

Stagnation-Point Flow of a Radiative Tangent Hyperbolic Nanofluid over a Nonlinear Surface with Variable Thermal Conductivity

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Abstract: This study investigates the boundary layer stagnation point flow of hydromagnetic tangent hyperbolic nanofluid over a nonlinear plane in a porous device under the influence of a prescribed power law thermal condition. The modelled partial differential equations outlining the flow problems are translated into ordinary ones using suitable similarity transformation variables. The resultant equations are numerically solved by the shooting technique together with Runge-Kutta Fehlberg scheme. The effects of the various physical parameters are graphically analyzed and deliberated. The obtained results are validated with related existing studies in the limiting position and found to be in perfect agreement. The results indicate that the hydrodynamic boundary layer structure together with velocity profile decelerate when Darcy and magnetic parameters increase whereas the temperature profile appreciates with these terms. Temperature ratio or wall heating parameter triggers a rise in the heat transfer while the thermal field grows with an increase in the magnitude of thermophoresis and Brownian motion terms. Also, the heat transfer at the surface is a decreasing function of these terms.

Keywords: Magnetohydrodynamic; Radiation; Porous medium; Tangent hyperbolic nanofluid.

1 Introduction

The suspension of nanoparticles in traditional fluids such as water, oil, ethylene glycol, etc is described as nanofluid. This composition is to improve the thermal properties of the fluid and at such the heat transfer capacity of the fluid is enhanced. Also, such a blend offers unique characteristics often needed in various industrial and engineering operations such as in transportation industries, pharmaceutical processes, cooling of engines and vehicles, etc. Choi and Eastman [1] initialized such a concept which, owing to crucial applications has been widely extended by various authors on Newtonian/non-Newtonian fluids. Abolbashari *et al.* [2] generated results for the transport of Casson nanofluid activated by a stretched sheet coupled with convective wall situation and slip property. Of recent, Alsaedi *et al.* [3] offered an analytical report on such subject with the use of Eyring-Powell nanofluid associated with Ohmic heating and thermophoresis effect. More reports on this concept can be found in Refs [4-6].

The non-Newtonian fluids provide huge engineering and technological applications that makes them a sought-after in the recent years. These wide applications can be found in food processing, oil drilling, pharmaceutical operations, bio-medical engineering, chemical engineering operations and so on. Various branches and mathematical models capturing the non-Newtonian fluids transport exists in the literature due to diverse physical differences in the constitutive relations of fluids. The tangent hyperbolic fluid characterizes shear-thinning properties of non-Newtonian fluid, its concept originated from This model is simple and attractive and its preference over other constitutive non-Newtonian models stems from the fact that its constitutive laws are deduced from the theory of molecular kinetic as against conventional empirical relations. Common examples of fluids that characterize the tangent hyperbolic properties include the biological and industrial fluids (e.g. blood, ketchup, paints, melts solution and polymers). Due to its importance, various researchers have reported such fluids on different geometries. For example, Ullah *et al.* [7] numerically reported the transport of such fluids prompted a stretchable sheet with internal heat source associated with Lie group analysis, Mahdy [8] discussed numerically tangent hyperbolic nanofluid flow configured in a stretchable cylinder with non-uniform temperature whereas Malik *et al.* [9] analyzed such fluids using the same configuration coupled with magnetic field impact and reported that a higher flow resistance occurred with an enlargement of the material parameter and magnetic field term.

Stagnation point flow describes a region of highest static pressure where at the same time the heat transfer and mass deposition attain maximum level whereas the velocity is zero. The applications of such a phenomenon can be encountered in the engineering and industrial operations including the cooling of electronic appliances by fans, thermal oil recovery, the extrusion of polymers in melt-spinning processes and so on. Pioneering such a study, Hiemenz [10] worked on a two-dimensional flat sheet. Afridi *et al.* [11] conducted an analysis on such phenomenon engaging a Newtonian fluid enclosed in a stretchable sheet with Ohmic heating and entropy generation. Animasaun [12], Fatunmbi and Adeniyi [13] engaged micropolar fluid to investigate such a concept on a steady stretchable sheet with an internal heat source.

In many engineering processes and material manufacturing which require high temperature (e.g Nuclear power plants, hot rolling, heat exchangers, electrical power generation), the knowledge of thermal radiation is important for the construction of relevant devices. Generally, the radiative heat flux can be modelled as linear or nonlinear types depending on the magnitude of temperature difference that exists within the flow. The linear type is operative in the case of low temperature difference whereas the nonlinear type is relevant for the situations of low and high temperature differences. Of recent, Al-Khaled *et al.* [14] conducted research showing the significance of such a concept in the transport of a reactive tangent hyperbolic fluid while Khan *et al.* [15] using the same fluid to study the impact of nonlinear radiation associated with an internal heat source in the neighborhood of a stagnation region. For wider applications of thermal radiation, the present study has incorporated the general nonlinear thermal radiation model [16-18].

