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Quadratic Mixed Convection Stagnation-Point Flow in Hydromagnetic Casson Nanofluid over a Nonlinear Stretching Sheet with Variable Thermal Conductivity

Ephesus Olusoji Fatunmbi[1,a], Samuel Segun Okoya[2,b]

[1]Department of Mathematics and Statistics, Federal Polytechnic, Ilaro, Nigeria. [2]Department of Mathematics, Obafemi Awolowo University, Ile-Ife, Nigeria.

[a]ephesus.fatunmbi@federalpolyilaro.edu.ng, [b]ssokoya@gmail.com

Corresponding author *ephesus.fatunmbi@federalpolyilaro.edu.ng

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Abstract. An analysis of nonlinear mixed convection transport of hydromagnetic Casson nanofluid over a nonlinear stretching sheet near a stagnation point is deliberated in this study. The flow is confined in a porous device in the presence of thermophoresis, Ohmic heating, non-uniform heat source with temperature-dependent thermal conductivity associated with haphazard motion of tiny particles. The transport equations are translated from nonlinear partial differential equations into ordinary ones via similarity transformation technique and subsequently tackled with shooting method coupled with Runge-Kutta Fehlberg algorithm. The significant contributions of the embedded parameters on the dimensionless quantities are graphically depicted and deliberated while the numerical results strongly agree with related published studies in the limiting conditions. It is found that a rise in the magnitude of Casson fluid parameter decelerates the fluid flow while enhancing the viscous drag and thermal profiles. The inclusion of the nonlinear convection term aids fluid flow whereas heat transfer reduces with growth in the thermophoresis and Brownian motion terms.

Introduction

The scrutinization of the boundary layer stagnation point-flow offers numerous industrial and engineering applications in diverse fields of human endeavours as commonly encountered in the cooling of electronic gadgets, thermal oil recovery, the extrusion of polymers in melt-spinning processes, nuclear reactors during emergency shutdown and so on, Seini and Makinde [1]. The stagnation-point describes a region of zero velocity, highest static pressure where at the same time the heat transport and mass deposition reach maximum level at this region. The initial study of such concept was conducted by Hiemenz [2] on a two-dimensional flat sheet while an improved and extended versions of such phenomenon on Newtonian/non-Newtonian fluids have been discussed by various authors on different geometries, assumptions and conditions. Few of these studies can be found in Refs [3-8] and the references therein.

More so, researches that characterize the non-Newtonian fluids are on the increase in the recent times due to extensive engineering and industrial applications obtainable from such studies ranging from food processing, crude oil extraction, biomedical engineering (such as fluid flow in brains and blood flows) to pharmaceuticals. Various models of non-Newtonian fluids have been proposed owing to the difficulty of capturing the fluids characteristics in a single model, for instance, the micropolar fluid, Maxwell fluid, Casson fluid, Eyring-Powell fluid, etc. Casson fluid demonstrates a shear thinning characteristics. It is prominent among others owing to its distinct property of zero viscosity at an infinite rate of shear while exhibiting an infinite viscosity at zero rate of shear, Mythili & Sivaraj [9]. This model was invented by Casson [10] to analyze the transport characteristics of pigment-oil suspensions of printing ink with intrinsic yield stress attribute. The suitability of this model to adequately describe the rheological behaviour of various ingredients such as paints, lubricants, jelly, tomato sauce, blood, honey, etc has been reported by various authors as found in Refs [11-13].

Conventional base fluids (water, oil, ethanol glycol, etc.) have been found to be of low thermal conductivity and thus offer low heat transfer rates. A new class of heat transfer fluid originated by Choi and Eastman [14] is the nanofluid. Such types of fluids are known to have better thermal conductivity as well as heat transfer coefficients in comparison to the traditional fluids, [15]. Nanofluid describes the suspension of nanometer particles in base fluids for an improved thermal conductivity in comparison to the traditional fluids. Such a composition offers an improved thermal conductivity required in various engineering/manufacturing devices such as in cooling reactors, cancer therapy, reducing cholesterol levels in the blood, etc. Moreover, they can be used as heat exchangers and lubricants, pharmaceutical industries, cooling of engines and vehicles, refrigerator and so on, Mebarek-Oudina [16]. These numerous applications have propelled researchers and scientists to study such concept with various assumptions and geometries (see [17-20]. In this study, the transport of a nonlinear mixed convection reactive Casson nanofluid is examined over a nonlinearly stretching sheet with effect of Ohmic heating.

