On Fractional Time Domain Modelingof Nigerian Monthly Rainfall Statistics

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ABSTRACT

That rainfall contributes substantially to the economic sustenance of Nigeria is an understatement. To help government in the continuous provision of necessary agricultural policies needful to sustain sufficient food production, there is a need to build an appropriate time domain model which can be used to forecast the future rate of monthly rainfall. As a result of this, autoregressive fractionally integrated moving average (ARFIMA) model was employed to model the sourced rainfall data to take care of the fractional nature of seasonal rainfalls. It was discovered that the data was not stationary and stationarity was achieved through 1st order differencing. The long lasting autocorrelation function of the data showed the presence of long memory structure, and The ACF and PACF of the differenced data suggested possible models for selection. Among the several fitted models, ARFIMA (1, 2, 3) appeared to be the best based on its minimum AIC, BICand MSE values. The model was used for 2 years monthly forecast and it was observed that there is going to be a slow and steady increase in the volume ofrainfall within the context of this research data.

Keywords: Autocorrelation function, ARFIMA, Partial autocorrelation function, stationarity, Rainfall

1. Introduction

Rainfall is a determinant factor of many natural occurrences and one of the indicators of climate change. It affects every facet of the ecological system. Hence the study of rainfall is important and cannot be overemphasized (Odjugo et al., 2010; National Research Council, 2010). Animals breeding period synchronize with rainfall period and rainfall events have been directly linked to sickness and disease particularly those of waterborne and vector-borne types. Crop planning, yields and harvest are influence by rainfall. Investments in agricultural produce and products are expected to be done in accordance with the knowledge of rainfall and other weather condition (Onyenechere, 2014; Okonkwo and Mbajiorgu, 2016).Rainfall can both be beneficial and harmful for humans, aside the beneficial aspect of rainfall, it can also be destructive in nature, natural disasters like floods and landslides are caused by rain (Ogbo et al., 2013).

The rainfall climatic conditions in Nigeria make the seasons more obvious. The south and north get the biggest rainfalls. The three different rainy seasons which can be observed in Nigeria are long rainy season, short rainy season and wet season.Rainy season starts in late March and continues up to the end of July. The rainiest, peak period is in June and affects most parts of southern Nigeria. More than 85% humidity and wet rainy weather in most southern parts of Nigeria is observed during this season in Nigeria (Azuwike et al., 2013). The northern part of Nigeria has the lowest amount of rainfall in a year sometimes as little as 120 to 130 rainy days in Kano and even lesser by 10 to 20 days in Katsina and Sokoto (Ologunorisa and Tersoo, 2016). The rain continues in most part of the country till even October and November, though Nigeria climate has its own variability. The peak of rainy season in Nigeria occurs in August with more than enough for South, East as well as the Northern region of the country (Adejuwon, 2014). According to Ilesanmi et al. (2014), Rainfall retreats truly towards the end of October and early November.

The negative impact of climate change such as temperature rise, erratic rainfall, sand storms desertification, low agricultural yield, human dehydration and sea level rise are real in the desert prone states of Nigeria. Environment degradation and the attendant desertification are major threats to the livelihood of the in habitants of the front-state of Nigeria. These lead to increase population pressure, intensive agricultural land-use, over grazing, bush burning just to mention few (Abaje et al., 2012).

Naturally, rain variability is of spatial and temporal forms and within these variations if by time series analysis, no significant trend is obtained then the rainfall is steady, otherwise, it has changed Several trend analysis researches have been conducted in times past globally and rainfall generally have been opined to be of no stable trend (Obasi and Ikubuwaje, 2012; Okonkwo and Mbajiorgu,2016). Since climate and rainfall are highly non-linear and complicated phenomenal, which requires vivid investigation and analysis, thus advancing this study has become germane.

Consequently in this research, attempt is made to model and forecast future values for Nigerian monthly rainfall data using Autoregressive fractionally integrated moving average (ARFIMA)which is capable of taking care the fragmentary nature of seasonal rainfall and equally sufficient for predicting the expected future rainfall using southwest, Nigeria as a focus.The motivation is derived from the fact that reliable and adequate estimates of rainfall are very essential for planning not only byindividual and the government but International agencies.

2. Materials and Methods

The data for this study was obtained on monthly basis from the Central Bank of Nigeria Statistical bulletin over a period of 15 years between the years 2002 – 2016.

Autoregressive Fractionally Integrated Moving Average (ARFIMA) developed by Hosking (1981) was employed in this research. It is a class of long memory models which explicitly accounts for persistence long term correlations incorporated in series data.

