, \dots, N \quad t = 1,2, \dots, T \tag{6}$$

Within the context of two-way error component, the regression disturbances term u_{it} can be described by the equation

$$u_{it} = \mu_i + \lambda_t + v_{it} \tag{7}$$

With μ_i representing individual-specific effect, λ_t representing time-specific effect and v_{it} the idiosyncratic remainder disturbance term, which is usually assumed to be well-behaved and independent from both the regressors x_{it} and μ_i. The two-way error component model can be written in matrix form as

$$y = X\beta + u \tag{8}$$

The disturbance term u in equation (15) can be written in vector form as

$$u = (I_{NT} \otimes \iota_{NT})v + (I_N \otimes \iota_T)\mu + (I_T \otimes \iota_N)\lambda + V \tag{9}$$

Where I_{NT} is an identity matrix of dimension NT, I_N is an identity matrix of dimension N, I_T is an identity matrix of dimension T, ι_{NT} is a vector of ones of dimension NT, ι_T is a vector of ones of dimension T, ι_N

is a vector of ones of dimension N, $\mu' = (\mu_1, \ldots, \mu_N)$, $\lambda' = (\lambda_1, \ldots, \lambda_T), V$ is the AR(1) covariance matrix of dimension T, $\otimes$ denotes the kronecker product and

$$Var(\mu_i) = \sigma_{\mu i}^2 = h\left(f_i'(\alpha)\right) \quad , i = 1, \ldots, N \tag{10}$$

According to [23], the function $h(\cdot)$ is an arbitrary strictly positive twice continuously differentiable function, α *is a* $P\,x\,1$ vector of unrestricted parameters and f_i is a $P\,x\,1$ vector of strictly exogenous regressors which determine the heteroscedasticity of the individual specific effects and the first element of f_i is one, and without loss of generality, $h(\alpha_1) = \sigma_\mu^2$.

Following [26], the variance-covariance matrix of u can be written as

$$E(uu') = \Sigma = \sigma_u^2(I_N \otimes \iota_T\, \iota_T') + (I_T \otimes \iota_N \iota_N')\sigma_\lambda^2 + \sigma_v^2 I_{NT} \otimes V$$

$$= (I_N \otimes \iota_T) diag[h(f_i'\alpha)](I_N \otimes \iota_T)' + (I_T \otimes \iota_N \iota_N')\sigma_\lambda^2 + \sigma_v^2 I_{NT} \otimes V$$

$$= diag[h(f_i'\alpha)] \otimes J_T + (I_T \otimes \iota_N \iota_N')\sigma_\lambda^2 + \sigma_v^2 I_{NT} \otimes V \tag{11}$$

Where J_T is a matrix of ones of dimension T, $diag[h(f_i'\alpha)]$ is a diagonal matrix of dimension NxN and V can be expressed as

$$V = E(VV') = \sigma_v^2 \left(\frac{1}{1-\rho^2}\right) V_1 \tag{12}$$

where V_1 is a symmetric matrix of order ρ^{T-N}

2.2.1 Conditional LM Test for $H_0 : \sigma_{\mu i}^2 \neq \sigma_\mu^2, \forall_i$ *and* $\sigma_{\lambda t}^2 = 0$ *but* $\sigma_{v_{it}}^2 \neq 0, \rho = 0$

In this section, we derive a conditional LM test for presence of individual heteroscedasticity in the absence of serial correlation.

Under normality of the disturbances, the log-likelihood function, L of a Lagrange multiplier follows that of a multivariate normal distribution. Thus,

$$L(\beta, \theta) = \frac{-NT}{2}\ln(2\pi) - \frac{1}{2} ln|\Sigma| \tag{13}$$

Where $\theta' = (\sigma_v^2, \sigma_\mu^2, \sigma_\lambda^2, \rho, \alpha')$ and $= y - x\beta$. In this case, we set $\tilde{\theta}' = (\sigma_v^2, \sigma_\mu^2, \sigma_\lambda^2, \rho)$. Thus, $L(\beta, \theta)$ becomes $(\beta, \tilde{\theta}')$ *and we set* $\eta_1 = (\beta', \sigma_v^2, \sigma_\mu^2, \sigma_\lambda^2, \rho)$.

In order to obtain the conditional LM statistic, we need to obtain the score statistic $D(\theta) = \frac{\partial L}{\partial \theta}$ and the Information matrix $I(\theta) = -E[\frac{\partial L^2}{\partial \theta \partial \theta'}]$. Following [26], we obtain $D(\theta)$ and $I(\theta)$ as

$$\frac{\partial L}{\partial \theta} - \frac{1}{2} tr\left[\Sigma^{-1}\left(\frac{\partial \Sigma}{\partial \theta}\right)\right] + \frac{1}{2}\left[u'\Sigma^{-1}\left(\frac{\partial \Sigma}{\partial \theta}\right)\Sigma^{-1} u\right] \tag{14}$$

$$-E\left[\frac{\partial L^2}{\partial \theta \partial \theta'}\right] = \frac{1}{2} tr\left[\Sigma^{-1}\left(\frac{\partial \Sigma}{\partial \theta}\right)\Sigma^{-1}\frac{\partial \Sigma}{\partial \theta'}\right] \tag{15}$$