Many engineering and material manufacturing processes such as nuclear power plants, hot rolling, electrical power generation, etc require high temperature. In such operations, the knowledge of thermal radiation plays a vital role for the construction of energy conversion devices. In cases where the magnitude of temperature difference is high within the flow, the modelling of the radiative heat flux as linear type becomes inoperative, thus, it is imperative to apply the more general nonlinear type to capture the effect of thermal radiation. For instance, Makinde and Animasaun [21] considered such effect on the transport of biconvection nanofluid with chemical reaction and thermo-migration of tiny particles. Al-Khaled *et al.* [22] investigated such a phenomenon on the motion of a reactive tangent hyperbolic fluid whereas Fatunmbi and Adeniyan [23] engaged micropolar fluid to report such concept while Khan *et al.* [24] carried out such investigation with an internal heat source in the neighborhood of a stagnation-point. Mixed convection transport over linearly stretchable surfaces offers crucial engineering and industrial applications as found in the cooling of nuclear reactor, metallurgical and extrusion activities, glass blowing, manufacturing, cooling and/drying of textile and paper materials, etc. The investigation of boundary layer flow coupled with heat and mass transfer along a stretchable sheet has been found useful in practical situations such as in extrusion processes, hot rolling, glass production, manufacturing of plastic sheets, continuous stretching of plastic films, Raza *et al* [25]. The initial study of Crane [26] on the case of linearly stretching surface has been widely extended by various authors [27-28]. However, in practical situations such as in annealing of copper wires and drawing of plastic sheet, linearity of the stretching sheet velocity is unrealistic as the sheet velocity can be nonlinear and/exponential. Such phenomenon of nonlinear stretching sheet was first reported by Gupta and Gupta [27] while various authors have deliberated such concept for both Newtonian and non-Newtonian fluids as found in Refs [29-33].

The combined free and forced convection flow is referred to as mixed convection which is often encountered in drying processes, cooling of fans and electronic appliances, solar power collectors, etc. Previous investigators assumed a linear density variation in the buoyancy force term which is usually applicable with low temperature difference. However, little is known on the case where the convection is associated to larger difference of thermal and concentration difference subject to haphazard motion and thermo-migration of tiny particles. For accurate prediction of the flow, heat and mass transfer in the boundary layer, the incorporation of nonlinear density variation with temperature and concentration becomes non-negotiable. In the light of this, a natural convective flow with nonlinear density variation with temperature on an isothermal sheet was carried out by Korovkin and Andrievskii [34], Prasad *et al.* [35] evaluated nonlinear convective transport of a Newtonian fluid in a porous medium while such investigation was analytically conducted via perturbation technique by Athira *et al.* [36] on a reactive Newtonian fluid with induced magnetic influence. Mandal and Mukhopadhyay [37] engaged micropolar fluid to discuss such a concept with radiation effect. Meanwhile, these studies were mostly investigated on density variation with temperature without considering that of concentration. Besides, the impact of temperature-dependent thermal conductivity, nonlinear thermal radiation, non-uniform heat source alongside thermophoresis and Brownian motion on such studies have been neglected in the thermal field which the present study aims to address.

[redacted]

Inspired by the above literature analysis and the significant industrial and engineering applications highlighted above, the primary concern of the current investigation is to scrutinize nonlinear mixed convection transport of a non-Newtonian Casson nanofluid past an impermeable nonlinear stretching sheet in the neighbourhood of a stagnation point. The flow is configured in a porous device with the impacts of nonlinear thermal radiation, Joule heating, non-uniform heat source, temperature-reliant thermal conductivity, thermo-migration and haphazard motion of tiny particles coupled with chemical reaction and activation energy. To the best of authors knowledge, such an investigation has not been publicized before in literature. The effects of the embedded parameters are computationally analyzed, graphically depicted and deliberated while comparisons with some related published studies in the limiting scenarios show good agreement with the present study.

