ON MODELLING RAINFALL PATTERN IN TWO DIFFERENT LOCATIONSWITH SAME SEASONAL ARIMA Models

AjibodeI.A. and OgunnusiO.N.
Department of Mathematics & Statistics, Federal Polytechnic, Ilaro, Nigeria.
E-mail Address: ilesanmi.ajibode@federalpolyilaro.edu.ng; ogunnusioluwatobi@gmail.com

ABSTRACT

The Forecast of rainfall on monthly and seasonal timeframe is not only scientifically challenging, but is also a means of planning for decision making purposes. Various research groups have attempted to forecast rainfall on a seasonal timeframe using different techniques. This research work describes the Box-Jenkins time series Seasonal ARIMA (Auto Regressive Integrated Moving Average) approach for the prediction of rainfall on monthly scale using Abeokuta and Ijebu Ode, Ogun State. Seasonal ARIMA$(1,0,2)(1,0,1)_{12}$ was identified the best model to forecast rainfall for Abeokuta and Ijebu Ode for the next four years with 95% confidence level by analyzing 17 years data(1999-2016) recorded on monthly basis by the Nigeria Meteorological Agency as extracted from Central Bank of Nigeria statistical bulletin. Previous data was used to formulate the seasonal ARIMA model and in the determination of model parameters. The preference evaluation of the adopted models in R was carried out on the basis of Akaike Information Criterion (AIC), MSE and Log-likelihood taking method of maximum likelihood estimation into consideration. The result indicated that the Seasonal ARIMA model provide consistent and satisfactory prediction for rainfall parameters on monthly scale as evidenced from the Shapiro-wilk normality test and Ljung-Box test of independence of residuals. Forecasts also indicated that there is higher rise of rainfall in the month of April, May, June, July and September in Abeokuta and Ijebu-Ode respectively between year 2017 to 2020, but with little variation of measurements.

Keywords: ARIMA, Rainfall, Auto correlation function (ACF), Partial autocorrelation function (PACF), Stationarity, AIC, Log-likelihood

INTRODUCTION

Forecasting is a significant method for inferring the future behavior of a particular field under the uncertainty conditions with the limited known weatherbehaviors. This scenario is well suited for many practical applications such as rainfall data, because of the importance of knowing about weather for agricultural purposes as well as for preventing natural disasters. Practically, rainfall data forecasting is a difficult task due to its' nonlinear behavior with high volatility and complex nature. Furthermore, rainfall is considered as the most important climatic element that influences agriculture. Therefore monthly rainfall forecasting plays an important
role in the planning and management of agricultural scheme and management of water resource systems.

Rainfall is the most important natural factor that determines the agricultural production in Nigeria, particularly in the South Western part of Nigeria. The variability of rainfall and the pattern of extreme high or low precipitation are very important for agriculture as well as the economy of the state. It is well established that the rainfall is changing on both the global and the regional scales due to global warming (Hulme et al, 1998, Kayano, 2008). As the moves to encourage agriculture to ensure food security continues to gain ground and acceptability, information on rainfall probabilities is vital for the design of water supply and supplemental irrigation schemes and the evaluation of alternative cropping and of soil water management plans. Such information can also be beneficial in determining the best adaptable plant species and the optimum time of seeding to reestablish vegetation on deteriorated rangelands. Much as long rainfall records are mostly available in many countries, little use is made of this information because of the unwieldy nature of the records (Mina and Sayedul, 2012).

However, the most important part of the hydrological cycle is rainfall (Ramana *et al.* 2013). It is the result of many complex physical processes that induce particular features and make its observation complex (Akrour*et al.* 2015). In the prediction of meteorological information the investigation and analysis of precipitation is so essential (Radhakrishnan, Dinesh 2006), and accurate forecast of precipitation is crucial for improved management of water resources, particularly in arid environment (Feng *et al.*2015).According to Somvanshi*et al.* (2006), rainfall isnatural climatic occurrences and its prediction remains a difficult challenge as a result of climatic variability. The forecast of precipitation is particularly relevant to agriculture, growth of plants and development, which profoundly contribute to the economy of Africa.

Available evidences show that climate change will be global, likewise its impacts, but the biting effects will be felt more by the developing countries especially those in Africa due to their low level of coping capabilities (Mshelia, 2005; Nwafor, 2007; Jagptap, 2007). Nigeria is one of such developing countries. Researchers have shown that

Nigeria is already being plagued with diverse ecological problems, which have been directly linked to the ongoing climate change (Adebayo 1998; Odjugo and Ikhouriia, 2003).
Babatolu (2002) studied spatial distribution of rainfall in Ondo State, Nigeria. Variation in rainfall receipt per rain-day and observed that there has been a progressive early retreat of rainfall over the whole country, and consistent with this pattern, reported a significant decline of rainfall frequency in September and October which, respectively coincide with the end of the rainy season in the northern and central parts of the country.

Chukwukere (2005) carried out a research on monthly rainfall at Isunjaba Imo State from (2000-2004), he revealed that there exists a trend for the period considered and it showed a regular cyclical movement. Climate variability has been noted to arise as a result of changing rainfall pattern, some regions have experienced marked decline in rainfall patterns depending on the location. For state whose economy largely depends on efficient and productive rain-fed agriculture, rainfall patterns and trends are often quoted as one of the major causes of several socio-economic problems like food insecurity in the state. (Ekwe, Joshua, Igwe, Osinowo,2014).

These studies focused more on climatic impacts. Studies that addressed climate trends in Nigeria cover either short period or small area (Anyadike 1992a; 1992b Clerk 2002; Nkeiruka and Apagu, 2005; Olaniran 2002; Odjugo, 2005; Nnodu et al., 2007). Whereas Singer and Avery (2007) revealed that it takes at least a century of weather data to evaluate climate trend for a reasonable conclusion to be drawn.

There is therefore the need to examine the rainfall pattern in Ogun state over a long time as also suggested by Nwafor (2006) so as to capture the long term changes as the current rainfall pattern in the state has been a concern to agriculturist who rely on it for their economic growth and some other social activities. Therefore, it is on this basis that this paper seeks to examine the trend of rainfall in Ogun State taking Ijebu Ode and Abeokuta as case studieswith the view to ascertain the feasibility of government's efforts towards improved agricultural produce in order to enhance food security in the state.