Under H_0, the variance covariance matrix of the disturbances as given by equation (18) becomes

$$\Sigma = diag[h(f_i'\alpha)] \otimes J_T + \sigma_v^2 I_{NT} \otimes I_T + \sigma_\lambda^2 I_N \tag{16}$$

And according to [34], the spectral decomposition and inverse of Σ respectively becomes

$$\Sigma = diag[Th(f_i'\alpha) + \sigma_v^2] \otimes \bar{J}_T + \sigma_v^2 I_{NT} \otimes I_T + \sigma_\lambda^2 I_N \tag{17}$$

$$\Sigma^{-1} = diag\left[\frac{1}{\Omega_i^2}\right] \otimes \bar{J}_T + \frac{1}{\sigma_v^2} I_{NT} \otimes E_T + \frac{1}{\sigma_\lambda^2} I_N$$

Where $\quad \Omega_i^2 = Th(f_i'\alpha) + \sigma_v^2$ (18)

Therefore,

$$\frac{\partial L}{\partial \rho} = D(\hat{\rho}) = -\frac{1}{2} tr\left[\Sigma^{-1}\left(\frac{\partial \Sigma}{\partial \rho}\right)\right] + \frac{1}{2}\left[\hat{u}'\Sigma^{-1}\left(\frac{\partial \Sigma}{\partial \rho}\right)\Sigma^{-1}\hat{u}\right]$$

$$= -\frac{1}{2} tr\left[diag\left[\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2}\right] \otimes \bar{J}_T Z + I_{NT} \otimes E_T Z + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z\right] + \frac{1}{2}\left[\hat{u}'\left(diag\left[\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^4}\right] \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T Z + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^4} I_{NT} Z\right)\hat{u}\right]$$

$$= -\frac{1}{2}\left[\frac{2(T-1)}{T}\sum_{i=1}^N \sum_{t=1}^T \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} - \frac{2(T-1)}{T} - tr\sum_{i=1}^N \sum_{t=1}^T \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2}\right] + \frac{\hat{\sigma}_v^2}{2}\left[\hat{u}'\left(diag\left[\frac{1}{\hat{\Omega}_i^4}\right] \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T Z + \frac{1}{\hat{\sigma}_\lambda^4} I_{NT} Z\right)\hat{u}\right]$$

$$since\ tr(Z) = 0\ and\ tr(\bar{J}_T Z) = tr(E_T)Z = \frac{2(T-1)}{T}$$

$$= -\frac{1}{2}\left[\frac{2(T-1)}{T}\sum_{i=1}^N \sum_{t=1}^T \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} - \frac{2(T-1)}{T} - 0\right] + \frac{\hat{\sigma}_v^2}{2}\left[\hat{u}'\left(diag\left[\frac{1}{\hat{\Omega}_i^4}\right] \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T Z + \frac{1}{\hat{\sigma}_\lambda^4} I_{NT} Z\right)\hat{u}\right]$$

$$since\ thre's\ no\ serial\ correlation\ of\ which\ its\ variance\ has\ been\ expressed\ as\ \hat{\sigma}_\lambda^2$$

$$= \frac{(T-1)}{T}\sum_{i=1}^N \sum_{t=1}^T \left(\frac{\hat{\Omega}_i^2 - \hat{\sigma}_v^2}{\hat{\Omega}_i^2}\right) + \frac{\hat{\sigma}_v^2}{2}\left[\hat{u}'\left(diag\left[\frac{1}{\hat{\Omega}_i^4}\right] \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T Z + \frac{1}{\hat{\sigma}_\lambda^4} I_{NT} Z\right)\hat{u}\right] \tag{19}$$

Equation (26) is the solution obtained after maximization of the first order condition, where $\hat{u} = y - x\hat{\beta}_{GLS}$ is the generalized least square residuals under $H_0, \hat{\Omega}_i^2 = Th(f_i'\hat{\alpha}) + \hat{\sigma}_v^2, where\ \hat{\alpha}$ is the ML estimator of $\alpha\ under\ H_0, and\ h'(f_i'\hat{\alpha})$ is the evaluated value of $\partial h(f_i'\hat{\alpha})/\partial f_i'\alpha$. All the components of the score test statistic $\frac{\partial L}{\partial \eta_1}(\cdot)$ evaluated at maximization of the first order condition are all equal to zero except $\frac{\partial L}{\partial \rho}$ [27].Thus, the partial derivatives under H_0 are expressed in vector form as

$$D(\hat{\eta}_1) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ D(\hat{\rho}) \end{pmatrix} \tag{20}$$