Problem Development and Formulation

To develop the governing equations modelling the problem under consideration, it is assumed that; the flow is incompressible, viscous and steady. The working fluid is a hydromagnetic Casson nanofluid configured in a two-dimensional vertically stretched sheet with zero mass flux at the sheet. An external magnetic field is applied perpendicular to the flow axis with non-uniform strength given as ����(����) = ����$_0$����$^{(����-1)/2}$ [38-41] without accounting for the induced magnetic field based on sufficiently low magnetic Reynolds number. The permeability of the porous medium is assumed to be non-uniform described as ��������(����) = ����$_1$����$^{(1-����)}$ [42-44]. The flow is in the neighborhood of a stagnation point in the direction of (����) while (����) axis is normal to it. The velocity components in the leading edge and normal directions are orderly given as (����, ����) as indicated in Fig. 1. The stretching sheet has the velocity ���� = �������� = ��������$^{����}$ while the velocity upstream is indicated as ����$_\infty$ = ��������$^{����}$ where ���� > 0, ���� and ���� orderly describe stretching rate, a constant which measures the magnitude of stagnation point flow and the nonlinear stretching parameter. The thermal field also incorporates temperature-reliant thermal conductivity, nonlinear thermal radiation, Ohmic and frictional heating and non-uniform heat source/sink associated with thermophoresis and Brownian motion (see Eq. 7). The Casson nanofluid density (����) variation with temperature and concentration in the momentum equation are taken to be nonlinear in nature and given by [37].

����(����) = ����(����$_\infty$) + �������� �������� �$_\infty$(���� − ����$_\infty$) + �$^{����2}$���� ��������^{2} �$_\infty$(���� − ����$_\infty$)^2+. .. (1)

����(����) = ����(����$_\infty$) + �������� �������� �$_\infty$(���� − ����$_\infty$) + �$^{����2}$���� ��������^{2} �$_\infty$(���� − ����$_\infty$)^2+. . ., (2)

The expansion of Eqs. (1-2) up to the second order respectively gives

����−����$_\infty$

$$\text{����} = -\text{����}_1(\text{����} - \text{����}_\infty) - \text{����}_2(\text{����} - \text{����}_\infty)^2, \quad (3)$$

$$\text{����} = -\text{����}_3(\text{����} - \text{����}_\infty) - \text{����}_4(\text{����} - \text{����}_\infty)^2. \quad (4)$$

[redacted]

Fig. 1 The Flow Configuration

With respect to the principles of the boundary layer approximations coupled with the above raised assumptions, the equations listed in Eqs. (5-8) describe the transport equations for the nonlinear mixed convection hydromagnetic Casson nanofluid [24, 36].

$$\frac{\text{��������}}{\text{��������}} + \frac{\text{��������}}{\text{��������}} = 0, \quad (5)$$

����� ������� ���� ���� �1 + 1 ���� � ���� 2 ���� ������� ������� � = � ������� 2 (��� �) ���� ∞ ������� �∞ ���� ���� + ���� ���� ������� + ���� ���� ������� 2 − ������� + ���� ������� ���� ���� ���� ���� (����)� (���� − ����∞) + (6)

$\text{����}[\text{����}_1(\text{����} - \text{����}_\infty) + \beta_2(\text{����} - \text{����}_\infty)^2] + \text{����}[\text{����}_3(\text{����} - \text{����}_\infty) + \text{����}_4(\text{����} - \text{����}_\infty)^2].$ ������� � + (������� ����)���� � ������� ������� � � ���� ����

(������� ����)���� ����� ������� ����∞ � ������� ������� + ���� ������� ������� � = ���� ������� ����� ������� ������� � + ���� ���� � ������� ������� �� + ������� ^{2}(����)(���� − ����$_\infty$)^2 +

(7)

���������� 16����⋆

��������$�^2$+

��������(����)(�������� 3

���� − ����� ��������

$����_\infty)^2$ + 3����⋆ ���� ��������� +

���������� �1 + �������� ����‴,

1�����

����� �= ����² ������� ����� ����∞�� ���2���� ������� ������� ��������+ ��������² + ��������²�. (8)

����������������� ��������

The temperature-dependent thermal conductivity is expressed as [45-46],

$$���� = \frac{����_\infty}{��������−����_\infty}[(��������� − ����_\infty) + ����(���� − ����_\infty)], (9)$$

the non-uniform heat source (����‴) appearing in the last term of Eq. (7) is modelled

as [47] ����‴ = ������������

$$\frac{����}{������������}(��������′ + ����^\star����)(��������� − ����_\infty), (10)$$

while the accompanied boundary conditions for equations (5-8) are:

$$���� = �������� = ��������^{����}, ���� = 0, ���� = ��������(= ����_\infty + ��������^{����}), ���� = �������� \quad ����h�������� ���� = 0,$$

$$���� \to ����_\infty = ��������^{����}, ���� \to ����_\infty, ���� \to ����_\infty, \quad �������� ���� \to \infty. (11)$$

Where $����_\infty$ in Eq. (9) describes the uniform value of the thermal conductivity at upstream while ���� is the variable thermal conductivity parameter based on the nature of the fluid. It is noted that when ���� = 0, the thermal conductivity assumes uniformity and hence, it does not depend on the temperature.