Empirical Study of SARIMA for Model Building Development
Box and Jenkins (1976) introduced a SARIMA model as an adaptation of an autoregressive integrated moving average (ARIMA) model, which they earlier proposed, to specifically explain the variation of seasonal time series. SARIMA modeling has been quite successful.Recently a few researchers modeled monthly rainfall using SARIMA methods. Nirmala &Sundhram (2010) fitted a SARIMA $(0,1,1) \times (0,1,1)_{12}$ model to monthly rainfall in Tamil Nadu. Abdul-Aziz *et al.*(2013) fitted a SARIMA $(0,0,0)\times(2,1,0)_{12}$ to the rainfall data in Ashanti region of Ghana. EtukandMohamed(2014) fitted a SARIMA $(0,0,0)(0,1,0)_{12}$ model to monthly rainfall in Gadaref, Sudhan. Anosh Graham *et al.(*2017)fitted SARIMA$(0,0,0)\times(0,1,0)_{12}$ model to monthly rainfall and identified best model and forecasted rainfall for next 5 years for Allahabad region, Uttar Pradesh (India). Hellen *et. al* (2014) *fitted* SARIMA$(1,0,1)\times(1,0,0)_{12}$ to forecast precipitation in Kenya.

MATERIALS AND METHODS
The data for this study was obtained on monthly basis from the Central Bank of Nigeria Statistical bulletin over a period of 15 years between the years 1999 – 2016 where rainfall series of Ijebu Ode and Abeokuta recorded by Nigeria Meteorological Agency was extracted. Data to be used for modeling was subjected to unit root test in order to calibrate the data for model building development. In order to validate this characteristic in time series data, we utilize Augmented Dickey-Fuller unit root test. The study test each time series individually to ensure non-stationarity at the levels of the data, and also run the unit root tests on the first differences to ensure either I(0), I(1) or I(2). The equation for the ADF is given as:

$$\Delta y_t = a_0 + a_1 y_{t-1} + \sum_{i=1}^{p} a_i \Delta y_{t-i} + \varepsilon_i \qquad (1)$$

We reject the null hypothesis of no unit root if the P-value of the Dickey Fuller test statistic is less than the risk value of 0.05.
Box-Jenkinsmethodology of Seasonal Autoregressive Integrated Moving Average model was adopted in this paper. The SARIMA model incorporate both non-seasonal and seasonal factors, written in multiplicative form as:
SARIMA $(p,d,q)\times(P,D,Q)_S$ (2)

Where; **p** is non seasonal AR order; **P** is seasonal AR order**; d** is non seasonal difference D is seasonal differencing **q** is non seasonal MA order; **Q** is seasonal MA order; **s** is time span of repeating seasonal pattern.
From equation (2) the Autoregressive model consists of two parts, AR part and an MA part. The notation of ARMA (p, q) refers to the model with p autoregressive terms and q moving average terms. This model contains the AR (p) and MA (q) models,

$$X_t = c + \varepsilon_t + \sum_{i=1}^{p} \emptyset_i X_{t-i} + \sum_{i=1}^{q} \theta_i X_{t-i} \quad (3)$$

Equation (3) can be written in lag operator form as

$$\left(1 - \sum_{i=1}^{p} \emptyset_i B^i\right) X_t \quad = \quad \left(1 + \sum_{i=1}^{q} \theta_i B^i\right) \varepsilon_t \quad (4)$$

The error terms $\varepsilon_t$are generally assumed to be independent identically distributedrandom variables (*i.i.d*) sampled from a normal distribution with zero mean:ε_t~N(0, σ^2) where σ^2 is the variance. These assumptions may be weakened but doingso will change the properties of the model. In particular, a change from the *i.i.d*assumption would make a rather fundamental difference.

ARMA is appropriate when a system is a function of a series of unobserved shocks (the MA part) as well as its own behaviour.

Generalization of the ARMA model result to the Autoregressive Integrated Moving Average (ARIMA). ARIMA (p,d,q) are applied when data show evidence of non-stationarity where an initial differencing step (corresponding to the "integrated" part of the model) can be applied to remove the non-stationarity.

A time series X_t is said to follow an integrated autoregressive moving average model if the dthdifference $W_t = \nabla^d X_t$is a stationary ARMA process.

Consider an ARIMA (p, 1, q) process where d = 1. With $W_t = X_t - X_{t-1}$ we have:

$$W_t = \emptyset_1 W_{t-1} + \emptyset_2 W_{t-2} + \cdots + \emptyset_p W_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q} \quad (5)$$

This can be written in lag form as:

$$\left(1 - \sum_{i=1}^{p} \emptyset_i B^i\right) X_t \quad = \quad \left(1 + \sum_{i=1}^{q} \theta_i B^i\right) \varepsilon_t \quad (6)$$

When data is seasonal in nature, i.e. the movement is usually due to the recurring events which takes place annually, or quarterly as the case may be, the plot of the series against time checks for non-seasonal or seasonal changes that reveal non-stationarity. Peaks in a series for every 12 months would indicate annual seasonality whereas peaks in a series of every 3 months would indicate quarterly seasonality. Seasonal models have pronounced regular ACF and PACF patterns with a periodicity equal to the order of seasonality.

Seasonal Autoregressive (SAR) model contains seasonal parameters at seasonal lags defined as:

$$X_t = \Phi X_{t-12} + \varepsilon_t \quad (7)$$

where $|\Phi| < 1$ and $\varepsilon_t$is independent of $X_{t-1}, X_{t-2}, \ldots$ It is obvious that $|\Phi| < 1$ensures stationarity.

Generally, a seasonal AR(P) model and a seasonal periods s is given as:

$$X_t = \Phi_1 X_{t-s} + \Phi_1 X_{t-2s} + \cdots + \Phi_p X_{t-ps} + \varepsilon_t \quad (8)$$

It is required that $\varepsilon_t$is independent of X_{t-1}, X_{t-2},… and, for stationarity, that the roots of $\Phi(x) = 0$ be greater than 1 in absolute value. The PACF can be used as a primary instrument for identifying seasonal autoregressive model.