Also, we obtain information matrix under the null hypothesis as follow

$$E\left[-\frac{\partial L^2}{\partial\eta_1\partial\eta_1{}'}\right]=\frac{1}{2}tr\left[\Sigma^{-1}\left(\frac{\partial\Sigma}{\partial\eta_1}\right)\Sigma^{-1}\frac{\partial\Sigma}{\partial\eta_1{}'}\right]$$

$$E\left[-\frac{\partial^2 L}{\partial\beta\partial\beta'}\right]=\frac{1}{2}tr[X'\Sigma^{-1}X\ \ ]^2$$

$$=\frac{1}{2}tr[X'\Sigma^{-1}X\ \ X'\Sigma^{-1}X\ \ ]=N\overset{lim}{\longrightarrow}\infty\left[\frac{X\prime\Sigma^{-1}X}{NT}\right]=I_{\beta\beta}(\hat{\eta}_1)$$

$$E\left[-\frac{\partial^2 L}{\partial\beta\partial\sigma_v^2}\right]=\frac{1}{2}tr\left[X'\Sigma^{-1}X\left(diag\left(\frac{1}{\hat{\Omega}_i^2}\right)\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^2}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^2}I_N\right)\right]=0$$

$$\left(since\ E\left[-\frac{\partial^2 L}{\partial\beta\partial\sigma_v^2}\right]\xrightarrow{N,T\longrightarrow\infty}0\right)$$

$$E\left[-\frac{\partial^2 L}{\partial\beta\partial\sigma_\mu^2}\right]=\frac{1}{2}tr\left[X'\Sigma^{-1}X\left(diag\left(\frac{1}{\hat{\Omega}_i^2}\right)\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^2}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^2}I_N\right)\right]=0$$

$$\left(since\ E\left[-\frac{\partial^2 L}{\partial\beta\partial\sigma_\mu^2}\right]\xrightarrow{N,T\longrightarrow\infty}0\right)$$

$$E\left[-\frac{\partial^2 L}{\partial\beta\partial\sigma_\lambda^2}\right]=\frac{1}{2}tr\left[X'\Sigma^{-1}X\left(diag\left(\frac{1}{\hat{\Omega}_i^2}\right)\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^2}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^2}I_N I'_N\right)\right]=0$$

$$\left(since\ E\left[-\frac{\partial^2 L}{\partial\beta\partial\sigma_\lambda^2}\right]\xrightarrow{N,T\longrightarrow\infty}0\right)$$

$$E\left[-\frac{\partial^2 L}{\partial\beta\partial\rho}\right]=\frac{1}{2}tr\left[X'\Sigma^{-1}X\left(diag\left(\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2}\right)\otimes\bar{J}_T Z+I_{NT}\otimes E_T Z+\frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2}I_{NT}\otimes Z\right)\right]$$

$$=\frac{T-1}{T}tr\ \left((X'\Sigma^{-1}X)diag\left(\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2}\right)+I_{NT}+\frac{1}{\hat{\sigma}_\lambda^2}I_{NT}\right)=0\ \left(since\ E\left[-\frac{\partial^2 L}{\partial\beta\partial\rho}\right]\xrightarrow{N,T\longrightarrow\infty}0\right)$$

$$(since\ tr(Z)=0,tr(\bar{J}_T Z)=tr(E_T Z)=2((T-1)/T)$$

$$E\left[-\frac{\partial^2 L}{\partial\sigma_v^4}\right]=\frac{1}{2}tr\left[diag\left[\frac{1}{\hat{\Omega}_i^2}\right]\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^2}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^2}I_N\ \right]^2=\left[diag\left[\frac{1}{\hat{\Omega}_i^4}\right]\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^4}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^4}I_N\ \right]$$

$$=\frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{1}{\hat{\Omega}_i^4}+\frac{NT-1}{\hat{\sigma}_v^4}+\frac{1}{\hat{\sigma}_\lambda^4}\right)=\frac{1}{2}(\hat{\sigma})_v^4$$

$$E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\sigma_\mu^2}\right]=\frac{1}{2}tr\left[\left(diag\left(\frac{1}{\hat{\Omega}_i^2}\right)\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^2}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^2}I_N\right)\left(diagT\left(\frac{1}{\hat{\Omega}_i^2}\right)\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^2}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^2}I_N\right)\right]$$

$$=\frac{1}{2}tr\left(diagT\left(\frac{1}{\hat{\Omega}_i^4}\right)\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^4}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^4}I_N\right)=0\ \left(since\ E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\sigma_\mu^2}\right]\xrightarrow{N,T\longrightarrow\infty}0\right)$$

$$E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\sigma_\lambda^2}\right]=\frac{1}{2}tr\left[\left(diag\left(\frac{1}{\hat{\Omega}_i^2}\right)\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^2}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^2}I_N\right)\left(diag\left(\frac{1}{\hat{\Omega}_i^2}\right)\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^2}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^2}I_N I'_N\right)\right]$$

$$=\frac{1}{2}tr\left(diag\left(\frac{1}{\hat{\Omega}_i^4}\right)\otimes\bar{J}_T+\frac{1}{\hat{\sigma}_v^4}I_{NT}\otimes E_T+\frac{1}{\hat{\sigma}_\lambda^4}I_N I'_N\right)=0\ \left(since\ E\left[-\frac{\partial^2 L}{\partial\sigma_v^2\partial\sigma_\lambda^2}\right]\xrightarrow{N,T\longrightarrow\infty}0\right)$$

$$E\left[-\frac{\partial^2 L}{\partial \sigma_v^2 \partial \rho}\right] = \frac{1}{2} tr\left[\left(diag\left(\frac{1}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N\right)\left(diag\left(\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z\right)\right]$$