The present study aims to carry out an investigation on the stagnation point flow in hydromagnetic tangent hyperbolic nanofluid towards over a nonlinear stretching sheet in a porous enclosure under the influence of variable thermal conductivity. The consequential applications of such study have motivated its analysis. More so, the study incorporates the influence of nonlinear thermal radiation, prescribed surface temperature and concentration boundary conditions.

2 Problem Development and Governing Equations

Consider a steady, two dimensional, incompressible and stagnation point flow of hydromagnetic tangent hyperbolic nanofluid over an impermeable and nonlinearly stretching sheet with the influence of temperature-dependent thermal conductivity. The coordinate system is taken to be (x, y) with the respective velocity components indicated as (u, v) where x axis signifies the flow direction with normal axis being y as shown in Fig. 1. The nonlinear stretching sheet has the velocity $u = U_s = hx^\alpha$ whereas the velocity at the upstream is $u = U_b = qx^\alpha$ where $h > 0, q$ and α are the stretching parameter, a constant which measures stagnation point flow potency and the nonlinear stretching term related to the surface stretching speed in that order. The magnetic field is imposed normal to the flow direction and it is taken as $B(x) = B_0x^{(\alpha-1)/2}$. In the energy equation, the heat transfer is being influenced by Joule heating, viscous dissipation, nonlinear thermal radiation as well as thermophoresis and Brownian motion. Note that *MBL, TBL* and *CBL* in Fig. 1 respectively describe the Momentum boundary layer, Thermal boundary layer and Concentration boundary layers.

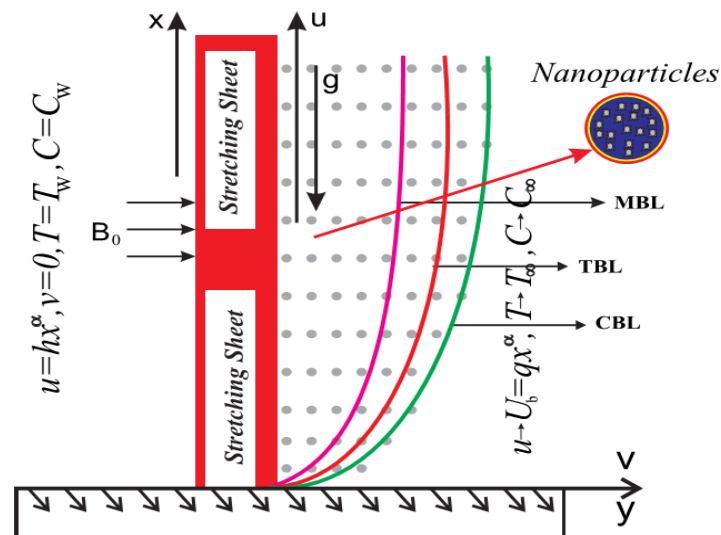


Fig. 1 The Flow Configuration

Incorporating these assumptions together with the principles of the boundary layer approximations, the following equations are the formulated boundary layer equations for the hydromagnetic tangent hyperbolic nanofluid.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = U_b \frac{dU_b}{dx} + \frac{\mu_f}{\rho_f} (1 - \beta) \frac{\partial^2 u}{\partial y^2} + \frac{\mu_f}{\rho_f} \sqrt{2\kappa\Gamma} \frac{\partial^2 u}{\partial y^2} \frac{\partial u}{\partial y} - \frac{\sigma B(x)^2}{\rho} (u - U_b) - \frac{\mu_f}{\rho_f K_p} (u - U_b), \tag{2}$$

$$(\rho c_p)_f \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + (\rho c_p)_p \left(\frac{\partial T}{\partial y} \right) \left[\frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right) + D_B \left(\frac{\partial C}{\partial y} \right) \right] + \sigma B(x)^2 (u - U_b)^2 + \frac{\mu_f}{K_p} (u - U_b)^2 + \frac{16\zeta^{\dot{a}}}{3a^{\dot{a}}} \frac{\partial}{\partial y} \left(T^3 \frac{\partial T}{\partial y} \right), \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial C^2}{\partial y^2} + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2} \right). \tag{4}$$

The accompanied boundary conditions for equations (5-8) are:

$$u = U_s = hx^\alpha, v = 0, T = T_w (= T_\infty + Bx^{m_1}), C = C_w (= C_\infty + Dx^{m_2}) \text{ when } y = 0, \tag{5}$$

$$u \rightarrow U_b = qx^\alpha, T \rightarrow T_\infty, C \rightarrow C_\infty, \text{ with } y \rightarrow \infty.$$

The expression for the thermal conductivity as relates to temperature is presented as,

$$k = \frac{k_\infty}{T_w - T_\infty} [(T_w - T_\infty) + \delta(T - T_\infty)]. \tag{6}$$

with k_∞ being the constant thermal conductivity while δ defines the thermal conductivity parameter.