The seasonal moving average model typically possesses a seasonal moving component. However, the seasonal moving average model is a common multiplicative model because the differencing factors are not multiplied directly by the moving average factors. A seasonal moving average model of order Q with seasonal period s is given as:

$$X_t = \varepsilon_t + \Theta_1 \varepsilon_{t-s} - \Theta_2 \varepsilon_{t-2s} - \Theta_3 \varepsilon_{t-3s} - \cdots - \Theta_Q \varepsilon_{t-Qs} \quad (9)$$

Box-Jenkins has generalized the ARIMA model to deal with seasonality and defines a general multiplicative **seasonal ARIMA** model in the form:

$$\emptyset(B)\Phi(B)(1-B)(1-B^{12})X_t = \theta(B)\Theta(B^{12})\varepsilon_t \quad (10)$$

where B denotes the backward shift operator, $\emptyset, \Phi, \theta$and Θ are polynomials for order p,P, q, and Q respectively. X_t is the observed time series and ε_t represent an unobserved white noise series. i.e a sequence of independently (uncorrelated) identically distributed random variables with zero mean and constant variance σ_ε^2.

Model Selection

Best fitted model among several iterated models was chosen based on Akaike Information Criterion (AIC) and Log-Likelihood technique. The AIC using maximum likelihood case is defined as;

$$AIC = 2\,log(maximum\ likelihood) + 2k \quad (11)$$

Where

$k = p + q + 1$ if the model contains a constant term and k=p+q otherwise. Theaddition of the 2(p+q+1) or 2(p+q) serves as a "penalty function" to help ensureselection of parsimonious models and to avoid models with too many parameters.

Residual Analysis and Diagnostic Check

The first diagnostic check is to inspect a plot of the residuals over time. If the model is adequate, we expect the plot to suggest a rectangular scatter around a zero horizontal level with no trends whatsoever.

The residual of a model can be written as:

Residual = actual – predicted (12)

If the model is correctly specified and the parameter estimates are reasonably close to the true values, then the residuals should have nearly the properties of white noise. They should behave roughly like independent, identically distributed normal variables with zero mean and common standard deviation. Deviations from these properties can help us discover a more appropriate model.

The Shapiro – Wilk test is a test is a test of **normality** in frequentist statistics. It was published in 1965 by Samuel Sanford Shapiro and Martin Wilk. It tests the null hypothesis that a sample $x_1, x_2 \ldots\ldots x_n$came from a normally distributed population.

The test statistic is:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \tag{12}$$

The Ljung-Box test whether any of agroup of autocorrelations of a time series are different from zero. This test for overall randomness based on a number of lags. The statistic is defined and written as:

$$Q = n(\hat{\rho}_1^2 + \hat{\rho}_2^2 + \ldots\ldots + \hat{\rho}_k^2) = n\sum_{k=1}^{n}\hat{\rho}_k^2 \tag{13}$$

The modified Q statistic is also defined as written as ;

$$Q_* = n(n+2)\left(\frac{\hat{\rho}_1^2}{n-1} + \frac{\hat{\rho}_2^2}{n-2} + \ldots\ldots + \frac{\hat{\rho}_k^2}{n-k}\right) = n(n+2)\sum_{k=1}^{n}\frac{\hat{\rho}_k^2}{n-k} \tag{14}$$

Where: n is the sample size

$\hat{\rho}_k^2$ is the sample autocorrelation at lag k, and h is the number of lags being tested. For significance level α, the critical region for rejection of the hypothesis of randomness is $Q_* > \chi^2_{1-\alpha,h}$ Where $\chi^2_{1-\alpha,h}$ is the α Quantile of the chi-squared distribution with h degrees of freedom.

Model with minimum AIC and few parameters is adjudged the best fit among several models estimated.

Results and Discussion

Table 4.1: Descriptive Statistics of Abeokuta and Ijebu Ode Rainfall Pattern

DescriptiveStat.	Abeokuta	Ijebu Ode
Minimum	0.00	0.00
1st Quarter	14.9	40.98
Median	88.4	108.90
Mean	105.3	141.57
3rd Quarter	169.2	228.43
Maximum	508.2	560.00

Source: R-Studio Output

Descriptive statistics of rainfall pattern of the two studied towns in Ogun state can be evidenced from table 4.1. analysis indicates that there is high downpour of rainfall in Ijebu Ode compared to Abeokuta in the first, second and third quarter of the studied years. This can also be seen from the maximum values of recorded rainfalls in the two towns as Ijebu recoded maximum value of 560mm compared to Abeokuta's recorded value of 508.2mm

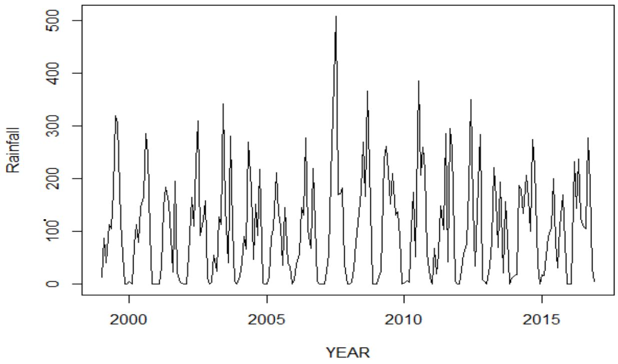

***Fig. 1:** Time series plot of Monthly Rainfall Pattern Over Abeokuta, Ogun State*

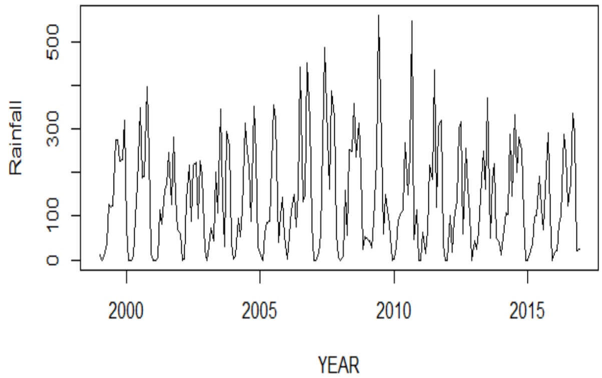

***Fig. 2:** Time series plot of Monthly Rainfall Pattern Over Ijebu Ode, Ogun State*

The time series plot of figure 1 and 2 shows the monthly rainfall pattern over Abeokuta and Ijebu Ode for the studied period. The series does not indicate unit root as it indicate an integration of order I(0).