The ARFIMA (p,d,q) model where "p" is the order of Autoregressive Component, "d" is the order of differencing used and "q" is the order of Moving average components in the model.The differencing parameter, d is not an integer but a real number where $0 < d < 1$.

Depending on the above definition, the ARFIMA model can be classified as follows:

2.1 Autoregressive (AR) Model

When the value of the current output $y_t$depends solely on p prior outputs and the current input (random shock), the AR model takes the form of

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + e_t \quad (1)$$

And its characteristics equation becomes

$$\Phi\ (B) y_t = e_t \quad (2)$$

Equation (1) is called an Autoregressive Model of order p, denoted by $AR(p)$ or $ARIMA(p, 0, 0)$.

2.2 Moving Average (MA) Models.

When the current output y_t depends solely on the current input and q prior inputs, the MA Model takes the form of

$$y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q} \quad (3)$$

$$y_t = \Theta(B) e_t \quad (4)$$

Equation (3) is called a Moving Average of order q, denoted by MA(q) or ARIMA $(0, 0, q)$.

2.3 Mixed Autoregressive and Moving Average (ARMA): When the current outputy_t depends on both the AR and MA process, the model is called an Autoregressive Moving Average Model, denoted by ARMA (p,q) or ARIMA (p, d, q) when stationary has been achieved through differencing. Stationary is achieved using

$$w_t = \nabla y_t = y_t - y_{t-1} \ (5)$$

It can be shown that $y_t - y_{t-1} = (1 - B) y_t$ for a difference of order one.

A difference of order two implies that the first order differenced series is differenced again resulting in

$$k_t = w_t - w_{t-1} = (y_t - y_{t-1}) - (y_{t-1} - y_{t-2}) = y_t - 2y_{t-1} + y_{t-2} \quad (6)$$

The result again is a new time seriesk_t, having two less observations than the original series y_t

This can be generated tod^{th} order differencing, where d is the order of differencing required to achieve mean stationarity. Thus, the general ARIMA model is given as

$$\phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q} \quad (7)$$

A general solution of equation (7) is expressed as its characteristic equation given as

$$(1 - B)^d (1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p) Y_t = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q) e_t \quad (8)$$

$$(1 - B)^d \emptyset(B) Y_t = \theta(B) e_t \quad (9)$$

2.4. Fractional integrated (FI) Models

The process of fractional integration was independently developed by Hosking (1981). They are models of Autoregressive Fractionally Integrated Moving Average (ARFIMA) where the differentiation is fractional. The

differencing parameter is not an integer but a real number. The ARFIMA processes produce long memory if the parameter of differentiation is in the range $0 < d < 1/2$, in which case the process is stationary and invertible.
According to the earlier works of Reisen et al., (2001); Jensen and Nielson (2014),The general expression for ARFIMA processes $\{y_t\}$ may be defined by the equation:

$$\emptyset(L)(1-L)^{d}(y_t - u) = \theta(L)\varepsilon_t \qquad (10)$$

Where
$E(\varepsilon_t) = 0, E(\varepsilon_t^2) = \sigma^2 E(\varepsilon_t \varepsilon_t)$ and where the fractional parameter d is possibly non integer. It will be seen that the process is weakly stationary for d< (1/2) and is invertible for d> - (1/2).
The characteristic equation of ARFIMA model specified that:

$$\emptyset(B)y_t = \theta(B)(1-B)^{-d}\varepsilon_t \qquad (11)$$

Where
$\emptyset(B) = 1 - \emptyset_1 — B - \cdots - \emptyset_P B^P$ and $\theta(B) = 1 + \theta_1 B + \cdots + \theta_q B^q$ are the autoregressive and moving average operators respectively; B is the backward shift operator and $(1-B)^{-d}$ is the fractional differencing operator given by the binomial expression

$$(1-B)^{-d} = \sum_{j=0}^{\infty} \frac{r(j+d)}{r(j+1)r(d)} B^J = \sum_{J=0}^{\infty} nj\, B^j \qquad (12)$$

2.5 Forecasting using ARFIMA Model.
Once an adequate and satisfactory model is fitted to the series, forecasts can be generated using the model. The one-step ahead forecast for time t+1 is given by:

$$y_t = \phi_1 y_t + \phi_2 y_{t-1} + \ldots + \phi_p y_{t-p+1} + e_{t+1} - \theta_1 e_t - \theta_2 e_{t-1} - \cdots - \theta_q e_{t-q+1} \qquad (13)$$

Except for the random shock e_{t+1} at time $t + 1$, all other parameters are estimated as roots of the characteristic equation expressed in equation (11).

