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Robust Scale Estimator-Based Control Charts for Marshall-Olkin Inverse Log-Logistic Distribution

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Abstract. The combination of Shewhart $\overline{X}$ and S-control charts developed to control the process variability based on the assumption that the underlying distribution of the quality characteristic is normal. The normality assumption is often violated when the underlying distribution of the characteristic under consideration is skewed; therefore, the use of the Shewhart $\overline{X}$ and S control charts on real-life data might lead to inaccurate estimation of control limits. The robust methods of estimation of control chart statistics can be used in such situations. In this paper, the robust scale estimator was used to estimate the mean, variance and median based on Marshall-Olkin Inverse log-logistic distribution. Monte Carlo simulation study was conducted using Marshall-Olkin Inverse log-logistic distribution to determine the performance of the proposed method in comparison with the Shewhart S and MAD methods. The proposed control limits showed an improvement over the Shewhart and stocktickerMAD control charts for non-normal process.

Keywords: Marshall-Olkin inverse log-logistic distribution, non-normality, process variability, performance evaluation, robust scale estimator.

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1. Introduction

The main objective of Statistical Process Control (SPC) is to monitor, improve process performance and reduce variability. SPC Charts are vital tools for studying process variability and detecting quality improvements or quality deteriorations. The major purpose of control chart is to detect the occurrence of assignable and common causes so that the necessary remedial action(s) may be taken before a large number of non-conforming products are manufactured (Chou *et al.* 2001). Shewhart control charts are based on the assumption that quality measurements are independently and identically normally distributed. However, the assumption of an independently and identically distributed normal population is invalid in many cases, especially encountered frequently in real-application. Therefore, if the quality characteristics under study are not normally distributed, the traditional way of designing the control chart may reduce the ability that a control chart detects the assignable causes (Chou *et al.* 2001). Specifying the control limits, which is a function of mean and variance, is of utmost importance in designing a control chart. Improper estimation of the process dispersion which results in narrower or wider limits can increase the probability of type I error or the probability of type II error (Shahriari *et al.* 2009). Most of the standard control charts use a mathematical model to estimate the mean, variance, median and other statistics from the available data to assess the action lines. By doing this, it is assumed that the underlying distribution fits the symmetrical normal pattern. It is best to make use of probability distribution that best fits the available data set, estimate the parameters of the distribution and calculate various statistics, based on the underlying distribution, needed to construct the control limits. This is the focus of this paper.

There are many robust control chart methods available in literature. Some of them are Rocke (1989, 1992) who proposed trimmed Mean and Inter Quartile Range chart and Median and Range chart to control pro-

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cess mean. Schoonhoven *et al.* (2011) used Tatum estimator and trimean value based on quantiles. Wu *et al.* (2002) considered the Median of the Absolute Deviation from the Median (MDM), the Average Absolute Deviation from the Median (ADM) and the Median of the Average Absolute Deviation (MAD) as alternatives for the sample standard deviation to investigate their effect on $\bar{X}$ control chart performance. Abu-Shawiesh (2009) used the Sample Median (MD) to estimate the process mean, μ, and MAD to estimate the dispersion σ. Figueiredo and Gomes (2009) used Bootstrap based median, range, total median and total range statistics to control process mean. Maddahi *et al.* (2011) presented a robust mean control chart based on M-estimators in presence of outliers. Adekeye (2012), Adekeye and Azubuike (2012) used sample mean with standard deviation estimated using MAD to construct control chart limits for non-normal process. Schoonhoven and Does (2013) used trimean (TM) and Adaptive Trimmed Standard deviation (ATS) to control process. Sindhumol and Srinivasan (2015) introduced a dispersion chart based on MMLE. Sindhumol *et al.* (2016a,b) developed control charts for location and dispersion based on trimmed mean and Modified Trimmed Standard Deviation (MTSD). Wang *et al.* (2017) proposed control charts for monitoring the lower Weibull percentiles under complete data and Type II censoring, Srinivasa Rao (2018) considered an exponentiated half logistic distribution to develop an attribute control chart for time truncated life tests with known or unknown shape parameter. Adewara and Aako (2018) derived the control limits of variable control charts based on percentiles of exponentiated lomax distribution.