$$= \frac{1}{2} tr\left(diag\left(\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^4}\right) \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^4} I_{NT} \otimes E_T Z\right)$$

$$= \frac{1}{2\hat{\sigma}_v^2} tr\left(diag\left(\frac{\hat{\sigma}_v^4}{\hat{\Omega}_i^4}\right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T Z + \frac{\hat{\sigma}_v^4}{\hat{\sigma}_\lambda^4} I_N \otimes E_T Z\right) = \frac{1}{2\hat{\sigma}_v^2} N \overset{lim}{\longrightarrow} \infty \left[\frac{I_N' Z}{N}\right]$$

$$E\left[-\frac{\partial^2 L}{\partial \sigma_\mu^4}\right] = \frac{1}{2} tr\left[\left(diagT\left(\frac{1}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N\right)\right]^2$$

$$= \frac{1}{2} tr\left[diagT^2\left[\frac{1}{\hat{\Omega}_i^4}\right] \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^4} I_N\right]$$

$$= \frac{1}{2} \sum_{i=1}^N \sum_{t=1}^T T^2\left(\frac{1}{\hat{\Omega}_i^4} + \frac{NT-1}{\hat{\sigma}_v^4} + \frac{1}{\hat{\sigma}_\lambda^4}\right) = \frac{1}{2}(\hat{\sigma})_\mu^4$$

$$E\left[-\frac{\partial^2 L}{\partial \sigma_\mu^2 \partial \sigma_\lambda^2}\right] = \frac{1}{2} tr\left[\left(diagT\left(\frac{1}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N\right)\left(diag\left(\frac{1}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N I'_N\right)\right]$$

$$= \frac{1}{2} tr\left(diagT\left(\frac{1}{\hat{\Omega}_i^4}\right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^4} I_N I'_N\right) = 0 \;\left(since\ E\left[-\frac{\partial^2 L}{\partial \sigma_\mu^2 \partial \sigma_\lambda^2}\right] \xrightarrow{N,T \to \infty} 0\right)$$

$$E\left[-\frac{\partial^2 L}{\partial \sigma_\mu^2 \rho}\right] = \frac{1}{2} tr\left[\left(diagT\left(\frac{1}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N\right)\left(diag\left(\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z\right)\right]$$

$$= \frac{1}{2} tr\left(diagT\left(\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^4}\right) \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^4} I_{NT} \otimes E_T Z\right) = 0 \;\left(since\ E\left[-\frac{\partial^2 L}{\partial \sigma_\mu^2 \partial \rho}\right] \xrightarrow{N,T \to \infty} 0\right)$$

$$E\left[-\frac{\partial^2 L}{\partial \sigma_\lambda^4}\right] = \frac{1}{2} tr\left[\left(diag\left(\frac{1}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N I'_N\right)\right]^2 = \frac{1}{2} tr\left(diag\left(\frac{1}{\hat{\Omega}_i^4}\right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^4} I_N I'_N\right) = \frac{1}{2}(\hat{\sigma})_\lambda^4$$

$$E\left[-\frac{\partial^2 L}{\partial \sigma_\lambda^2 \rho}\right] = \frac{1}{2} tr\left[\left(diag\left(\frac{1}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N I'_N\right)\left(diag\left(\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z\right)\right]$$

$$= \frac{1}{2} tr\left(diag\left(\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^4}\right) \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^4} I_{NT} \otimes E_T Z\right) = 0 \;\left(since\ E\left[-\frac{\partial^2 L}{\partial \sigma_\lambda^2 \partial \rho}\right] \xrightarrow{N,T \to \infty} 0\right)$$

$$E\left[-\frac{\partial^2 L}{\partial \rho^2}\right] = \frac{1}{2} tr\left[\left(diag\left(\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2}\right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z\right)\right]^2$$

$$= \frac{1}{2} tr\left(diag\left(\frac{\hat{\sigma}_v^4}{\hat{\Omega}_i^4}\right) \otimes \bar{J}_T Z \bar{J}_T Z + I_{NT} \otimes E_T Z + \frac{\hat{\sigma}_v^4}{\hat{\sigma}_\lambda^4} I_{NT} \otimes Z'Z\right) = \frac{1}{2} N \overset{lim}{\longrightarrow} \infty \left[\frac{Z'Z}{N}\right]$$