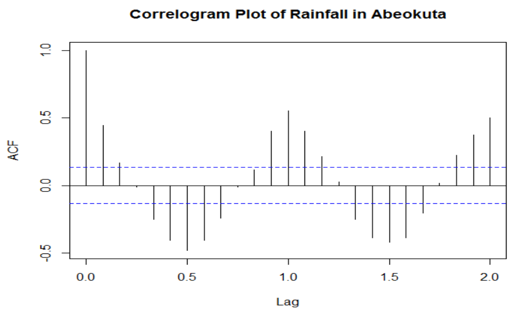

***Fig. 3:**ACF plot of Monthly Rainfall Pattern over Abeokuta*

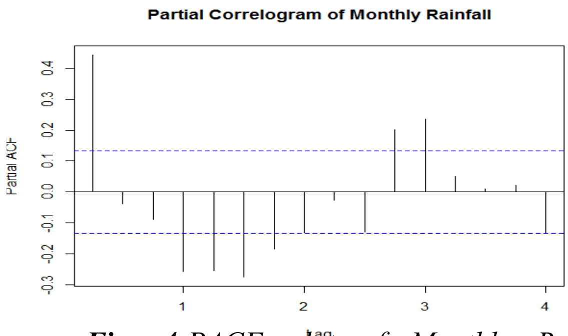

***Fig. 4:**PACF plot of Monthly Rainfall Pattern over Abeokuta*

The ACF and PACF plots in figure 3 and 4 for rainfall pattern over Abeokuta indicate significant spikes from lag 1, 2, 4 to 24 except for some few lags. There was a decay at lags 3,9,15 and 21 of the ACF plots which serves as an element of seasonality

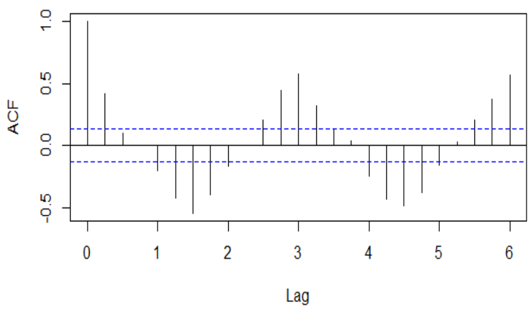

***Fig. 5:**ACF plot of Monthly Rainfall Pattern over Ijebu Ode*

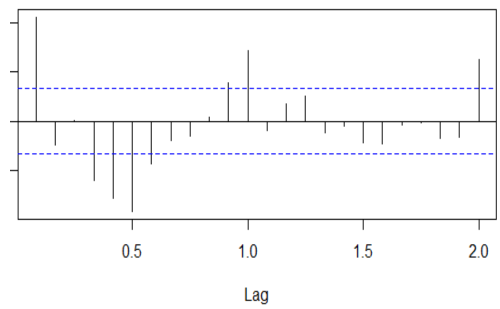

***Fig. 6:**PACF plot of Monthly Rainfall Pattern over Ijebu Ode*

Correlogram and Partial correlogram studie of Ijebu Ode also indicates significant spikes from lag to lag and evidence of calibration of SAR and SMA into the models.
However, the stationarity was further confirmed using the Augmented Dickey-Fuller test approach for the rainfall pattern of Abeokuta and Ijebu Ode, Ogun State.

Table 2: Augmented Dickey-Fuller Test for Series Stationarity @Level

Towns	Dickey-Fuller test statistic	Lag order	P-value
Abeokuta	-3.5842	12	0.03588
Ijebu Ode	-3.5917	11	0.03517

Source: R-Studio Output

The stationarity of the series was further confirmed in table 2, with ADF values of -3.5842 and -3.3917 and associated P-values of 0.03588, 0.03517 < 0.05 significance level for the respective two towns. However, with the series stationarity, we then go ahead with model identification. At this point, it is very important to identify various order for the AR(p), MA(q), $SAR_{(P)}$ and $SMA_{(Q)}$ components. With few iterations on this model building strategy, we hope to arrive at the best possible model for the two series.

The orders of AR(p) and MA(q) were chosen based on the significant spikes from the ACF and PACF plots. The significant spikes in ACF of lag 1, lag 2, 4, and lag 5 suggest MA of order 1, 2 4, or 5 in figure 5. Also, the PACF with significant spikes of lag 1, lag 2, lag 5, and lag 6 suggest AR of order 1, 2 up to 5 in figure 6 respectively. The tail off of the ACF plot was found at lag 9, 15 and 21 also suggests an $SMA_{(Q)}$ of order 1, likewise the order of Seasonal Autoregressive found at the tail off of the PACF of the respective rainfall over Abeokuta and Ijebu Ode respectively. These significant spikes suggested the models built for selection in table 3 below.