[redacted]

Similarly, ����/����⋆ in Eq. (10) corresponds to space/temperature dependent heat source in that order. The other symbols in the governing equations are described in the nomenclature. The underlisted dimensionless variables are introduced into the transport Eqs. to translate them into ordinary differential Eqs. [32-33].

���� = �����(����+1)��������−1 / 2���� ����, ���� = �2������������+1 / ��������−����∞, �������� = 16����⋆����∞3 / 3����⋆����∞, �������� ���� = ����+1 ����(����), ����(����) = ����−����∞ / ��������, �������� = ���� / �������������, ����(����) = ����−����∞ / ��������−����∞, ���� = ������������ (��������)��������������(������−����∞)

��������, ���� = − ��������$_{����\infty}$, ���� = ��������$_{0^2}$ (��������$_{����}$)$_{����}$����$_{\infty}$ ����$_{����}$, ���� = ����$_{����}$, �������� = ���� ��������$_{1}$

����$_{\infty}$, �������� = ����$_{����}$(����$_{����}$)$_{����}$ ����$_{����}$ = ����$_{����}$ ��������$_{����}$ = 2��������$_{3}$(����$_{����}$−����$_{\infty}$)����^{3} ��������$_{����}$, ��������$_{����}$ = 2��������$_{1}$(����$_{����}$−����$_{\infty}$)����^{3} (����+1)����$_{����2}$, �������� = ����$_{����}$$_{����}$, ����$_{1}$ = ����$_{2}$(����$_{����}$−����$_{\infty}$) ����$_{1}$, ��������$_{����}$, �������� = (��������$_{����}$)$_{����}$������(����$_{����}$−����$_{\infty}$) (12)

(����+1)����$_{����2}$, ����$_{1}$ = ��������$_{����}$ ��������$_{����2}$, ���� = ��������$_{����}$ �������� = ����$_{����2}$ ����$_{����}$(����$_{����}$−����$_{\infty}$), ����$_{2}$ = ����$_{4}$(����$_{����}$−����$_{\infty}$) ����$_{1}$. (��������$_{����}$)$_{����}$����$_{����}$,

Where ���� is the similarity variable, ��������$_{����}$ defines Reynolds number, ���� is the dimensionless temperature and ���� is the dimensionless nanoparticles concentration.

Similarly, ��������, ��������, ����, ��������, ����$_{����}$, ����, ��������$_{����}$, ��������, ����$_{1}$, ��������$_{����}$, ����$_{1}$, ����, ��������, �������� and ����$_{2}$ respectively indicates radiation parameter, thermophoresis term, velocity ratio, Darcy number, temperature ratio, magnetic field term, local Grashof number, Schmidt number, nonlinear thermal convection, local solutal Grashof number, mixed convection, ratio of concentration to buoyancy force, Brownian motion, Eckert number, and nonlinear mass convection parameters.

Incorporating the dimensionless variables (12) in the governing equations (5-8) and taking cognizance of equations (9-10) results to the underlisted:

�1 + ^{1}$_{����}$� ����''' + ��������'' − $\frac{����+1}{2����}$(����'^{2} − ����^{2}) − 2(����+��������) ����+1 (����' − ����) + ����$_{1}$����(1 + ����$_{1}$����) + ����$_{2}$��������(1 + ����$_{2}$����) = 0, (13)

��������/3�������� [1 + �������� + ��������(1 + (����$_{����}$ − 1)����)^{3}]����'' + 1 (������������'^{2} + ������������'����') + 2��������(����+��������) ��������(����$_{����}$ − 1)����'^{2}(1 + (����$_{����}$ − 1)����)^2 + ���������' − 2���� ����+1 ����'����� + ��������(����+1)(��������' + ����*����) = 0, (14)