Thus, information matrix under the null hypothesis can be obtained as a symmetric matrix of the form

$$I(\hat{\eta}_1) = \begin{pmatrix} \beta'\beta & \beta\sigma_v^2 & \beta\sigma_\mu^2 & \beta\sigma_\lambda^2 & \beta\rho \\ \beta\sigma_v^2 & \sigma_v^4 & \sigma_v^2\sigma_\mu^2 & \sigma_v^2\sigma_\lambda^2 & \sigma_v^2\rho \\ \beta\sigma_\mu^2 & \sigma_v^2\sigma_\mu^2 & \sigma_\mu^4 & \sigma_\mu^2\sigma_\lambda^2 & \sigma_\mu^2\rho \\ \beta\sigma_\lambda^2 & \sigma_v^2\sigma_\lambda^2 & \sigma_\mu^2\sigma_\lambda^2 & \sigma_\lambda^4 & \sigma_\lambda^2\rho \\ \beta\rho & \sigma_v^2\rho & \sigma_\mu^2\rho & \sigma_\lambda^2\rho & \rho^2 \end{pmatrix} \tag{21}$$

$$= \begin{pmatrix} I_{\beta\beta}(\hat{\eta}_1) & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(\hat{\sigma})_v^4 & 0 & 0 & \frac{1}{2\hat{\sigma}_v^2} N \overset{lim}{\longrightarrow} \infty \left[\frac{I_N' Z}{N}\right] \\ 0 & 0 & \frac{1}{2}(\hat{\sigma})_\mu^4 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(\hat{\sigma})_\lambda^4 & 0 \\ 0 & \frac{1}{2\hat{\sigma}_v^2} N \overset{lim}{\longrightarrow} \infty \left[\frac{I_N' Z}{N}\right] & 0 & 0 & \frac{1}{2} N \overset{lim}{\longrightarrow} \infty \left[\frac{Z'Z}{N}\right] \end{pmatrix} \tag{22}$$

Thus, a conditional LM statistic under the specified H_0 is given as

$$LM_{\rho|\alpha} = D(\hat{\rho})'\left[(I_{NT}(\hat{\eta}_1))^{-1}|_{\rho\rho}\right]D(\hat{\rho}) \tag{23}$$

Setting $H_{NT}^{\rho} = diag\left(\frac{1}{\sqrt{NT}} I_{k,} \frac{1}{\sqrt{NT}}, \frac{1}{\sqrt{N}}, \frac{1}{\sqrt{T}}, \frac{1}{\sqrt{NT}}\right), LM\ statistic\ also\ becomes$

$$LM_{\rho|\alpha} = \left[D(\hat{\rho})_{H_{NT}^{\rho}}\right]' [H_{NT}^{\rho}(I_{NT}(\hat{\eta}_1))H_{NT}^{\rho})^{-1}|_{\rho\rho}] \tag{24}$$

$$H_{NT}^{\rho}\big(I_{NT}(\hat{\eta}_1)\big)H_{NT}^{\rho} \xrightarrow{N,T\to\infty} I(\hat{\eta}_1)$$

Thus, the LM statistic becomes

$$LM_{\rho|\alpha} = D(\hat{\rho})'\left[(I(\hat{\eta}_1))^{-1}|_{\rho\rho}\right]D(\hat{\rho}) \tag{25}$$

Where $(I(\hat{\eta}_1))^{-1}|_{\rho\rho} = \frac{1}{2} N \overset{lim}{\longrightarrow} \infty \left[\frac{1}{N} Z' \left(I_N - \frac{I_N I_N'}{N}\right) Z\right]$

Under H_0, LM statistic is asymptotically distributed as $\chi_1^2\ as\ N, T \to \infty$

2.3. **Monte Carlo Simulation Scheme**: The design of Monte-Carlo experiments will assume a uniform distribution and follow closely that of [26] , Garba *et al.* (2013) and [27]. The differences however lie in the utilization of two-way error component model with four exogenous variables. Three sizes of cross-sectional units (i.e N=20, 40 and 60), three time periods (i.e T = 10, 40 and 100), an homoscedastic situation and two degrees of heteroscedasticity (i.e moderate and severe) were used for simulation with each of the combinations iterated 1000 times and the assessment of the three estimators considered based on the Absolute bias Variance, (ABIAS), Mean square error (MSE) and the Root Mean Square (RMSE) of parameters estimates.

The AB of parameter $\hat{\beta}_k$ estimated over r replicates is defined by

$$AB(\hat{\beta}_k) = \frac{1}{r}\sum_{j=1}^{r} \left| \hat{\beta}_{kj} - \beta_k \right| \tag{26}$$

The variance of the estimator $\hat{\beta}_k$ is defined as

$Var(\hat{\beta}_k) = \frac{1}{r}\sum_{j=1}^{r}(\hat{\beta}_{kj} - \beta_k)$

(27)

$$MSE(\hat{\beta}_k) = \frac{1}{r}\sum_{j=1}^{r}(\widehat{\beta}_k - \beta_k)^2 \qquad (28)$$

$$RMSE(\hat{\beta}_k) = \sqrt{\frac{1}{r}}\sum_{j=1}^{r}(\widehat{\beta}_k - \beta_k)^2 \qquad (29)$$

Where $\hat{\beta}_k$ indicates the k^{th} parameter being estimated for $j = 1,2,\dots,r\ (number\ of\ iterations)$

After evaluating the above criteria for each of the estimator, their performances were ranked and the best method identified.

3. RESULTS AND DISCUSSION

The results for the smallest iterated space and time combinations of N=20 and T=10 is hereby presented. Small values of N and T was chosen to demonstrate that the researched model and size of Lagrange multiplier tests also work well even for small samples and once the research opinion being investigated worked at that sample level, it would definitely works well asymptotically. This is in line with the opinion expressed by [27].