Similarly, the permeability of the porous medium in Eq. (6) is indicated as $K_p = K_0 x^{1-\alpha}$, where K_0 is a constant. The underlisted dimensionless variables are introduced into the main equations [19-20].

$$\eta = \sqrt{\frac{h(\alpha+1)x^{\alpha-1}}{2\nu}} y, \psi = \sqrt{\frac{2\nu h x^{\alpha+1}}{\alpha+1}} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, Nr = \frac{16\zeta^{\dot{a}} T_\infty^3}{3a^{\dot{a}} k_\infty}, Re = \frac{U_s x}{\nu_f}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, Nt = \frac{(\rho c_p)_p D_T (T_w - T_\infty)}{(\rho c_p \nu)_f T_\infty}, H = \frac{q}{h}, Da = \frac{\nu}{h K_0} \tag{7}$$

$$\theta_w = \frac{T_w}{T_\infty}, Pr = \frac{\mu_f c_p}{k_\infty}, M = \frac{\sigma B_0^2}{h\rho}, Sc = \frac{\nu}{D_B}, We = \Gamma \sqrt{\frac{h^3 x^{3r-1} (\alpha+1)}{\nu_f}}, Ec = \frac{U_w^2}{C_p (T_w - T_\infty)}$$

$$Nb = \frac{(\rho c_p)_p D_B (C_w - C_\infty)}{(\rho c_p \nu)_f}.$$

The substitution of quantities in (12) into the governing Eqs. (5-8) yields:

$$((1 - \beta) + \beta Wef''') f''' + ff''' - \frac{2\alpha}{\alpha+1} (f'^2 - A^2) - \frac{2}{\alpha+1} (M + Da)(f' - A) = 0, \tag{8}$$

$$\left[1 + \delta\theta + Nr(1 + (\theta_w - 1)\theta)^3\right]\theta'' + 3Nr(\theta_w - 1)\theta'^2(1 + (\theta_w - 1)\theta)^2 + Pr\left(f\theta' - \frac{2m_1}{\alpha + 1}f'\theta\right) + Pr(N_t\theta'^2 + N_b\theta'\phi') + \frac{2}{\alpha + 1}(M + Da)(f' - H)^2 Pr + \delta\theta'^2 = 0, \tag{9}$$

$$\phi'' + \frac{N_t}{N_b}\theta'' + Sc\left(f\phi' - \frac{2m_2}{\alpha + 1}f'\phi\right) = 0. \tag{10}$$

The boundary situations transform to:

$$\begin{aligned} f'(0) = 1, f(0) = 0, \theta(0) = 1, \phi(0) = 1. \\ f'(\infty) = A, \theta(\infty) = 0, \phi(\infty) = 0. \end{aligned} \tag{11}$$

Table 1: Description of the Symbols

Symbols	Description	Symbols	Description
C	Concentration	Nb	Brownian motion parameter
C_w	Concentration at the sheet	a^*	Mean absorption coefficient
C_∞	Concentration at the free stream	Nt	Thermophoretic parameter
Da	Darcy parameter	Pr	Prandtl number
D_B	Brownian diffusion coefficient	D_T	Thermophoretic diffusion coefficient
Ec	Eckert number	Sc	Schmidt number
T_w	Temperature at the sheet	T	Temperature
u	Velocity in x direction	T_∞	Temperature at free stream
H	Velocity ratio parameter	U_s	Velocity at the sheet
K_1^2	Chemical reaction rate	v	Velocity in y direction
M	Magnetic field term	We	Weissenberg number
	Temperature exponent	x, y	Cartesian coordinates
Greek letters		Greek letters	
μ_f	Base fluid viscosity	ρ_f	Base fluid density
γ_1	Chemical reaction	$\dot{\theta}$	Temperature relative parameter
ν_f	Kinematic viscosity	κ	Power law exponent
θ	Dimensionless temperature	θ	Dimensionless temperature
θ_w	Temperature ratio term	θ_w	Temperature ratio term
$\zeta^{\dot{a}}$	Stefan-Boltzmann constant	Γ	Relaxation time
σ	Electrical conductivity	$(\rho c_p)_f$	Effective heat capacity of fluid
m_2	Concentration exponent	$(\rho c_p)_p$	Effective heat capacity of nanoparticles

The quantities of interest for the engineers are the coefficient of skin friction C_{fx} , the Nusselt number Nu_x and the Sherwood number Sh_x . These quantities are orderly presented as follows.