3. Results and Discussion

3.1 Results

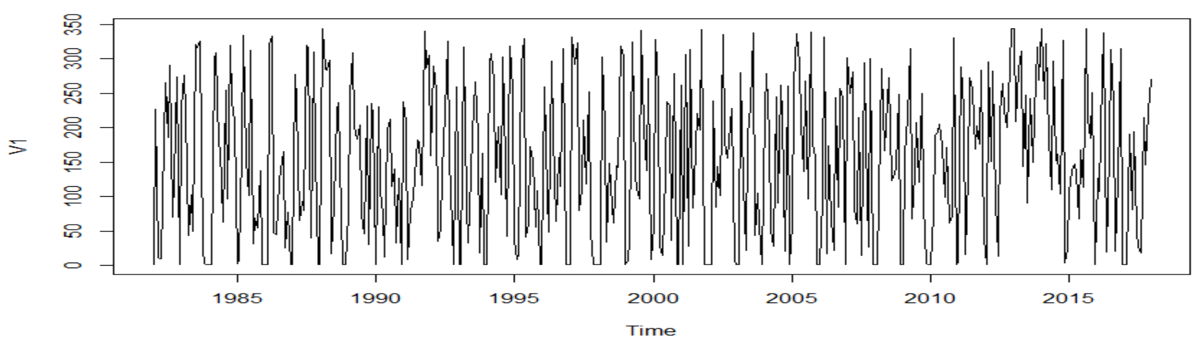

Figure 1: Time Plot of Monthly Rainfall

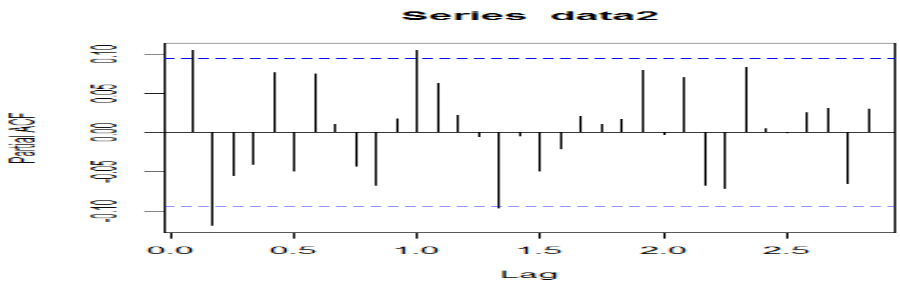

Figure 2 PACF Plot of Rainfall Data

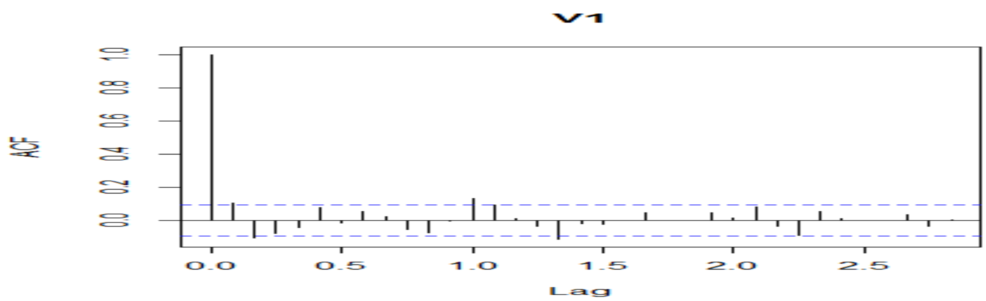

Figure 3 ACF Plot of Rainfall Data

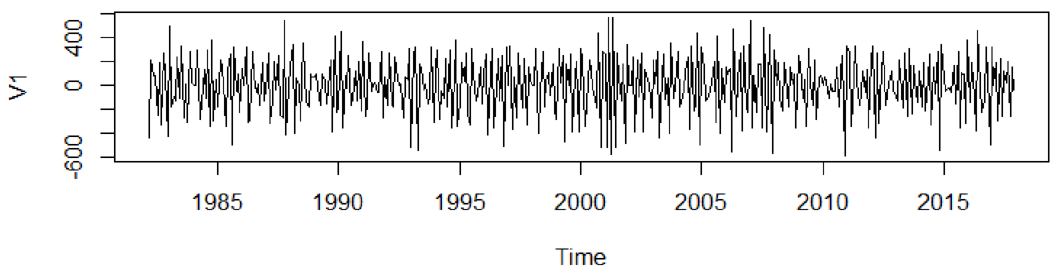

Figure 4 Time Plot of the 1^{st} Differenced Data

Table 1: Augmented Dickey Fuller Test

Dickey-Fuller	Lag order	p-value
-40.634	**0**	**0.01**

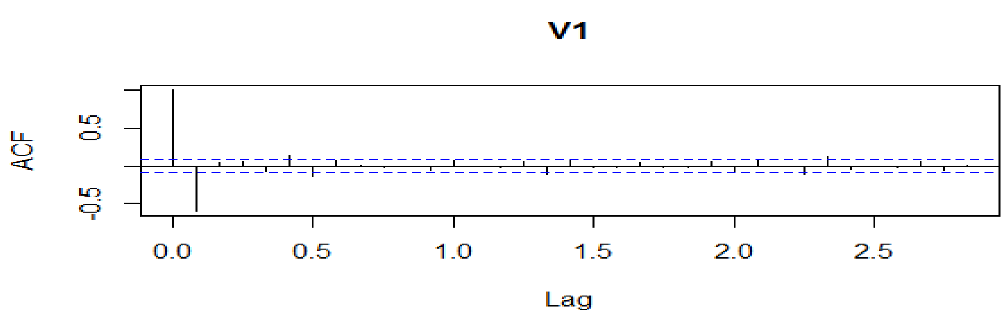

Figure 5: ACF Plot of 1^{st} Differenced Rainfall Data

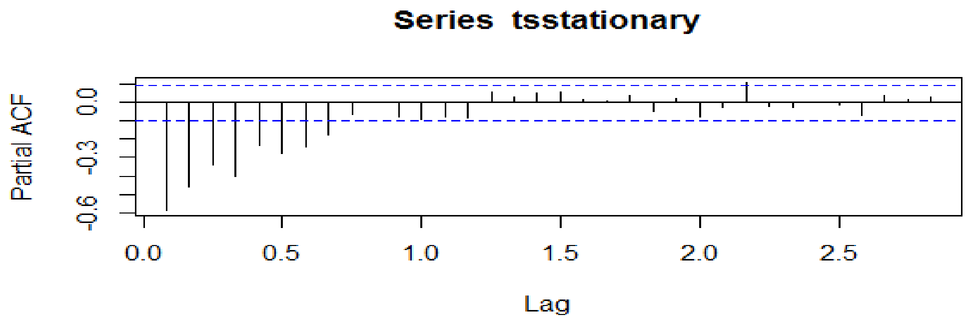

Figure 6: PACF Plot of 1st Differenced Rainfall Data

Table 2: Model Selection Criteria

Model	AIC	BIC	LOG LIKELIHOOD	MSE(σ^2)
(1,2,3)	**4100.42**	**6674.42**	**-2044.21**	**13200.4**
(2, 2, 1)	4372.04	6946.04	-2180.02	23738.9

Table 3: Parameter Estimation for the Chosen Model