This paper presents robust location and dispersion charts based on Marshall-Olkin Inverse Log-logistic Distribution. The proposed method is used to construct control limits and its performance is also compared with existing control charts.

2. Marshall-Olkin inverse log-logistic distribution

Marshall-Olkin Inverse Log-logistic Distribution (MOILLD) is a two-parameter distribution derived by generalizing Inverse Log-logistic Distribution using the Marshall-Olkin G family of distribution.

The probability density function (pdf) of the Inverse Log-logistic (ILL) distribution as defined by Para and Jan (2017) is giving by

$$f(x,\gamma) = \frac{\gamma}{x^{\gamma+1}(1+x^{-\gamma})^2}, \qquad x>0, \gamma>0, \tag{1}$$

its corresponding cumulative distribution function (cdf) is given by

$$F(x,\gamma) = \frac{1}{1+x^{-\gamma}}, \qquad x>0, \gamma>0, \tag{2}$$

and the survival function is given by

$$\bar{F}(x,\gamma) = \frac{x^{-\gamma}}{1+x^{-\gamma}}, \qquad x>0, \gamma>0, \tag{3}$$

where γ is the shape parameter.

The survival function of Marshall-Olkin G family of distribution is given by

$$\bar{G}(x) = \frac{\alpha\bar{F}(x)}{1-\bar{\alpha}\bar{F}(x)}, \tag{4}$$

where $\alpha>0$, $\bar{\alpha}=1-\alpha$ and $-\infty<x<\infty$. The corresponding probability density function (pdf) is given by

$$g(x) = \frac{\alpha f(x)}{\left(1-\ \bar{\alpha}\bar{F}(x)\right)^2}, \tag{5}$$

where $\alpha>0$, $\bar{\alpha}=1-\alpha$ and $-\infty<x<\infty$.

Marshall-Olkin Inverse Log-logistic Distribution (MOILLD) is derived by inserting Equation (3) in Equation (4) to get the survival function and Equations (1) and (3) in Equation (5) to get the corresponding density function.

The survival function of MOILLD is given by

$$\bar{G}\,(x) = \frac{x^{-\gamma}}{(1+x^{-\gamma})-(1-\alpha)x^{-\gamma}}.$$

Therefore,

$$\bar{G}(x) \;=\; \frac{\alpha x^{-\gamma}}{1+\alpha x^{-\gamma}}, \qquad x>0,\gamma>0,\alpha>0. \tag{6}$$

The cumulative distribution function (cdf) is given by

$$G\,(x) = 1-\bar{G}(x) = 1-\frac{\alpha x^{-\gamma}}{1+\ \alpha x^{-\gamma}}$$

$$G\,(x) = \frac{1}{1+\ \alpha x^{-\gamma}}, \qquad x>0,\gamma>0,\alpha>0. \tag{7}$$

and the corresponding probability density function (pdf) of MOILLD is given by

$$g(x) = \frac{\alpha\gamma}{x^{\gamma+1}\,(1+\ \alpha x^{-\gamma})^{2}}, \qquad x>0,\gamma>0,\alpha>0. \tag{8}$$

Figure 1 presents the pdf plot of MOILLD for different values of the parameters γ *and* α. It is evident that the distribution of the Marshall-Olkin inverse Log-logistic random variable X is positively skewed and it is unimodal. The skewness tends to zero as the parameters values increase simultaneously.