$$C_{fx} = \frac{2\tau_w}{\rho_\infty U_s^2}, Nu_x = \frac{xq_w}{k_\infty(T_w - T_\infty)}, Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}, \tag{12}$$

$$\tau_w = \mu_\infty \left[\frac{\partial u}{\partial y} + \frac{\Gamma}{\sqrt{2}} \left(\frac{\partial u}{\partial y} \right)^2 \right]_{y=0}, q_w = - \left[\left(k_\infty + \frac{16T^3 \sigma^{\dot{a}}}{3k^{\dot{a}}} \right) \frac{\partial T}{\partial y} \right]_{y=0}, q_m = - \left(D_B \frac{\partial C}{\partial y} \right)_{y=0}, \quad (13)$$

where τ_w indicates surface shear stress, q_w represents surface heat flux and q_m denotes the surface mass flux in that order. Using Eqs. (12) and (18), the dimensionless C_{fx} is written in Eq. (19) while that of Nu_x and Sh_x are respectively presented in Eq. (20).

$$Re_x^{\frac{1}{2}} C_{fx} = 2(\alpha + 1)^{\frac{1}{2}} \left[(1 + \beta) f''(0) + \frac{We}{2} f''^2(0) \right], \quad (14)$$

$$Re_x^{-\frac{1}{2}} Nu_x = - \left(\frac{\alpha + 1}{2} \right)^{\frac{1}{2}} \left[1 + Nr(1 + (\theta_w - 1)\theta(0))^3 \right] \theta'(0), Re_x^{\frac{1}{2}} Sh_x = - \left(\frac{\alpha + 1}{2} \right)^{\frac{1}{2}} \phi'(0). \quad (15)$$

3 Numerical Method with validation

The system (13) to (15) subject to (16) depicts a boundary value problem with high degree of nonlinearity, thus, an exact solution to the system seems difficult to assess. In this view, an approximate numerical solution is sought by a famous shooting technique coupled with Runge-Kutta-Fehlberg method. Based on the popularity of this method as described and applied by many authors ([21-24]), we do not give its details here. We set $\beta = We = \zeta_1 = \zeta_2 = N_t = N_b = m_1 = m_2 = 0.1, \theta_w = 1.3, Q = 0.01, Da = M = \delta = Nr = \lambda_1 = \lambda_2 = G = r = N = 0.2, Sc = \alpha = 1.0$ and $Pr = 2.0$ as parametric values unless listed otherwise in the plots. To crosscheck the credibility of the results obtained in the current study, a comparative analysis of $f''(0)$ for variations in M with previously reported data of Mabood and Das [22] and Xu & Lee [23] is reported in Table 2 and a comparative study of $(-\theta(0))$ for different Pr with previously published results of Ali [24] alongside with Mabood and Shateyi [25] is recorded in Table 3 which validate our code. The comparisons of solutions are shown in Table 2 and 3, reasonable agreement is obtained with earlier computations which confirm the validity of the present numerical solutions.

Table 2: Values of $f''(0)$ in comparison with [22-23] as M changes when $\beta = We = H = Da = \lambda_1 = \lambda_2 = N = 0, Pr = \alpha = 1.0$.

M	[22]	[23]	Present Study
0	1.000008	--	1.00000
	-1.4142135	-1.41421	-1.41421
	-2.4494987	-2.4494	-2.44949
	-3.3166247	-3.3166	-3.31662
	-7.1414284	-7.1414	-7.14143
	-10.049875	-10.0498	-10.04989
	-22.383029	-22.3830	-22.38302
	-31.638584	-	-31.63863

Table 3: Comparison of values $(-\theta'(0))$ with [24] and [25] for changes in Pr when $m_1 = m_2 = \alpha = 1.0$ and $\delta = Nr = Ec = Q = G = M = H = Da = 0$.

Pr	[24]	[25]	Present Study
0.72	0.8058	0.8088	0.808637
.0	0.9691	1.0000	1.000000
.0	1.9144	1.9237	1.923683
.0	3.7006	3.7207	3.720674

4 Presentation of Results

The physical impacts of the selected parameters on the various dimensionless quantities are presented graphically (Figures 2-14) and discussed in this section. The plot capturing the reactions of the velocity field versus η is presented in Figs. 2. In clear terms, there is a decelerated flow as the magnetic field (M) parameter enlarges. Such a trend is induced by the action of the Lorentz force occasioned by the interaction of the transversely applied magnetic field and the electrically conducting tangent hyperbolic fluid.