Coefficients	Estimates	Standard Error	|Z| value
ϕ_1	-0.1909331	0.0664154	0.0040424 **
ϕ_2	-0.2250016	0.0543875	3.5186e-05 ***
θ_3	-0.1396907	-0.1396907	0.0073097 **
ε_1	-0.6109583	0.0495681	< 2.22e-16 ***

$$\mathbf{y_t} = \boldsymbol{\varepsilon_t} + \emptyset_1 \mathbf{y_{t-1}} + \emptyset_2 \mathbf{y_{t-2}} + \emptyset_3 \mathbf{y_{t-3}} \qquad \textbf{(14)}$$
$$\mathbf{y_t} = \boldsymbol{\varepsilon_t} - 0.1909331\mathbf{y_{t-1}} - 0.2250016\mathbf{y_{t-2}} - 0.1396907\mathbf{y_{t-3}} \qquad \textbf{(15)}$$

Table: 4 Rainfall Forecasts

FORECASTS	EXACT SD	LIMITING SD
68.9972	214.2406	215.7454
46.0386	217.8270	218.8541
71.7023	296.9442	300.4228
49.7650	299.9741	302.8081
75.8285	361.0369	367.0366
54.1398	363.3183	368.6420
80.2511	415.7982	424.9715
58.6195	417.4417	425.9898
84.6618	464.6554	477.7430
63.0096	465.7666	478.3453
88.9281	509.2863	527.1084
67.2142	509.9416	527.4278
92.9769	550.6271	574.0885