Table 3: Iterated SARIMA Models for selection using Abeokuta and Ijebu Ode Rainfall Data

	ABEOKUTA				IJEBU ODE			
S/N	**Models**	**AIC**	**LL**	**(σ^2)**	**Models**	**AIC**	**LL**	**(σ^2)**
1	$(3,0,1)x(0,0,1)_{12}$	2511.92	-1248.96	6112	$(4,0,5)x(1,0,1)_{12}$	2553.12	-1263.56	6582
2	$(0,0,3)x(0,0,1)_{12}$	2532.82	-1260.41	6797	$(2,0,2)x(1,0,1)_{12}$	2554.42	-1269.21	7071
3	$(1,0,2)x(0,0,1)_{12}$	2533.55	-1260.78	6819	$(2,0,1)x(1,0,1)_{12}$	2554.81	-1270.40	7117
4	$(1,0,1)x(0,0,1)_{12}$	2532.11	-1261.05	6837	$(2,0,0)x(0,0,1)_{12}$	2623.01	-1306.51	10417
5	$(1,0,0)x(0,0,1)_{12}$	2530.18	-1261.09	6840	$(3,0,2)x(1,0,1)_{12}$	2556.04	-1269.02	7077
6	$(1,0,3)x(0,0,1)_{12}$	2534.81	-1260.41	6797	**$(1,0,2)x(1,0,1)_{12}$**	**2553.83**	**-1269.92**	**7127**
7	$(0,0,2)x(0,0,1)_{12}$	2532.09	-1261.04	6835	$(1,0,1)x(0,0,1)_{12}$	2622.49	-1306.25	10392
8	$(2,0,0)x(0,0,1)_{12}$	2514.90	-1252.45	6262	$(0,0,2)x(0,0,1)_{12}$	2622.49	-1306.25	10397
9	$(2,0,1)x(1,0,0)_{12}$	2516.83	-1252.41	6260	$(0,0,2)x(1,0,0)_{12}$	2594.88	-1292.44	9019
10	$(2,0,2)x(0,0,1)_{12}$	2474.70	-1230.35	4726	$(1,0,1)x(1,0,0)_{12}$	2595.43	-1292.71	9056
11	**$(1,0,2)x(1,0,1)_{12}$**	**2469.36**	**-1227.68**	**4704**	$(3,0,2)x(0,0,1)_{12}$	2610.51	-1297.26	9559

***LL** = Log likelihood;* ***AIC**=Akaike Information Criterion;* **σ^2**= *Variance*

Source: R-Studio Output

Table 4.3 above shows different SARIMA models of various orders. In order to choose the best model, we look for the model with the least AIC,σ^2 and higher log-likelihood. Analysis indicates that SARIMA order $(1,0,2)x(1,0,1)_{12}$ is adequate for both rainfall observations over Abeokuta and Ijebu Ode respectively with different AIC's, Log-likelihood and Variance out of all the eleven iterated models. In addition, SARIMA order $(1,0,2)x(1,0,1)_{12}$ gave lower AIC of 2469.36, Log-likelihood -1227.68 and variance 4704 for Abeokuta rainfall series while it gave AIC 2553.83, Log-likelihood -1269.92 and variance 7127 for Ijebu Ode rainfall series. Applying the rule of parsimony which says the simplest model should be chosen provided all necessary conditions are met, this resulted in choosing the SARIMA order whose parameters were tested for significance in table 4 below.

Table 4: Best Fitted SARIMA Model Estimates

Towns	Coefficients	Estimates	Standard Error	*Z value*	Pr(>|Z|)
Abeokuta	ϕ_1	0.815447	0.133935	6.0884	0.0000***
	θ_1	-0.675717	0.143635	-4.7044	0.0000***
	θ_2	-0.179510	0.066093	-2.7160	0.00661***
	Φ_1	0.987586	0.014364	68.7523	0.0000***
	Θ_1	-0.850756	0.081320	-10.4618	0.0000***
	C	104.47963	14.59574	7.1582	0.0000***
	ϕ_1	0.529317	0.275362	1.92226	0.054573*

Ijebu Ode	θ_1	-0.4151	0.272321	-1.52432	0.12743
	θ_2	-0.23077	0.073785	-3.1276	0.0018***
	Φ_1	0.946229	0.028933	32.70384	0.0000***
	Θ_1	-0.65905	0.076737	-8.58834	0.0000***
	C	139.1506	16.59823	8.383457	0.0000***

**** indicates the significance of the coefficients @1% level*
** indicates the significance of the coefficients @5% level*
Source: R-Studio Output

The model specification for SARIMA(1,0, 2)x(1,0,1)$_{12}$ in table 4.3 is written in form of backshift operator as;
$(1-\phi_1 B)(1-\Phi_1 B^{12})(1-B)Y_t = (1-\theta_1 B-\theta_2 B^2)(1-\Theta_1 \mathrm{B}^{12})\varepsilon_t$ (15)
Expanding in linear form, we have
$Y_t+\phi_1 Y_{t-1}+\Phi_1 Y_{t-12} = C+\varepsilon_t+\theta_1\varepsilon_{t-1}+\theta_2\varepsilon_{t-2}+\Theta_1\varepsilon_{t-12}$ (16)
Substituting the coefficients for Abeokuta rainfall pattern, we have:
$Y_t+0.8155Y_{t-1}+0.9876Y_{t-12} = 104.48+\varepsilon_t-0.6757\varepsilon_{t-1}-0.1795\varepsilon_{t-2}-0.8508\varepsilon_{t-12}$ (17)
Substituting the coefficients for Ijebu Ode rainfall pattern, we also have:
$Y_t+0.5293Y_{t-1}+0.9462Y_{t-12} = 139.1506+\varepsilon_t-0.415\varepsilon_{t-1}-0.2308\varepsilon_{t-2}-0.65905\varepsilon_{t-12}$ (18)

From table 4, it shows that estimated parameters of AR(1), MA (2) SAR(1) and SMA(1) are statistically significant since their corresponding P-values are <0.05 level of significance. Only MA(1) was found to insignificantly contribute to the model taking model building development of rainfall over Ijebu Ode into consideration. This indicates that the model coefficient is efficient in forecasting rainfall over the two towns of Abeokuta and Ijebu Ode respectively.

Since it is essential to check whether the model is correctly specified, that is, whether the model assumptions are supported by the data, the residual of the fitted model was subjected to normality check by plotting the ACF of residuals, and dependence test by plotting the P-values for Ljung-Box test statistic.