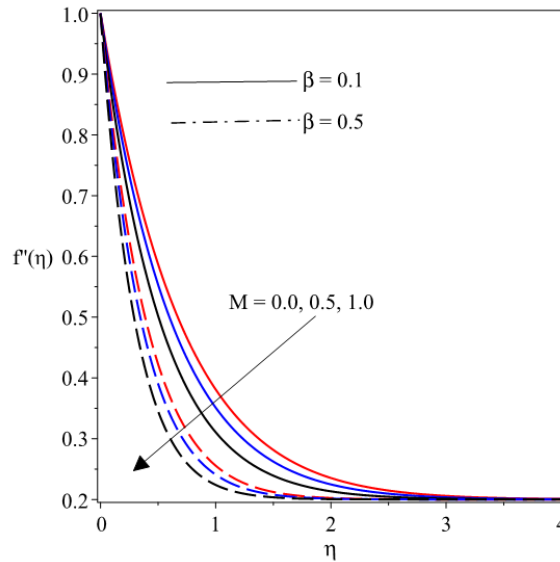


Fig. 2 Plots of velocity distribution with variations in M & β

Hence, a hike in M improves the Lorentz force and consequently compel a reduction in the flow regime. The power law exponent β in the presence of M also reduces the flow and has a diminishing influence on the velocity regime as depicted in Fig. 2. This pattern resulted from an enhanced strength of viscosity as the magnitude of β enlarges. Figure 3 informs that the increasing trend in M raises the temperature in the presence of β . This can be attributed to the additional friction created in the flow field as a result of Lorentz force due to magnetic field influence. Similarly, an increase in β created an expansion in the thermal boundary layer and consequently compel the temperature to increase as shown in Fig. 3.

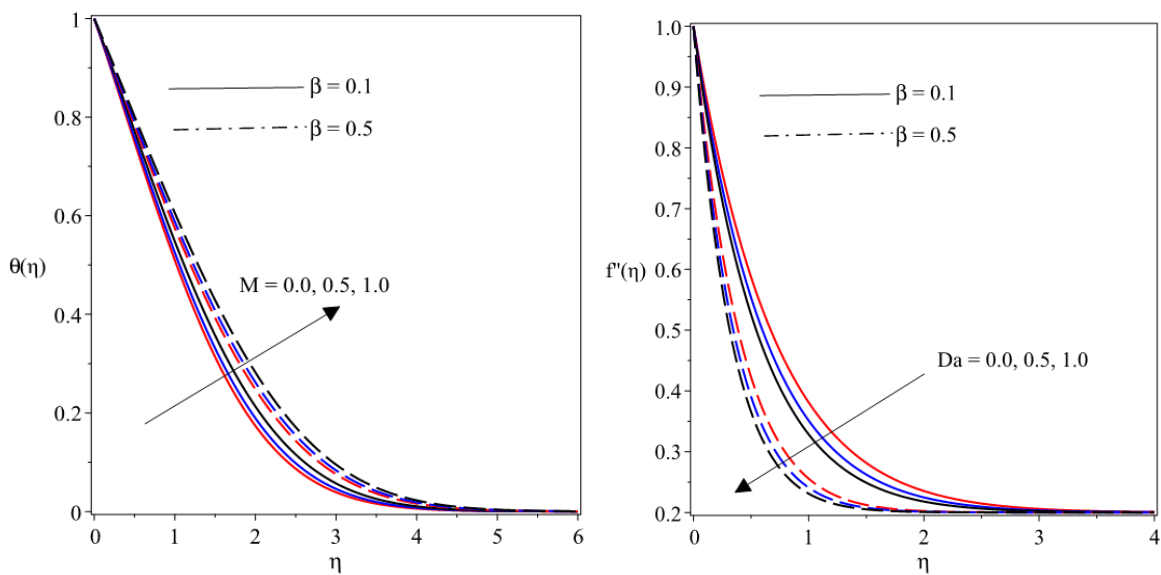


Fig. 3 Temperature distribution M & β Fig. 4 Velocity profiles for variations in Da & β

Figure 4 reveals that the fluid motion decreases with rising values of the Darcy number Da in the existence of β . An increase in Da creates a resistance for the fluid flow as noticed in this figure. It is observed

that the fluid flow is higher for non-porous medium, that is, $Da = 0$ than for porous medium ($Da \neq 0$). Contrarily, the temperature field is enhanced as Da is increasing in magnitude as clearly shown in Figure 5. This is due to the thickness of the thermal boundary layer for growth in Da .