Figures

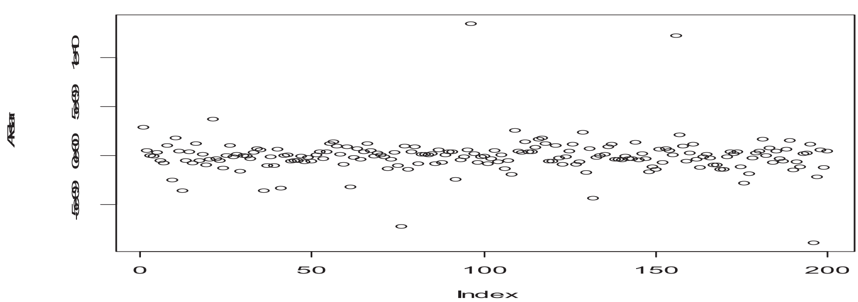

Fig 1: Homoscedastic and zero serial correlation plot of audit fees

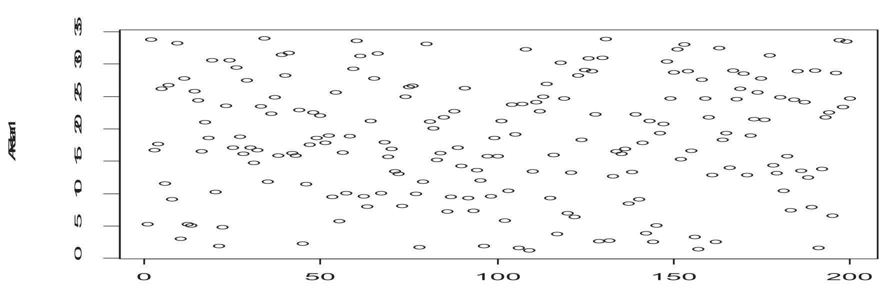

Fig 2: Moderate Heteroscedastic and zero serial correlation plot of audit fees

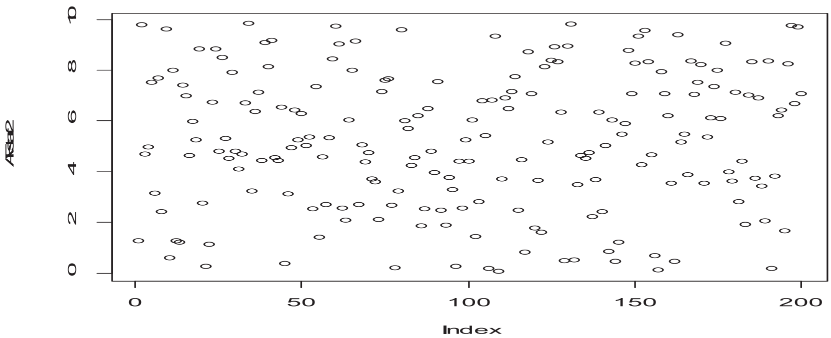

Fig 3: Severe Heteroscedastic and zero serial correlation plot of audit fees

Tables

Panel Data Regression (Audit Fees) Models for Replicates of 1000

Table 1: PDRM when N=20 and T=10 for Homoscedastic and Zero Serially correlated Errors

PARAMETERS	POLS			WITHIN			GLS		
	Coefficient	|t|	MSE	Coefficient	|t|	MSE	Coefficient	|t|	MSE
β_1	-210511.8	11723.7	88628956480	-3.674524	49.93313	24.864	-1.16e-32	1.12e-32	2.6671e-64
β_2	-0.001402	0.000526	6.046822	-0.00000002	0.0000003	0.09227631	-7.69e-41	7.44e-41	1.8196e-74
β_3	0.0000045	0.00002	6.056896	0.00000000079	0.00000011	0.09227633	2.49e-42	2.41e-42	1.8227e-74
β_4	-0.000000077	0.0000000077	0.000000077	-0.00000000024	0.000000077	0.000000077	-7.68e-43	0.000000077	0.000000077
β_5	-0.00044	0.00017	6.053501	-0.000000077	0.0000001	0.09227632	-2.42e-41	2.35e-41	1.8216e-74

$\boldsymbol{POLS}: \boldsymbol{R^2} = \mathbf{0.37124}, \boldsymbol{\bar{R}^2} = 0.3583, \boldsymbol{F} = 10.7552\ , \boldsymbol{DF}(\mathbf{4}, \mathbf{195}), \boldsymbol{P - value} = \mathbf{0.025}$
$\boldsymbol{WITHIN}: \boldsymbol{R^2} = 0.6374, \boldsymbol{\bar{R}^2} = 0.6300\ , \boldsymbol{F} = 18.807\ , \boldsymbol{DF}(\mathbf{4}, \mathbf{195}), \boldsymbol{P - value} = \mathbf{0.0000}$ $\boldsymbol{GLS}: \boldsymbol{R^2} = 0.46712, \boldsymbol{\bar{R}^2} = 0.4562, \boldsymbol{F} = 17.16815\ , \boldsymbol{DF}(\mathbf{4}, \mathbf{195}), \boldsymbol{P - value} = \mathbf{0.0000}$

Table 2: PDRM when N=20 and T=10 for Moderate Heteroscedastic and Zero Serially correlated Errors