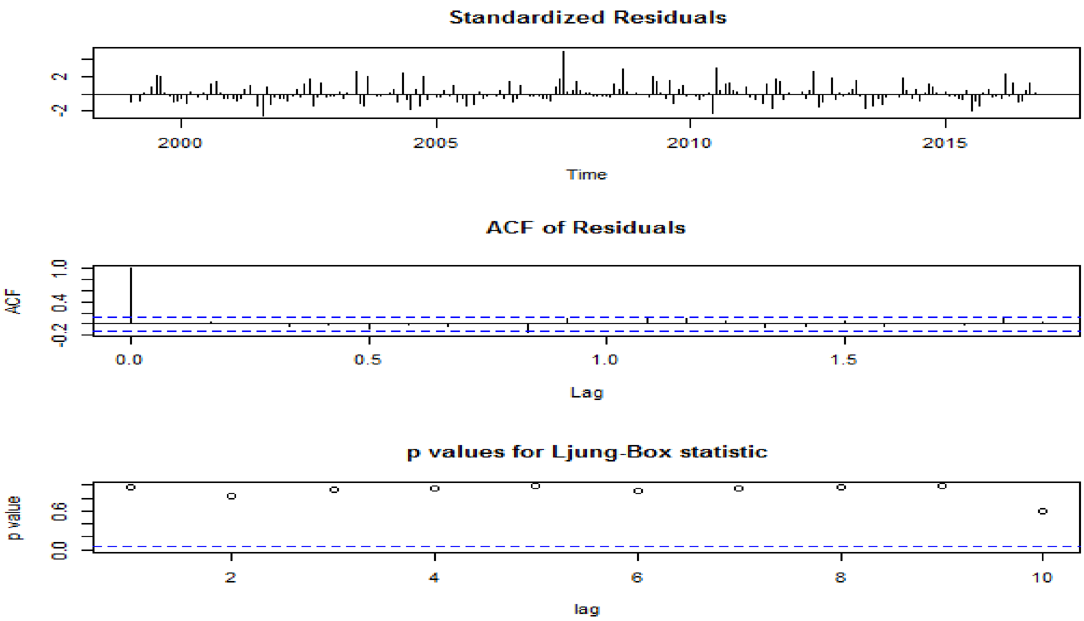

Fig. 7: ***Adequacy Check for SARIMA(1,0,2)x(1,0,1)$_{12}$ using Standardized Residuals, ACF Of Residuals and P-values for Ljung-Box Statistics for Rainfall over Abeokuta***

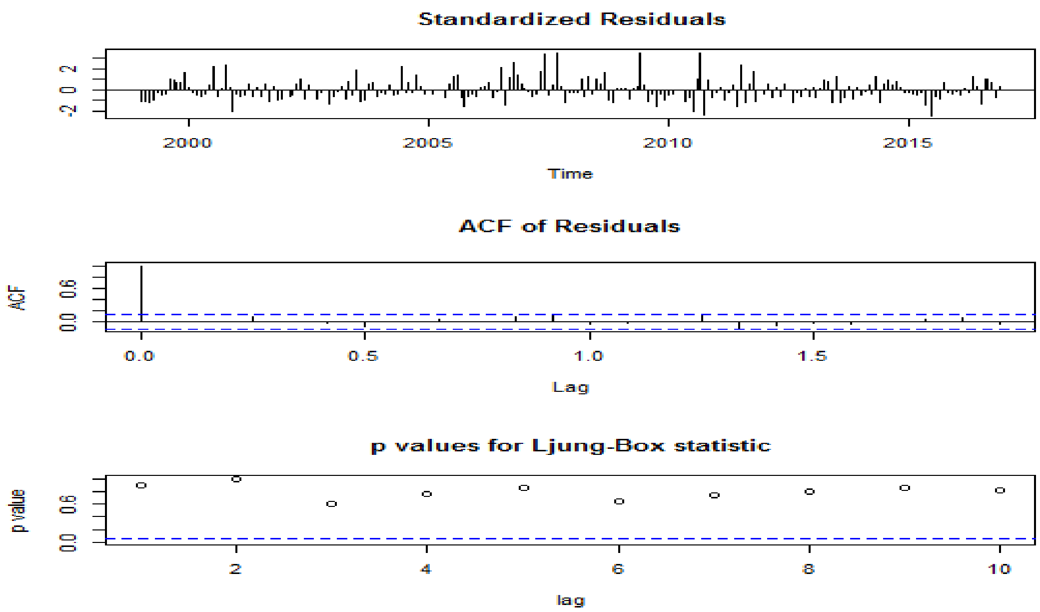

Fig. 8: ***Adequacy Check for SARIMA(1,0,2)x(1,0,1)$_{12}$ using Standardized Residuals, ACF Of Residuals and P-values for Ljung-Box Statistics of Rainfall Over Ijebu Ode***

Figure 7 and 8 depicts the standardized residuals, ACF of residuals and P-values for the Ljung-Box statistic. This indicates that the models has captured goodness of fit since the spikes of the ACF of residuals were found to be insignificant and were found within the upper and lower bound and are white noise. The P-values for the Ljung-Box statistics shows an evidence of efficient and parsimonious fit since the dots are above the bounds.

3.1 Further Diagnostic Check on SARIMA(1,0, 2)X(1,0,1) Model

Before we accept a fitted model and interpret its findings, it is essential to check whether the model is correctly specified, that is, whether the model assumptions are supported by the data. If some key assumptions seem to be violated, then a new model should be specified, fitted and checked again until a model that provides an adequate fit to the data is found. Here, the quality of the model is further assessed.

Table 4.5: Shapiro-Wilk Test of Normality

TOWNS	SHAPIRO-WILK	P-VALUE
Abeokuta	0.93593	0.3956
Ijebu Ode	0.949	0.6425

Source: R-Studio Output

The Shapiro-Wilk test of normality in table 4.5 above has a test statistic of 0.93593 for Abeokuta rainfall and 0.949 for Ijebu Ode, with corresponding P-values of 0.3956 and 0.6425 > 0.05 level of significance where normality of residuals of the best fitted SARIMA(1, 0, 2)x(1,0,1) is not rejected at 1%, 5% and 10% significance levels. This indicates that the residuals are Independently and Identically Normally Distributed.