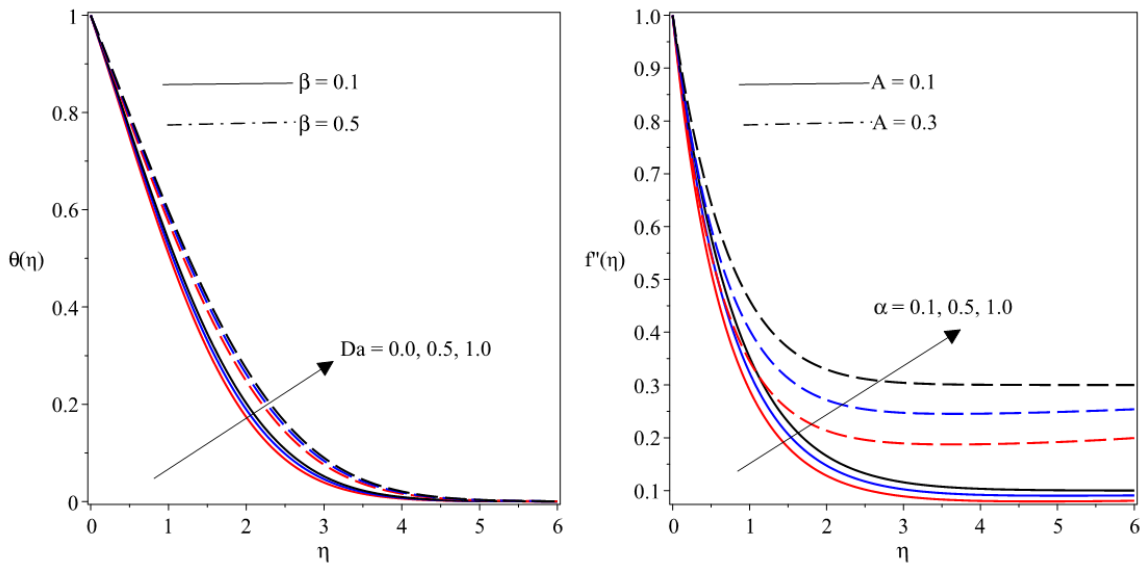


Fig. 5Temp. versus η for variation in Da & β **Fig. 6** Velocity field for varying A & α

The nonlinear stretching parameter α and the velocity ratio parameter boost the momentum boundary layer thickness and thus accelerate the motion of the fluid as demonstrated in Fig. 6. The upstream velocity is moving faster than the sheet velocity, hence the reduction in the drag and accelerate flow is recorded due a hike in the velocity ratio term.

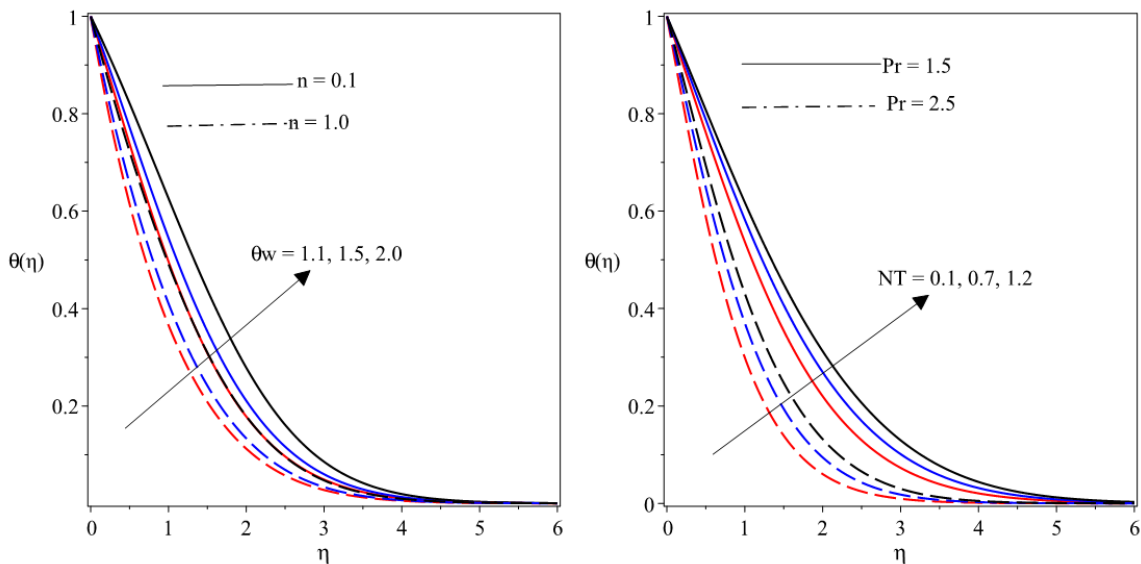


Fig. 7 Temp field for variation in θ_w & n **Fig. 8** Tempfield for different values of NT & Pr

Figure 7 portrays the impact of the temperature parameter and the temperature exponent term on the surface temperature. Here, it is seen that the thermal boundary layer is energized due to a rise in θ_w whereas the temperature exponent term n diminishes the thermal boundary layer and consequently lowers the average temperature. Similarly, a rise in θ_w connotes a higher sheet temperature as compared to the upstream temperature, hence, the surface temperature rises.