71.1751	550.7271	574.2282
96.7598	550.8701	619.3341
74.8507	589.2421	619.3727
100.2410	589.0997	663.2871
78.2085	625.4900	663.2886
103.3891	624.9755	706.2642
81.2201	659.6050	706.2771
106.1780	658.7111	748.4941
83.85952	691.7221	748.5614
108.5831	690.4323	790.1552

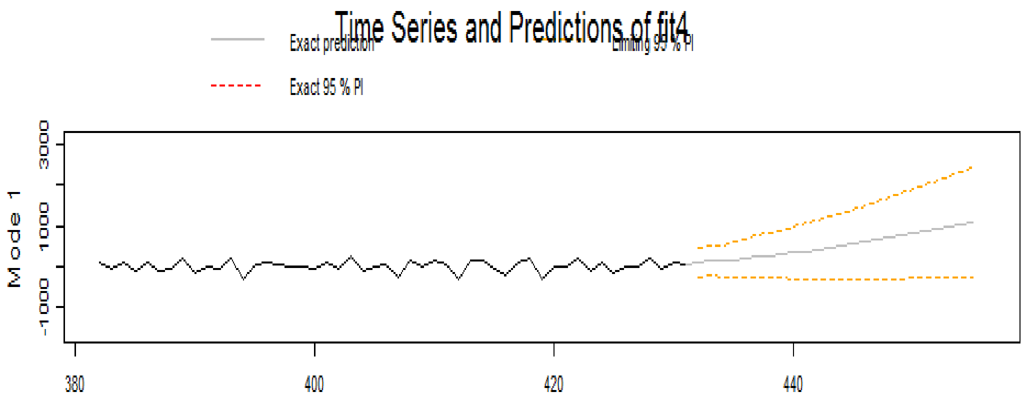

Figure 7: Time series Predictions of Fitted Model

3.2 Discussion

From figure 1 above, the time series plot of monthly rainfall indicates non- stationarity, with both upward and downward movements which contains some seasonal and irregular features. However, in order to achieve stationarity, the series was subjected to both ACF and PACF plots as shown in figures 2 and 3 respectively; these started high and declined slowly to suggest a first order differencing for stationarity purpose. Figure 4 shows that the data is stationary after the first difference and in order to ascertain this, Augmented Dickey Fuller test was carried out and the results presented in table 1 gives a statistic value of -14.794 with P-value of 0.000 which is less than 0.05 and hence conclude that the series is stationary in its mean and variance at first order difference. To further ascertain the stationarity condition of the rainfall data,the ACF and PACF of the differenced series were then carried out as reflected in figures 5 and 6respectively. The autocorrelation continues to decrease as the lag increases, confirming that there is no linear association between observations separated by larger lags.Also there is a slow decay in the PACF up to lag 8. Thus, with the achieved correlogram, it can be ascertained that the data is stationary over time.

Since the data is ready and satisfies all the assumptions of time domain modeling, the order of appropriate model to be fitted to the data is then determined for the three variables concerned (i.e. p, d, and q) which are non-negative integers that refer to the order of the autoregressive, integrated, and moving average parts of the model respectively. The orders were chosen based on the significant spikes from the ACF and PACF plots of figures 5 and 6. The significant spikes in ACF of lag 1 and lag 6 suggest MA of order 1 or 6. Also, the PACF with significant spikes up to lag 8 suggest AR of order 1 to 8.

The results in table 2 suggests that ARFIMA (1, 2, 3) is the best specified model with the least values of AIC, BIC, MSE and also with the highest log likelihood value. The parameters estimated for the chosen model is as presented in table 2 and the fitted model capable of forecasting the volume of Nigeria rainfalls is specified in equation (15).
Based on the set objectives of this research, forecasting was done using the fitted ARFIMA model for 24 months.A close look at figure 7 indicates that there is going to be a slow and steady rise in the incidence of rainfall in Abeokuta, Ogun state Nigeria with limiting bounds of between 215.7454 to 790.155 volumes as presented in table 4.

4. Conclusion

Having used necessary and suitable methods in line with the set goals of this research, there is no doubt that the main purpose has been fully realized.

Therefore, based on the results obtained by the empirical analysis of the data collected, the following conclusions are therefore arrived at:

- That ARFIMA model (1, 2, 3) is the most appropriate fit for Nigerian rainfall data.
- That the order of integration in the rainfall ARFIMA model is not fractional due to the integer value of the differencing parameter 'd'.
- That ARFIMA model is able to capture the long-range dependence, which cannot be expressed by stationary ARIMA.
- Thatthere is going to be a slow and steady rise in the incidence of rainfall in Abeokuta due to its forecast trend.

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