Table 7: Ljung-Box Test

Towns	Chi-Squared	Degree of Freedom	P-value
Abeokuta	0.0019299	1	0.965
Ijebu Ode	0.017075	1	0.896

Source: R-Studio Output

The small Chi-square statistic and large p-values in the Box test for Abeokuta and Ijebu Ode rainfall suggested the acceptance null hypothesis that all of the autocorrelation functions are zero. In other words, we can conclude that there is no (or almost nil) evidence for non-zero autocorrelations in the residuals of our fitted model. This also indicates that the model has captured the dependence in the series. The AIC and Log-likelihood deal with the fit and parsimony of the model which provides a measure of efficient and parsimonious prediction. In addition, the SARIMA(1, 0, 2)x(1,0,1)$_{12}$ models for both towns can be confirmed to be adequate for predicting the pattern of rainfall under study.

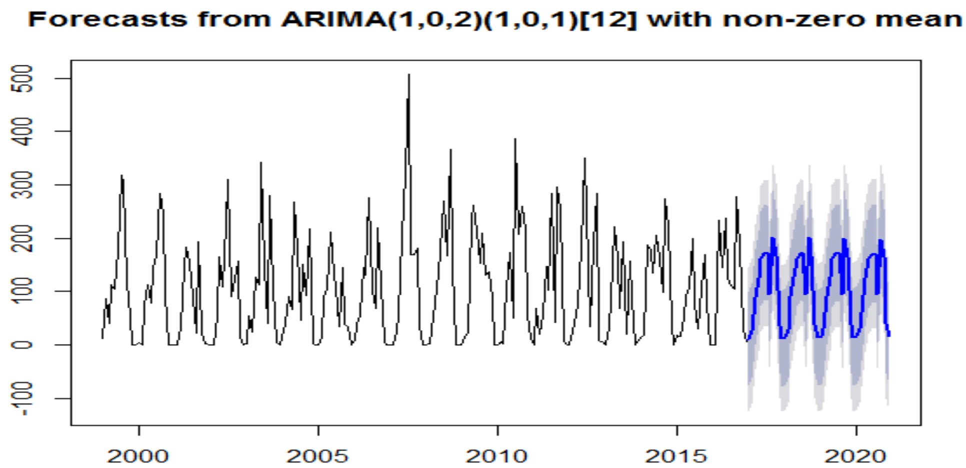

Fig 9: Four (4) Years Forecast from the fitted SARIMA model (1, 0, 2)x(1,0,1)$_{12}$ for Abeokuta Rainfall

Fig.9 shows the four years forecast. The two shaded zones of forecast represent the 80% and 95% (lower and upper side) projection of prediction intervals. A closer look indicates that there is always a rise in rainfall in the months of April, May, June, July and September on yearly basis. The higher rise in the month of September and October year 2010 may be as a result of climatic change experienced in the state taking Abeokuta Township as a study.

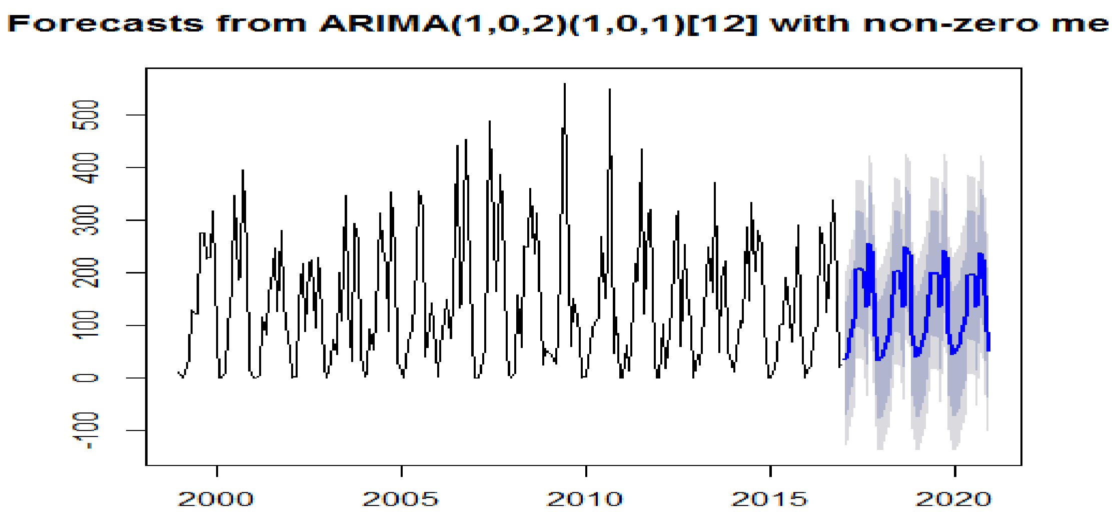

Fig 10: Four (4) Years Forecast from the fitted SARIMA model (1, 0, 2)x(1,0,1)$_{12}$ for Ijebu Ode Rainfall

Forecasts of the fitted SARIMA model (1,0,2)x(1,0,1)$_{12}$ of Abeokuta rainfall indicates that there are evidence of external forces observed in the forecast values which resulted in the high forecast values. It indicates that there will be high level of rainfall between April to October, year 2020.

CONCLUSION

Having used all necessary and suitable methods in line with the aim and objectives of this study, the following conclusions are therefore arrived at:

That SARIMA order (1,0,2)x(1,0,1)$_{12}$ was found to be most appropriate for fitting model for the two towns. The resulting same order of AR, MA, SAR and SMA might be due to the fact that rainfall follows the same pattern in those towns since they were observed in the same state.

That there is high increase in the rainfall for the future up to 2020 in the two towns. The upward fluctuating trend in the rainfall might be the reason for the change in the rainfall pattern in the said towns which might be as a result of external forces that could be observed along with the rainfall.

That high rainfall is said to be experienced in the month of April, May, June, July, September and October in the respective forecasted years.

The variability in the forecasted rainfall pattern of the two towns of Ijebu Ode and Abeokuta may be as a result of the climate change.