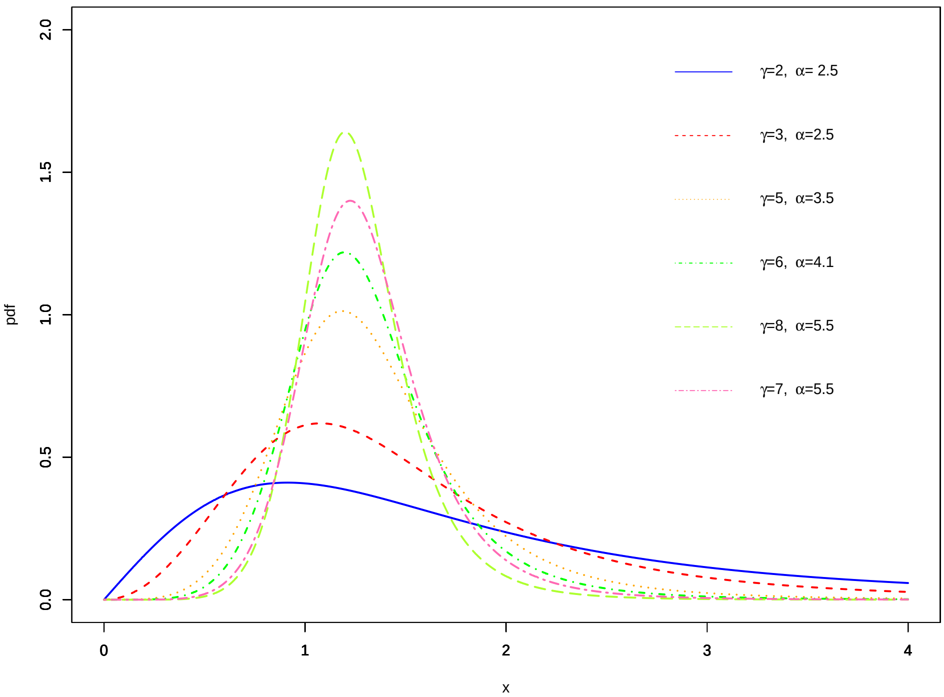

Figure 1: Probability density plot of MOILLD

The argument for using the Marshall-Olkin Inverse Log-logistic model was borne out from the fact that:

(1) The Marshall-Olkin Inverse Log-logistic distribution is simpler and efficient in modeling skewed distribution. Other distributions characterized by three or more parameters may be complex.
(2) The quantile, mean, variance and mode of the Marshall-Olkin Inverse Log-logistic distribution are very simple to handle.

It should be noted, that practitioners of control charts may not be experts in process control, thus, the proposed simpler, but efficient control charts based on Marshall-Olkin Inverse Log-logistic distribution can be used to monitor the stability and performance of skewed process data.

To use the estimates of the parameters of the MOILLD to construct control limits, we will determine the mean, variance and quantile of the distribution which are the important estimates required in constructing the control limits in a 3-sigma approach which we adopt in this work.

2.1 *Moments*

Let X be a random variable that has the MOILLD, then, the rth non-central moments was obtained to be

$$E(X^r) = \int_0^\infty x^r g\left(x\right)dx = \int_0^\infty x^r \frac{\alpha\gamma}{x^{\gamma+1}\left(1+\ \alpha x^{-\gamma}\right)^2}\,dx$$

$$= \int_0^\infty \frac{\alpha\gamma}{x^{\gamma-r+1}\left(1+\ \alpha x^{-\gamma}\right)^2}\,dx$$

After simplification using Maple 17 software mathematics function,

$$E(X^r) = \alpha^{\frac{r}{\gamma}}\frac{\pi r}{\gamma}\mathrm{csc}(\frac{\pi\left(\gamma-r\right)}{\gamma}), \qquad r>0, \gamma>0, \alpha>0, \tag{9}$$

where csc(.) is the cosecant function. From Equation (9), the first two moments about the origin for MOILLD were obtained by substituting r =1 and r =2, respectively. Thus, the first moment which is the mean is

$$E(X) = \alpha^{\frac{1}{\gamma}}\frac{\pi}{\gamma}\mathrm{csc}(\frac{\pi\left(\gamma-1\right)}{\gamma}), \tag{10}$$

and the second moment is

$$E(X^2) = 2\alpha^{\frac{2}{\gamma}}\frac{\pi}{\gamma}\mathrm{csc}(\frac{\pi\left(\gamma-2\right)}{\gamma}). \tag{11}$$