PARAMETERS	POLS			WITHIN			GLS		
	Coefficient	|t|	MSE	Coefficient	|t|	MSE	Coefficient	|t|	MSE
β_1	-210511.88	75029.66	886335308 28	-36745.24	30382190	2701081702	-1.15e-32	4.85e-33	2.67e-64
β_2	-0.001402	0.000499	86.30345	-0.0002479	0.202400	85.26617	-7.69e-41	3.23e-41	9.12e-75
β_3	0.0000454	0.0000016	86.28207	0.0000079	0.006554	85.26166	2.49e-42	1.04e-42	9.13e-75
β_4	-0.000014	0.000005	86.28295	-0.000002	0.002021	85.26185	-7.68e-43	3.23e-43	9.13e-75
β_5	-0.0004423	0.000158	86.28927	-0.0000 7.72	0.063841	85.26318	-2.42e-41	1.02e-41	9.12e-75

$\boldsymbol{POLS}: \boldsymbol{R^2} = 0.31224, \boldsymbol{\bar{R}^2} = 0.2981, \boldsymbol{F} = 8.6572\ , \boldsymbol{DF(4, 195)}, \boldsymbol{P - value = 0.0129}$
$\boldsymbol{WITHIN}: \boldsymbol{R^2} = 0.6077, \boldsymbol{\bar{R}^2} = 0.5996\ , \boldsymbol{F} = 18.607\ , \boldsymbol{DF(4, 195)}, \boldsymbol{P - value = 0.0000}$ $\boldsymbol{GLS}: \boldsymbol{R^2} = 0.49782, \boldsymbol{\bar{R}^2} = 0.4875, \boldsymbol{F} = 17.16815\ , \boldsymbol{DF(4, 195)}, \boldsymbol{P - value = 0.0000}$

Table 3: PDRM when N=20 and T=10 for Severe Heteroscedastic and Zero Serially correlated Errors

PARAMETERS	POLS			WITHIN			GLS		
	Coefficient	|t|	MSE	Coefficient	|t|	MSE	Coefficient	|t|	MSE
β_1	-210511.8	78874.80	8863188447 6	1116400	30382190	1246328000000	-1.16e-32	8.57e-33	2.67e-64
β_2	-0.001402	0.000525	30.21541	0.007437	0.020201	112.2198	-7.69e-41	5.71e-41	9.15e-75
β_3	0.000045	0.000017	30.21548	-0.000240	0.006554	112.3615	2.49e-42	1.85e-42	9.17e-75
β_4	-0.000014	0.000005	30.2159	0.000074	0.002021	112.3557	-7.68e-43	5.71e-43	9.17e-75
β_5	-0.000442	0.000165	30.21887	0.002346	0.063841	112.3138	-2.42e-41	1.80e-41	9.16e-75

$\boldsymbol{POLS}: \boldsymbol{R^2} = 0.24527, \boldsymbol{\bar{R}^2} = 0.22979, \boldsymbol{F} = 8.5672\ , \boldsymbol{DF(4, 195)}, \boldsymbol{P - value = 0.1696}$
$\boldsymbol{WITHIN}: \boldsymbol{R^2} = 0.6182, \boldsymbol{\bar{R}^2} = 0.6104\ , \boldsymbol{F} = 28.607\ , \boldsymbol{DF(4, 195)}, \boldsymbol{P - value = 0.0000}$ $\boldsymbol{GLS}: \boldsymbol{R^2} = 0.51986, \boldsymbol{\bar{R}^2} = 0.5100, \boldsymbol{F} = 17.16815\ , \boldsymbol{DF(4, 195)}, \boldsymbol{P - value = 0.0000}$

Table 4: Ranks of the PDRM Techniques Using ABIAS Criterion for Homoscedastic, Varying Degree of Heteroscedasticity and Serial Correlation levels for the various Sample Sizes Considered.

Space	Time	Serial Correlation Level	Homoscedasticity/Heteroscedasticity Degree	POLS	WITHIN	GLS
			Homoscedastic	1	2	3
	10	0	Moderate Heteroscedastic	2	1	3
20			Severe Heteroscedastic	2	1	3
	40					
		0	Homoscedastic	1	2	3
			Moderate Heteroscedastic	2	1	3
			Severe Heteroscedastic	1	2	3
	100	0	Homoscedastic	1	2	3
			Moderate Heteroscedastic	2	1	3
			Severe Heteroscedastic	2	1	3
			Homoscedastic	1	2	3
	10	0	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
	40	0	Homoscedastic	2	1	3
			Moderate Heteroscedastic	1	2	3
40			Severe Heteroscedastic	1	2	3
	100	0	Homoscedastic	1	2	3
			Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	2	1	3
	10	0	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
60						
	40	0	Homoscedastic	1	2	3
			Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
	100	0	Homoscedastic	1	2	3
			Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
Sum of the Ranks				**34**	**47**	**81**

Table 5: Estimated Size of Conditional LM Test in Linear Heteroscedasticity when T=10