����+1 (����' − ����)^2 + ������������''^{2} �1 + ^{1}$_{����}$� + ^{1}$_{��������}$ ��������'^{2} + 2(1+��������) ����'' + ��������/�������� ����'' + ����������������' = 0. (15)

While the boundary conditions translates to:

$$����'(0) = 1, ����(0) = 0, 0(0) = 1, ����(0) = 1.$$

$$����'(\infty) = ����,\ ����(\infty) = 0,����(\infty) = 0. \quad (16)$$

Furthermore, the expressions for the coefficient of skin friction ($����_{��������}$) as well as the Nusselt number ($��������_{����}$) and the Sherwood number ($����h_{����}$) are respectively presented in Eq. (17). These expressions are the physical quantities of engineering interest applicable in this study.

$$����_{��������} = 2����_{����}�����_{����}����_{����}{}^{2}�^{-1},$$
$$��������_{����} = ��������_{����}[����_{\infty}(����_{����} - ����_{\infty})]^{-1},$$
$$����h_{����} = ��������_{����}[����_{����}(����_{����} - ����_{\infty})]^{-1}, \quad (17)$$

where

$$����_{����} = ����_{����}�1 + \frac{1}{����}� �^{��������}{}^{2},\ ����_{����} = -\frac{16����^{3}_{����}}{3����^{\star}}�\ ��������\ ������_{\infty} + �����_{����}������ ��\ ��������^{�}_{����=0},\ ����_{����} = -��������^{�}_{����=0}\ ��������^{�}_{����=0}, \quad (18)$$

here $����_{����}$ corresponds to surface shear stress whereas $����_{����}(����_{����})$ typifies surface heat (mass) flux in that order. With the substitution of Eqs. (12) and (18) into (17), the quantities in Eq. (17) respectively results to the following the dimensionless terms as presented in Eq. (19).

$$\bar{����}��������� = ����''(0),\ ����\hat{}\ ����_{����} = -����'(0),\ ����\bar{h}���� = -����'(0). \quad (19)$$

[redacted]

Where

$$\bar{����}��������� = ��������_{����}\ 2(����+1)^{\frac{1}{2}}�1+\frac{1}{�����},\ ����\hat{}\ ����_{����} = \frac{��������_{����-}{}^{\frac{1}{2}}�����}{�^{����+1}{}_{2}�}\ \frac{���_{����}{}^{2}����_{��������}}{}\ ,\ ����\bar{h}_{����} = \frac{��������_{����-}{}^{\frac{1}{2}}����h}{�^{����+1}{}_{2}�}\ {}^{2}�1+���������� ��_{����}{}^{3}�\ .\ (20)$$

3 Numerical Method with Validation

Due to high nonlinearity nature of the boundary value problem (13-16), a numerical technique via shooting method together with Runge-Kutta-Fehlberg algorithm has been employed for the solutions. Notable authors have applied and described in details the effectiveness of this technique. For detail explanation of this technique (see [32, 48-50]). For the computations, the following values have been carefully selected as default parametric values $���� = ���� = ���� = 0.2$, $�������� = 0.3$, $�������� = 0.1$, $����_1 = ����_2 = ���� = 0.5 = ����_1 = �������� = ��������$, $�������� = 2.0$, $����_{����} = 1.5$, $���� = ����^{\star} = 0.1 = ��������$, $�������� = 0.44$, $���� = 0.3$, $���� = 1.0$ except if stated otherwise in the various graphs. The code for the solutions developed in this study have been verified by comparing the computational values of some chosen parameters with related published works found in literature for limiting scenarios. Table 1 gives the record of the Nusselt number ���� � ��������� as compared with Grubka and Bobba [51] for variations in temperature exponent term ���� and Prandtl number ��������. The comparison shows a good relationship as depicted in that table. More so, Table 2 shows the variations in the nonlinear stretching term ���� with respect to the skin friction coefficient ���̅���������� as compared with Cortell [30] and Fatunmbi *et al.* [32] in the limiting conditions. A strong relationship exists in the obtained results with those reported by those authors as typified in Table 2. These comparisons confirmed the validity of the current numerical solutions.