Using Equations (10) and (11), the variance of MOILLD is obtained to be

$$Var(X) \;=\; E(X^2) -\; [E(X)]^2 = 2\alpha^{\frac{2}{\gamma}}\frac{\pi}{\gamma}\mathrm{csc}\left(\frac{\pi\left(\gamma-2\right)}{\gamma}\right) \;-\left(\alpha^{\frac{1}{\gamma}}\frac{\pi}{\gamma}\mathrm{csc}\left(\frac{\pi\left(\gamma-1\right)}{\gamma}\right)\;\right)^2$$

$$= \alpha^{\frac{2}{\gamma}}\frac{\pi}{\gamma}\left(2\mathrm{csc}\left(\frac{\pi\left(\gamma-2\right)}{\gamma}\right) \;-\frac{\pi}{\gamma}\mathrm{csc}^2\left(\frac{\pi\left(\gamma-1\right)}{\gamma}\right)\right). \tag{12}$$

2.2 *Quantile and random number generation from MOILLD*

The random numbers from MOILLD distribution are generated by solving the equation obtained on equating the cumulative distribution function of the distribution to a number q. Let q be a uniform variate on the interval (0, 1), the random numbers from the MOILLD is given by:

$$x_q = \left[\frac{1}{\alpha}\left(\frac{1}{q} \;-- 1\right)\right]^{-\frac{1}{\gamma}}. \tag{13}$$

If q = 0.25, q = 0.5 and q = 0.75, the resulting solutions are the first quartile (Q_1), median (Q_2) and third quartile (Q_3) respectively. Hence, we have

$$Q_1 = x_{0.25} = \left(\frac{3}{\alpha}\right)^{-\frac{1}{\gamma}}, \tag{14}$$

$$Q_2 = x_{0.5} = \left(\frac{1}{\alpha}\right)^{-\frac{1}{\gamma}}, \tag{15}$$

$$Q_3 = x_{0.75} = \left(\frac{5.3333}{\alpha}\right)^{-\frac{1}{\gamma}}. \tag{16}$$

2.3 *Maximum likelihood estimation of MOILLD*

Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample of size n from MOILLD, then the likelihood function is given by

$$L(x/\alpha, \gamma) = \prod_{i=1}^{n} \frac{\alpha\gamma}{{x_i}^{\gamma+1}\,(1+\ \alpha {x_i}^{-\gamma})^2}; x > 0, \gamma > 0, \alpha > 0. \tag{17}$$

By taking logarithm of Equation (12), the log-likelihood function

$$l = logL(x/\alpha, \gamma) = nlog(\alpha\gamma) - (\gamma + 1)\sum_{i=1}^{n} \log(x_i) - 2\sum_{i=1}^{n} \log(1 + \alpha {x_i}^{-\gamma}). \tag{18}$$

To obtain the MLE's of α and γ, differentiate the log-likelihood function in Equation (18) with respect to α and γ. Thus, we have

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - \frac{2n\ \sum_{i=1}^{n}\ {x_i}^{-\gamma}}{1+\ \alpha \sum_{i=1}^{n}\ {x_i}^{-\gamma}}. \tag{19}$$

and

$$\frac{\partial l}{\partial \gamma} = \frac{n}{\gamma} - nln\left(\sum_{i=1}^{n} x_i\right) + \frac{2n\alpha \sum_{i=1}^{n}\ {x_i}^{-\gamma} ln\left(\sum_{i=1}^{n} x_i\right)}{1+\ \alpha \sum_{i=1}^{n}\ {x_i}^{-\gamma}}. \tag{20}$$

To find the estimate of α and γ, we set Equations (19) and (20) to zero. The two estimates were obtained numerically by maximizing the log-likelihood function using the Newton-Raphson method.