σ_v^2	σ_λ^2	α	ρ	N=20	N=40	N= 60
		Conditional LM Test for Heteroscedasticity and Zero Serial Correlation				
2	2	1	0	0.00461	0.00445	0.00440
	2	2	0	0.00433	0.00432	0.00430
	6	1	0	0.00402	0.00401	0.00400
	6	2	0	0.00399	0.00389	0.00380
6	2	1	0	0.00406	0.00404	0.00401
	2	2	0	0.00398	0.00396	0.00393
	6	1	0	0.00452	0.00471	0.00490
	6	2	0	0.00499	0.00489	0.00480

The specified models from POLS, Within and GLS estimators from tables 1-3 respectively are given as follows:

$AF_{POLS} = -210{,}511.8 - 0.001402PBT + 0.0000045TA - 0.000014TL - 0.00044SHF$
(30.1)

$AF_{WITHIN} = -3.6745 - 0.00000002PBT + 0.0000000008TA - 0.0000000002TL - 0.00000008SHF$ (30.2)
$AF_{GLS} = -1.16\text{e} - 32 \ - 7.69\text{e} - 41PBT + 2.49\text{e} - 42\ TA - 7.68\text{e} - 43TL - 2.42\text{e} - 41SHF$ (30.3)

$AF_{POLS} = -210{,}511.88 - 0.001402PBT + 0.0000454TA - 0.000014TL - 0.0004423SHF$
(31.1)

$AF_{WITHIN} = -36{,}745.24 - 0.0002479PBT + 0.0000079TA - 0.0000024TL - 0.00007.72SHF$
(31.2)

$AF_{GLS} = -1.15\text{e} - 32 - 7.69\text{e} - 41PBT + 2.49\text{e} - 42TA - 7.68\text{e} - 43TL - 2.42\text{e} - 41SHF$
(31.3)

$AF_{POLS} = -210{,}511.8 - 0.001402PBT + 0.0000454TA - 0.000014TL - 0.0004423SHF$
(32.1)

$AF_{WITHIN} = 1{,}116{,}400 + 0.007437PBT - 0.000240TA + 0.000074TL + 0.002346SHF$
(32.2)

$AF_{GLS} = -1.16\text{e} - 32 - 7.69\text{e} - 41PBT + 2.49\text{e} - 42TA - 7.68\text{e} - 43TL - 2.42\text{e} - 41SHF$
(32.3)

Figures 1-3 show the pattern of movement for the individual audit fees of all the banks and it can be observed that the plots are more dispersed when error is heteroscedastic than when it is homoscedastic. Based on the rank results presented in table 4 using estimated values of Absolute Bias (ABIAS), GLS technique ranked highest with a rank sum of 81 compared to that of Within and OLS with a rank sum of 47 and 34 respectively. This implied that GLS technique is expected to have given the best estimate for the specified Panel Data Regression Model and this is in line with the works of Garba *et.al* (2013) where variance was used to rank the results of similar PDRMs. However, the theoretical concept of our researched model does not support the empirical structure of the kind of models fitted through GLS, hence the adoption of a within model which will guarantee a positive value for the banks audit fee in line with prior opinion. This is also in line with [35] that within transformation implements the LSDV model better because the regression on de-meaned data yields the same results as estimating the model from the original data and a set of (N-1) indicator variables for all but one of the panel units. Thus, equation (48.2) is the ideal model that best fitted the specified audit fee model and all other model validity check such as $R^2, F\ and\ t$ statistic do not proof otherwise. Table 5 presents significance values for the empirical size of the Conditional LM test for heteroscedasticity and zero serial correlation. This was achieved at 5% level of significance when N = 20, 40 and 60 at T = 10. Adapting [26], both the bi-dimensional remainder error and time specific error terms $(\sigma_v^2, \sigma_\lambda^2)$ take values (2, 2), (2, 6), (6, 2) and (6, 6) in each experiment. These correspond to cases where the percentages of the total variance due to both errors are 25% and 75% accordingly. On the other hand, α is assigned values 0, 1 and 2 with $\alpha = 0$ denoting homoscedastic individual specific error while $\alpha = 1$ and 2 denote moderate and severe heteroscedastic errors respectively. These results show that the sizes of all the tests are significant at 5% for the specified linear heteroscedasticity function. These results conformed with that of [27] where similar tests have been used to examined heteroscedasticity and spatial correlation in a two-way random effect model.

4. CONCLUSION

Based on the results obtained by the empirical analysis of simulated data, the following conclusions are therefore arrived at:

1. That among the models presented, (32.2) is the only recommended model that satisfy the concept of our specified PDRM. The model is thus presented as
   $AF_{WITHIN} = 1{,}116{,}400 + 0.007437PBT - 0.000240TA + 0.000074TL + 0.002346SHF$
2. The model implies that heteroscedasticity in Nigerian commercial banks is observed to be severe. Therefore, all operational policies of such banks which include payment of external auditor's remuneration should be treated as such.
3. That OLS completely breaks down and can only give rise to unreliable inference in the presence of severe heteroscedasticity. It only performs better for small samples and as sample size increases, OLS derailed even in the absence of heteroscedasticity.
4. That Conditional LM test has good size and power under the adopted linear functional form of heteroscedasticity and errors variance of 25% and 75%.

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