Table 1 Computational values of ���� � �������� with respect to variations in ���� and �������� as compared with published data

����	Grubka & Bobba [51]			Present study		
	�������� = 1	�������� = 10	�������� = 100	�������� = 1	�������� = 10	�������� = 100
-2.0	-1.0000	-10.0000	-100.0000	-1.00000	-10.00000	-100.00000
-1.0	0.0000	0.0000	0.0000	0.00012	0.00000	0.00000
0.0	0.5820	2.3080	7.7657	0.58201	2.30800	7.76565
1.0	1.0000	3.7207	12.2940	1.00000	3.72067	12.29408
2.0	1.3333	4.7969	15.7120	1.33333	4.79687	15.71197
3.0	1.61534	5.6934	18.5516	1.61538	5.69338	18.55154

Table 2 Variations in ���� when other parameters are zero with respect to ���̅���������� as compared with published data

����	Cortell [30]	Fatunmbi *et al.* [32]	Present
0.0	0.627547	0.627624	0.627563
0.2	0.766758	0.766901	0.766945
0.5	0.889477	0.889602	0.889552
1.0	1.000000	1.000052	1.000008
3.0	1.148588	1.148637	1.148601
10.0	1.234875	1.234913	1.234882

4 Presentation and Analysis of Results

This section analyzes graphically the significant contributions of the main parameters on the various dimensionless quantities (velocity ����′(����), temperature ����(����), concentration ����(����), skin friction coefficient ���̅����������, Nusselt number ���� � �������� and Sherwood number ����ℎ����). These contributions are presented in Figures 2-19 with appropriate discussions.

[redacted]

Fig. 2 Plots of velocity profile for changes in ���� **Fig. 3** Velocity field for variations in ����

The implication of varying the magnetic field term ���� on the profile of velocity is described in Fig. 2 in the presence of Casson fluid term ����. The figure depicts the shrinking nature of the hydrodynamic boundary structure with a hike in ���� as well as ����. This decelerated flow due to ���� as observed in Fig. 2 is occasioned by the draglike Lorentz force produced by the interaction of the applied magnetic field and the electrically conducting Casson nanofluid. Hence, a hike in ���� raises the strength of the Lorentz force such that the motion of the fluid drags. Likewise, the reduction in the fluid flow owing to a rise in ���� indicates that growth in ���� compels a reduction in the transport field due to a fall in the yield stress as ���� increases which in turn lowers the fluid flow. Besides, a rise in ���� strengthens the plastic dynamic viscosity above the Casson fluid viscosity and at such, the flow is resisted as further depicted in Fig. 3. It is to be remarked that the non-Newtonian attribute vanishes when ���� $\rightarrow \infty$ and at such, the fluid purely exhibits Newtonian fluid property. Also, in relation to the fluid flow, the velocity profile depletes with respect to higher values of the nonlinear stretching term ����. For the linearly stretching scenario, the velocity field is higher than that of nonlinearly stretching case as depicted in figure 3. The plot depicting the velocity field versus ���� for varying mixed convection term ����$_1$ in the existence of the velocity ratio term ���� is sketched in Fig. 4. The impact of ����$_1$ is to boost the velocity field owing to a decline in the viscous force as ����$_1$ increases. Also, the velocity field heightens when ���� is raised as noticed in Fig. 4. This accelerated flow occurs due to the fact that the upstream velocity is higher than that of the wall velocity.

Fig. 4 Plots of velocity profile for variations in ����$_1$ **Fig. 5** Velocity profile for changes in

[redacted]

Fig. 6 Velocity field for variations in ����$_1$ **Fig. 7** Concentration field for variations in ��������

From Fig. 5, it is observed that the transport field accelerates when the ratio of the concentration to buoyancy forces (����) increases while the converse is the case for a rise in the Darcy term (��������). The thermal nonlinear mixed convection parameter (����$_1$) boosts the velocity profile as showcased in Fig. 6. Here, a hike in ����$_1$ leads to a rise in (����$_{����}$ − ����$_\infty$) and at such, the velocity field accelerates. The nanoparticles concentration field escalates with higher values of the thermophoresis parameter �������� as demonstrated in Fig. 7. On the contrary, the concentration field decays with a hike in Brownian motion parameter ��������. Furthermore, growth in the Schmidt number �������� shrinks the solutal boundary layer structure which in turn dictates a reduction in the concentration profile as found in Fig. 8.