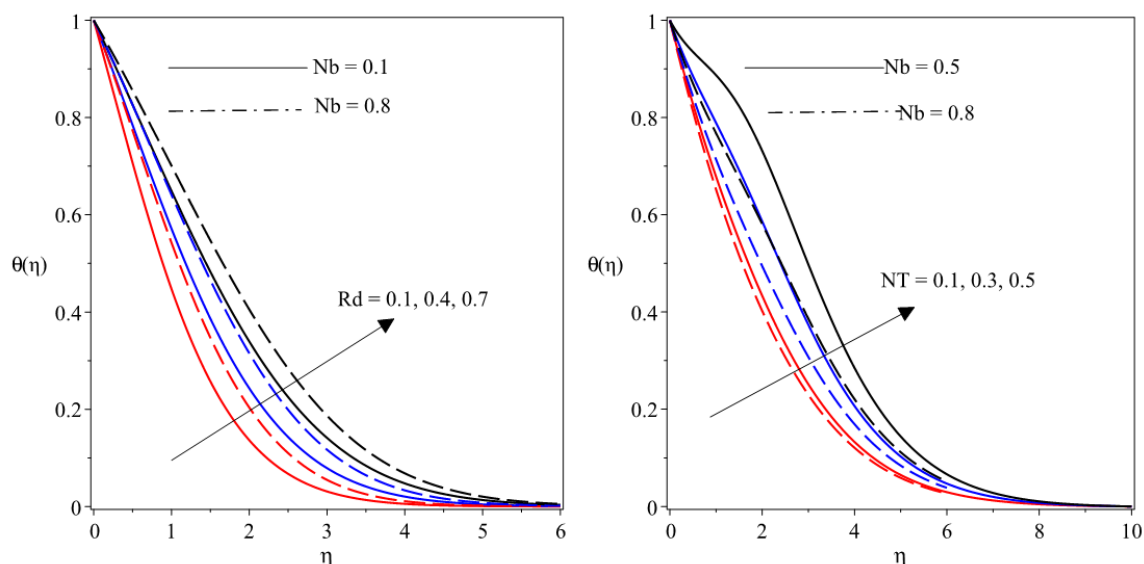


Fig. 9 Temp field for variation in Rd & Nb **Fig. 10** Temp field for different values of NT & Nb

Figure 9 depicts the influence of radiation parameter and Brownian motion term on the temperature profiles. A rise in the magnitude of Rd enhances the surface temperature as clearly shown in this figure. Similarly, haphazard motion of tiny particles compels a thickened boundary layer structure and thereby boost the temperature profiles as described in Fig. 9. In the same vein, the influence of the thermophoresis parameter is to cause a thickness in the thermal boundary structure which in turn boost the temperature across the boundary layer as clearly displayed in Fig. 10.

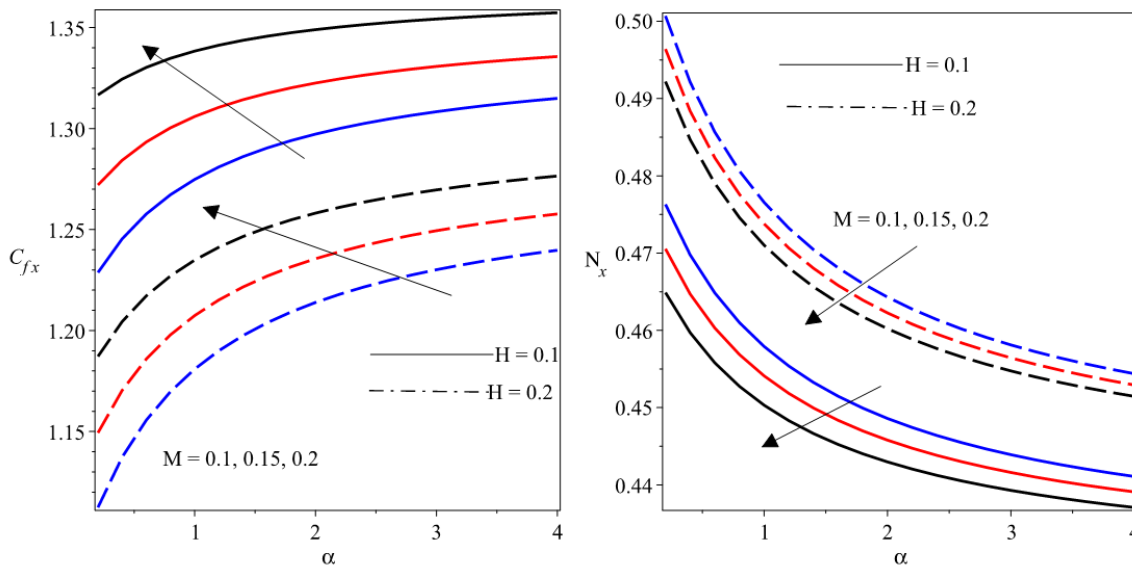


Fig. 11 Combined effects of M & H on C_{fx} **Fig. 12** Combined effects of M & H on N_{ux}