3. Proposed $\bar{X}$ and median absolute deviation (MAD) charts for MOILLD

Let X_{ij} be an independent and identical random sample of size *n* taken over *m* subgroup, *i* = 1, 2, ..., *n* and *j* = 1, 2, ..., *m*. If the underline distribution is normal, then an unbiased estimator of *S* is $C_4\sigma$, where

$C_4 = \left(\frac{2}{n} - 1\right)^{\frac{1}{2}} \left[\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}\right]$ and the standard deviation of S is $\sigma\sqrt{1 - C_4^2}$. Therefore, the 3-sigma control limits for S- chart is given by

$$\begin{cases} UCL = C_4\sigma + 3\sigma\sqrt{1 - C_4^2} \\ CL = C_4\sigma \\ LCL = C_4\sigma - 3\sigma\sqrt{1 - C_4^2} \end{cases} \tag{21}$$

When the distribution is skewed and non-normal, the standard deviation can be estimated using the Median Absolute Deviation (MAD) of the data to construct control limits for a skewed distribution. However, an estimate may not give an accurate and precise value of the parameter required, and hence the validity of the inference and conclusions drawn can be challenged. The proposed method uses MOILLD to model the data, estimate the parameters of the distribution using maximum likelihood estimation and then estimate the standard deviation using MAD based on the underlying distribution. The mean and median of the distribution were calculated using Equations (10) and (15), respectively. When MAD from MOILLD is used as an estimate of variability, then

$$\widehat{\sigma} = b_n \overline{MAD^*} \tag{22}$$

where $\overline{MAD^*} = \frac{1}{m} \sum_{j=1}^{m} MAD_j$ is the median absolute deviation for MOILLD and b_n is a function of the sample size *n*.

$$MAD_j = Median \left| X_{ij} - MD \right|. \tag{23}$$

where $MD = x_{0.5} = \left(\frac{1}{\alpha}\right)^{-\frac{1}{\gamma}}$ from Equation (15). Thus, the control limits and the central line for the Shewhart $\bar{X}$ and MAD charts are given as follows;

The MAD - control chart limits are

$$\begin{cases} UCL_s = B_4 b_n \overline{MAD^*} \\ CL_s = b_n \overline{MAD^*} \\ LCL_s = B_3 b_n \overline{MAD^*} \end{cases} \tag{24}$$

and the corresponding control limits for the $\overline{X}$ chart when the sigma is unknown are

$$\begin{cases} UCL = \mu + 3\frac{b_n \overline{MAD^*}}{\sqrt{n}} \\ UCL = \mu \\ LCL = \mu - 3\frac{b_n \overline{MAD^*}}{\sqrt{n}} \end{cases} \tag{25}$$

where $\mu = E\left(X\right) = \alpha^{\frac{1}{\gamma}} \frac{\pi}{\gamma} \csc(\frac{\pi(\gamma-1)}{\gamma})$ as given in Equation (10).

Let $A_6 = 3\frac{b_n}{\sqrt{n}}$, then Equation (25) will be reduced to:

$$\begin{cases} UCL = \mu + A_6 \overline{MAD^*} \\ UCL = \mu \\ LCL = \mu - A_6 \overline{MAD^*} \end{cases} \tag{26}$$

The value of the control chart constants required for the calculation of the proposed control limits in Equations (24) and (26) can be found in Adekeye *et al.* (2012). Also, the value of the constants B_3, B_4, and C_4 can be found in any standard statistical quality control textbook (Montgomery, 2012).

3.1 *Performance evaluation*

This paper employed Control Limit Interval (CLI) and Average Run Length (ARL) to evaluate the performance of the control charts. The CLI is the difference between the control limits value. Therefore, the CLI for the S-control charts will be determined using the expression:

$$CLI_s = \left[6\sqrt{1 - C_4{}^2}\right] b_n \overline{MAD^*} \quad (27)$$

and the CLI for the textitX control chart will be given by the expression:

$$CLI_s = \frac{6b_n}{\sqrt{n}} \overline{MAD^*} \quad (28)$$

A control chart is declared to be superior if its Control Limit Interval is the smallest out of all the control charts under consideration.

The Average Run Length (ARL) is the average number of points that must be plotted before a point showed out-of-control signal. It is defined as the inverse of the probability that any point exceeds the control limits. The ARL of Shewhart control chart in the in-control process state can be varied considerably as the standard deviation of the distribution of run length is high (Sindhumol and Srinivasan, 2015). When the process is in-control, the in-control ARL of Shewhart control chart is expected to be close to 370.4, while the out-of-control ARL should be small.