Fig. 8 Concentration field for varying �������� **Fig. 9** Temperature field for variations in ��������

[redacted]

Fig. 10 Temperature profile for varying ���� **Fig. 11** Temperature field for variations in ����*

Physically, the Schmidt number indicates the relative thickness of the momentum boundary layer to the species concentration boundary layer. In this view, the Schmidt number �������� varies inversely to the coefficient of mass diffusivity and at such, a rise �������� propels a decrease in the nanoparticles concentration boundary layer and consequently compels a reduction in the nanoparticles concentration profiles. The thermal boundary layer structure expands with rising values of the thermophoresis term �������� as indicated in Fig. 9. This can be attributed to a rise in the temperature gradient as �������� increases. A raise in the magnitude of the Brownian motion �������� also strengthens the temperature profile. The Brownian motion describes an irregular movement exhibited by the nanoparticles suspended in a base fluid. In respect to this haphazard motion, there is a higher kinetic energy owing to the enhanced movement of the molecules of both the nanoparticles as well as base fluid which leads to an improved surface temperature. The reaction of the space-dependent heat source parameter ���� with respect to temperature distribution in the existence/non-existence of the radiation term �������� is sketched in Fig. 10. Advancing the values of ���� causes an improvement in the surface temperature in the presence or absence of ��������. However, in the absence of ��������, the temperature is lower than in its presence. Obviously, the thermal boundary structure is energized with a rise in �������� and consequently propel a boost in the temperature profile as depicted in Fig. 10. The implication of the temperature-dependent heat source ����* on the thermal field is plotted in Fig. 11 in the existence or otherwise of the Eckert number ��������. In the presence of ����*, an additional heat is created which is responsible for the hike in temperature profile. Moreover, the inclusion of ��������, the thermal field also escalates due to friction between the fluid particles. Eckert number corresponds to the ratio of flow kinetic energy to that of the boundary layer enthalpy difference. In this regard, a raise in �������� enhances the production of heat and at such, compels an a rise in the temperature field.

[redacted]

Fig. 12 Temperature profile for variations in �������� **Fig. 13** Temperature profile for varying ����

The impact of the temperature ratio term �������� is to improve the temperature profile as clearly described in Fig. 12. The temperature parameter �������� corresponds to the ratio of the sheet temperature to that of the upstream temperature, i.e. �������� = ��������/����∞. Hence, a rise in �������� corresponds to higher temperature at the stretching sheet and at such, a rise in the surface temperature. The graph of temperature versus ���� for variations in the Casson fluid parameter ���� for linear (���� = 1) and nonlinear stretching sheet (���� ≠ 1) is captured in Fig. 13. There is an improvement in the temperature distribution with higher values of ���� for both linear/nonlinear stretching sheet. However, higher surface temperature occurs with nonlinear stretching sheet as found in Fig. 13. Figure 14 informs about the reaction of temperature profile to the changes in the nonlinear stretching term ���� for variations in the wall temperature exponent parameter ����. The thermal boundary structure expands with ���� in the existence of higher ���� and thus, temperature distribution heightens. This trend is however reversed with higher values of ���� as temperature field depreciates as found in Fig. 14.

Fig. 14 Temperature field for variations in ���� **Fig. 15** Sherwood profile for �������� & ��������

The reaction of the mass transfer (����h����) with respect to changes in the Schmidt number �������� for variations in the thermophoresis term �������� and Brownian motion �������� is demonstrated in Fig. 15. Clearly, the presence of �������� boosts mass transfer (����ℎ����) whereas the an increase in �������� reduces ����ℎ���� as depicted in that figure.

[redacted]

Fig. 16 Variations of ���� & ���� on ����̅��������� **Fig. 17** Variations of �������� & ���� on ����̅���������

The drag force (����̅���������) is strengthened with higher values of the Casson fluid term ���� and the nonlinear stretching term ���� as illustrated in the Fig. 16. However, for fixed values of ���� and ����, a rise in the nonlinear convection parameter ����$_1$ reduces ����̅���������. Likewise, a hike in the Darcy term �������� raises ����̅��������� in the existence or otherwise of the velocity ratio parameter ���� as depicted in Fig. 17. Meanwhile, the inclusion of ���� decreases the skin friction coefficient as noticed in this figure. Meanwhile, the presence of the wall temperature exponent term ���� in the thermal field causes an improvement in the heat transfer mechanism (���� � ��������) whereas in the presence of thermophoresis and Brownian motion terms, the heat transfer at the sheet surface reduces as demonstrated in Fig. 18. Similarly, ���� � �������� depreciates with higher values of radiation term �������� and the thermal conductivity parameter ���� as displayed in Fig. 19.