In Fig. 11, an increase in M promotes a higher skin friction coefficient. In this case, the drag phenomenon is increased to higher magnitudes of M whereas a rise in the stretching parameter H lowers the skin friction coefficient C_{fx} as showcased in Fig. 11. On the contrary, the heat transfer denoted as the Nusselt number N_{ux} depreciates with a hike in M while it accelerates with a boost in H as shown in Fig. 12.

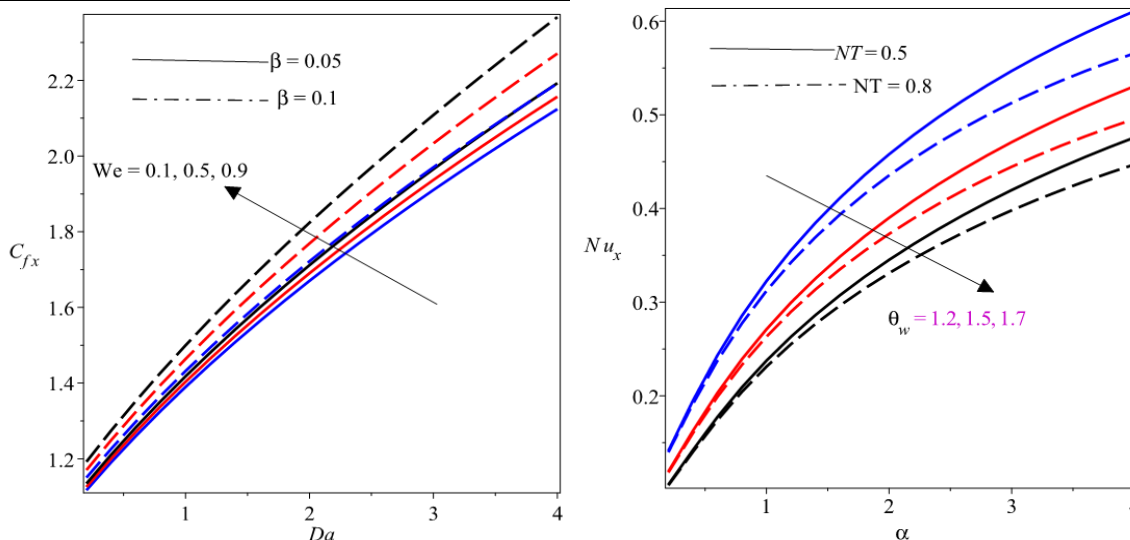


Fig. 13 Combined effects of We & β on C_{fx} **Fig. 14** Combined effects of θ_w & NT on Nu_x

A rise in the Weissenberg number We cause the skin friction coefficient to be increased whereas the opposite is the case for higher β as depicted in Fig. 13. This means that an increment in the strength of Weissenberg number (We) with the power law exponent accelerates the coefficient of skin friction as demonstrated in Fig. 13. In Fig. 14, both θ_w and NT act to reduce the rate of heat transfer across the surface of the sheet.

5 Conclusion

A physical model is developed for the thermo-solutal transport of a tangent hyperbolic nanofluid in the neighbourhood of a stagnation point from a nonlinear stretchable sheet enclosed in a porous device. Also included in the model are the impacts of nonlinear thermal radiation and activation energy with MHD. The transformed dimensionless nonlinear boundary value problem has been solved with the Runge-Kutta Fehlberg coupling shooting technique. Validation with previous studies reveals perfect correlation in the limiting cases. A parametric study of the impact of selected variables on the transport characteristics has been constructed graphically. The simulations have shown that:

- Increment in the Darcy Da , power law exponent β and magnetic field M terms lead to the diminution of the flow field.
- Rise in the velocity ratio term H upsurges fluid velocity while that peters out the temperature.
- There is an improvement in the tangent hyperbolic fluid temperature with the increased values of the radiation Nr , heat source Q , thermophoresis N_t and Eckert number Ec parameters.
- Skin friction coefficient is strengthened due to higher magnetic M , power law exponent β and Darcy parameters Da while a reversed trend occurred with a rise in H .
- Nusselt number appreciates in respect of the Prandtl number Pr , temperature ratio term θ_w and Nr while the opposite is the case for N_t , N_b and Ec .
- The mass transfer enlarges for incremental values of Da, M whereas it falls with thermophoresis term NT

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