4. Results

4.1 *Simulation study*

Two data sets were simulated and arranged in 3, 5 and 10 sample sizes with 30 subgroups using the Marshall-Olkin Inverse Log-logistic distribution with parameters $\alpha = 3.5, \gamma = 2.8$ and $\alpha = 4, \gamma = 3.5$. The $\bar{X}$ and MAD^* control limits for different sample sizes with 30 subgroups were computed. The proposed control limits in Equations (24) and (26) were computed together with control limit interval and average run length to evaluate the performance. The results are presented in Tables 1 and 2 alongside with the $\bar{X}$ and S and $\bar{X}$ and MAD for mean and dispersion charts respectively.

Table 1: Control Limits Interval (CLI) and Average Run Length (ARL) for mean charts

Parameter	Method	n=3		n=5		n=10		n=20	
		CLI	ARL	CLI	ARL	CLI	ARL	CLI	ARL
$\alpha = 3.5$ $\gamma = 2.8$	$\bar{X}\&S$	2.464	15	1.807	10	1.238	5	0.8591	3.3333
	$\bar{X}\&MAD$	2.513	15	1.570	10	1.001	5	0.6783	3.0000
	$\bar{X}\&MAD^*$	3.0360	23.971	1.8972	7.7194	1.2091	3.0037	0.8196	1.9254
$\alpha = 4.0$ $\gamma = 3.5$	$\bar{X}\&S$	2.149	30	1.576	10	1.080	5	0.7493	3.7500
	$\bar{X}\&MAD$	2.241	30	1.401	10	0.893	5	0.6051	3.0000
	$\bar{X}\&MAD^*$	2.6967	28.0287	1.6851	8.3012	1.0740	3.2023	0.7280	2.0140
$\alpha = 4.5$ $\gamma = 5.8$	$\bar{X}\&S$	2.0325	30	1.4905	10	1.0215	5	0.7088	3.7500
	$\bar{X}\&MAD$	2.1539	∞	1.3459	10	0.8578	5	0.5815	3.0000
	$\bar{X}\&MAD^*$	2.5828	35.162	1.6139	9.5286	1.0286	3.5340	0.6972	2.1484

4.2 *Real life application*

The data on asthma patients' length of stay (in days) on admission retrieved from the records of Asthmatic in-patients of Ogun State Hospital, Ijebu-Ode was used to assess the performance of the charts. The asthma patients' data is an individual observation data. To use Shewhart charts for analyzing individual observation data, one natural idea is to "create" grouped data, by grouping observations collected at consecutive time points (Qiu, 2014). 185 patients' data were retrieved and grouped into 37 subgroups of 5 units as presented in Table 3.

The Jarque Bera test for normality was obtained to be 39.908 with a p-value of 2.2e-16. It implies that the data is not normally distributed. Furthermore, the skewness coefficient was computed to be 1.1257; hence, the data is a positively skewed data.

Table 2: Control Limits Interval (CLI) and Average Run Length (ARL) for dispersion charts

Parameter	Method	n=3		n=5		n=10		n=20	
		CLI	ARL	CLI	ARL	CLI	ARL	CLI	ARL
$\alpha = 3.5$ $\gamma = 2.8$	S	1.622	∞	1.320	15	0.905	5	0.6191	1.8750
	MAD	1.863	∞	1.222	30	0.755	2	0.4954	1.6667
	MAD^*	2.2508	6.6591	1.477	1.9817	0.9126	1.2810	0.5987	1.1292
$\alpha = 4.0$ $\gamma = 3.5$	S	1.415	∞	1.151	15	0.789	7.5	0.5399	2.1429
	MAD	1.662	∞	1.090	30	0.674	2	0.4420	1.6667
	MAD^*	1.9992	5.1242	1.3119	1.5361	0.8106	1.1250	0.5317	1.0511
$\alpha = 4.5$ $\gamma = 5.8$	S	1.3384	∞	1.0887	30	0.7463	7.5	0.5107	2.3077
	MAD	1.5968	∞	1.0478	30	0.6474	2.143	0.4247	1.6667
	MAD^*	1.9147	3.8371	1.2565	1.2576	0.7763	1.0465	0.5092	1.0163