Fig. 18 Effects of ���� & �������� on ���� � ��������� **Fig. 19** Impact of �������� & �������� on ���� � ��������

5 Conclusion

A numerical analysis has been performed on the transport of a quadratic mixed convection hydromagnetic Casson nanofluid in the neighbourhood of a stagnation point. The flow is configured in a two-dimensional nonlinear vertically stretchable sheet enclosed in a porous medium with the impact of nonlinear thermal radiation coupled with variable thermal conductivity and non-uniform [redacted]

heat source/sink. The nonlinear boundary value problem has been numerically tackled with the Runge-Kutta Fehlberg coupling shooting technique. The solutions are presented graphically and deliberated while comparisons with earlier studies show good agreement in the limiting situations. The following points have been observed in the analysis carried out in this study:

• A rise in the Casson fluid parameter ���� strengthens the skin friction coefficient and thermal field while it shrinks the hydrodynamic boundary structure and the fluid flow decelerates. • Growth in the velocity ratio parameter ���� upsurges fluid velocity whereas the viscous drag effect declines with it while the converse is true for the nonlinear stretching term ����. • The fluid flow decelerates with rising values of the magnetic field term ���� and Darcy parameter �������� whereas there is an improvement in the Casson nanofluid temperature with the increased values of the radiation ��������, temperature ratio term ��������, thermophoresis �������� and Eckert number �������� parameters.

• Mass transfer improves with higher values of Schmidt number ��������. Also, the concentration profile advances with thermophoresis term �������� while the reverse is the case for the Brownian motion ��������. • Skin friction coefficient is strengthened due to higher velocity ratio and Darcy terms as the Nusselt number appreciates in respect of the wall temperature exponent tern ���� while it diminishes with thermophoresis �������� and Brownian motion parameters ��������.

Nomenclature

����, ����: Cartesian coordinates [����]

����, ����: Velocity components [��������$^{-1}$]

����: Fluid temperature [����]

����: Species concentration [����������������¯3]

��������: Species concentration at the sheet [����������������¯3]

����$_{\infty}$: Free stream species concentration [����������������$^{-}$3]
����$_{����}$: Fluid temperature at wall [����]
����$_{\infty}$: Free stream temperature [����]
����: Acceleration due to gravity [��������$^{-2}$]
����$_{����}$: mass diffusivity [����^{2}����$^{-1}$]
����$'''$: Non-uniform heat source/sink [��������$^{-3}$����$^{-1}$]
����$_{����}$: Permeability of the porous media [����^{2}]
����$_{0}$: Magnetic field strength [����/����]
����$_{����}$: Heat flux at the surface of the plate [��������$^{-2}$]
����$_{����}$: Specific heat at constant pressure [����/������������]
����$_{����}$: Heat flux at the surface [��������$^{-2}$]
����$_{����}$: Mass flux at surface [��������$^{-2}$]
����: Dimensionless stream function
��������$_{����}$: Local Grashof number
��������$_{����}$: Local solutal Grashof number
��������: Prandtl number
��������: Schmidt number
��������: Eckert number
��������: Darcy number
����: Magnetic field parameter
����: Velocity ratio term
����$_{����}$ Temperature ratio parameter

Greek symbols

����: Ratio of concentration to buoyancy force
����$_{����}$: Density of the fluid [������������$^{-3}$]
����: Stream function [����^{2}����$^{-1}$]
[redacted]
����: electric conductivity [��������$^{-1}$]
����$_{����}$: Dynamic viscosity of the fluid [������������$^{-1}$����$^{-1}$]
����$_{����}$: Kinematic viscosity [����^{2}����$^{-1}$]
����$_{1}$: Coefficient of linear thermal expansion [����$^{-1}$]
����$_{2}$: Coefficient of nonlinear thermal expansion [����$^{-1}$]
����$_{3}$: Coefficient of linear solutal expansion [����$^{-1}$]
����$_{4}$: Coefficient of nonlinear solutal expansion [����$^{-1}$]
����: Dimensionless scaling transformation variable
����(����): Dimensionless temperature
����(����): Dimensionless nanoparticles species concentration

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