That Abeokuta rainfall pattern have higher predictive ability compared to that of Ijebu-Ode when subjected to the same condition of observation based on their respective AIC's and significance of their coefficients.
Based on the findings in this research work, we hereby recommend that the developed models could be used for water resources planning and management. Sound water management planning and cropping system design can be achieved with an understanding of the statistical properties of long-term records of major climatic parameters like rainfall which will improve economy growth.

Also, the fitted model is recommended for use by policy makers in Ogun State ministry of Agriculture and other national planning bodies that rely on rainfall for carrying out their activities.
In the academics, the fitted model can be studied and improved upon, thereby forming the basis for further research through inclusion of exogenous variable such as relative humidity in the absence of white noise.
Since Nigeria is now highly dependent on agricultural production and tourism, proper planning is required to avoid adverse impacts of rainfall on these sectors which can in turn lead to deteriorating the economy if not well managed.

REFERENCES

Abdul-AzizA.R., Anokye M., Kwame A., Munyakazi L. &Nsowah-Nuamah N.N.N. (2013). Modeling and Forecasting Rainfall Pattern in Ghana as a Seasonal Arima Process: The Case of Ashanti Region, *International Journal of Humanities and Social Sciences* 3: February 2013 (2013).

Anosh Graham and Ekta Pathak Mishra(2017). Time series analysis model to forecast rainfall for Allahabad region. *Journal of Pharmacognosy and Phytochemistry* 6(5):1418-1421.

Anyadike, R.N.C. (1992b). Regional variations in fluctuations of seasonal rainfall over Nigeria Theor. Appl. Climatol. 45.:285-292.

Babatolu J.S., and Akinnubi RT (2014). Influence of Climate Change in Niger River Basin Development Authority Area on Niger Runoff, Nigeria. *J Earth Sci Clim Change 5: 230. doi: 10.4172/2157-7617.1000230* .

Babatolu S.J. (2002). Climate Change and its Implications for Water Supply in the Nigeria in the Niger Basin Development Authority Area of Nigeria, Ph.D. Thesis., University of Ilorin, Ilorin, Nigeria.

Chatfield, C.(2004). The analysis of time series, an introduction, sixth edition; New Yark, Chapman &Hall CRC.

Clark, C.R., (2002). Climate Change or Climate Fluctuations?. J. Arid Environ 1. 1:18-33

Ekwe, Joshua, Igwe, and Osinowo (2014). Mathematical Study Of Monthly And Annual Rainfall Trends In Nasarawa State, Nigeria. *IOSR Journal of Mathematics (IOSR-JM). Vol. 10, Issue 1 Ver. III. PP 56-62*

Enete I.C. and. Ebenebe I.N. (2009). Analysis of rainfall distribution over Enugu during the little dry season (1990-2005). *European Journal of Statistics and Probability Vol.3, No.3, pp.12-18.*

Etuk E.H., Mohamed T.M. (2014). Time Series Analysis of Monthly Rainfall data for the Gadaref rainfall station, Sudan, by SARIMA Methods, *International Journal of Scientific Research in Knowledge* 2(7): 320-327.

Hellen W.K., John M.K, George O.O, Walter O.Y. (2014). Forecasting Precipitation Using SARIMA Model: A Case Study of Mt. Kenya Region. IISTE Journal of Mathematical Theory and Modelling Vol.4, No.11, 2014. p 50-58

Jagtap S. (2007). Managing Vulnerability to extreme weather and climate events: Implications for agriculture and food security in Africa. *Proceedings of the International Conference on Climate Change and Economic Sustainability held at Nnambid Azikiwe University, Enugu,* Nigeria. 12-14 June 2007.

Hulme, M., Osborn, T.J. and Johns, T.C. (1998). Precipitation sensitivity to global warming: Comparison of observations with HADCM2 simulations. *Geophysical Re-search Letters*, 25, 3379-3382.

Kayano, M.T. and Sansígolo, C. (2008). Interannual to decadal variations of precipitation and daily maximum and daily minimum temperatures in southern Brazil. *Theoretical and Applied Climatology*, 97, 81-90. doi:10.1007/s00704-008-0050-4

Kenitzer S., Rosenfeld J., Heideman K. (2007). Bulletin of the America Meteorological Society: Annual Report 2006. Boston 88.4 (Apr 2007) p. 1–27.

Mina Mahbub Hossain and SayedulAnam. (2012). Identifying the dependency pattern of daily rainfall of Dhaka station in Bangladesh using Markov chain and logistic regression model *Agricultural Sciences*, Vol.3, No.3, 385-391 http://dx.doi.org/10.4236/ as.2012.33045.

Mshelia A.D. (2005). Adaptation strategies to climate change. *Journal of Energy and Environment,* 18(3): 74-81.

Nirmala, M., Sundhram, S.M.(2010). A Seasonal ARIMA Model for Forecasting monthly rainfall in Tamil Nadu, National *Journal on Advances in Building Science and Mechanic,* 1(2): 43-47.

NkeirukaE .N., Apagu B. (2005). Rainfall Analysis for Environmental Applications in Maiduguri, Nigeria. *Global Journal of Environmental Sciences* Vol. 4(2) 2005: 155-159

Nwafor J.C., (2007). Global climate intensity in Sub-Saharan Africa. *Paper presented at the International Conference on Climate Change and Economic Sustainability held at Nnambid Azikiwe University, Enugu,* Nigeria, 12-14 June 2007.

Odjugo P.A.O, Ikhuoria A.I. (2003). The impact of climate change and anthropogenic factors on desertification in the semi-arid region of Nigeria. *Global Journal of Environmental Science,* 2(2): 118-126.

Olaniran, O.J. (2002). Changing patterns of rain-days in Nigeria. *GeoJournal.* 22(1): 99: 107-137.

Osarumwense O.I. (2013). Applicability of Box Jenkins SARIMA model in rainfall forecasting: A case study of Port-Harcourt south Nigeria. *Canadian Journal on Computing in Mathematics, Natural Sciences, Engineering and Medicine.* Vol. 4. ISS. 1 p. 1–4.

Singer S.F., Avery D.T. (2007). Unstoppable Global Warming: Every 1000 Years. *New York: Rowman and Littlefield Publishers* Inc,, P. 114.