Table 3: Asthma patients' length of stay on admission

	X1	X2	X3	X4	X5
1	2	66	23	7	27
2	1	30	42	7	55
3	12	23	24	7	66
4	10	22	52	2	44
5	32	2	28	22	12
6	3	17	37	7	2
7	4	2	49	6	1
8	56	3	3	33	32
9	63	13	4	7	43
10	51	23	3	7	2
11	3	22	6	7	65
12	34	56	4	21	1
13	42	4	45	2	21
14	65	7	2	2	3
15	55	4	6	3	4
16	66	8	6	1	5
17	6	12	7	45	42
18	1	4	9	33	61
19	3	8	29	38	6
20	1	9	22	33	21
21	2	2	2	42	41
22	5	48	2	45	23
23	7	7	3	3	32
24	72	7	7	45	71
25	23	55	3	65	22
26	21	3	4	5	33
27	45	67	5	6	52
28	44	16	15	12	3
29	42	6	7	8	4
30	42	3	7	21	5
31	10	12	7	42	11
32	12	8	7	33	65
33	2	5	6	5	9
34	1	4	6	4	1
35	1	8	87	4	10
36	4	43	7	6	13
37	4	1	7	7	10

Table 4: Control Limits Interval (CLI) and Average Run Length (ARL) for mean and dispersion charts

	$\bar{X}$ Charts		S Charts	
	CLI	ARL	CLI	ARL
[SH]	56.5136	∞	41.2787	∞
[MAD]	41.8826	∞	32.6070	7.4
[MAD*]	46.1786	3.0833	35.9516	18.5

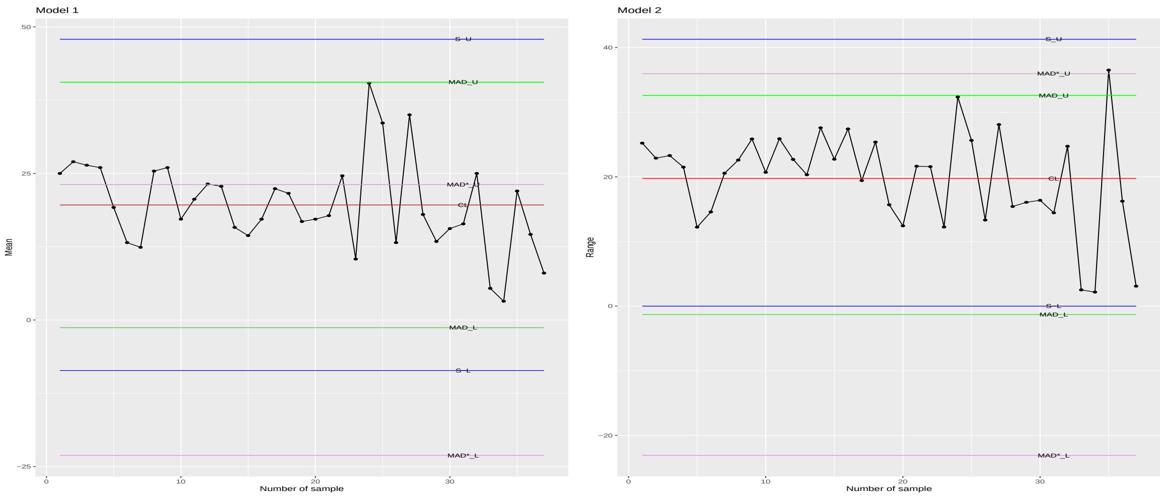

Figure 2: Mean and S- charts for asthma patients' length of stay using Shewhart S-, MAD and MAD* control charts

5. Discussion of results

The evaluation parameters, the CLI and the ARL used in this study revealed the following

(1) The CLI for mean chart of the proposed MAD* is wider compare to the other methods for n= 3 and 5, and shorter than that of $\bar{X}$/S chart for n=10 and 20.

(2) The CLI for dispersion chart of the proposed MAD* is wider compare to the other methods for n= 3, 5 and 10, and shorter than that of S-chart for n=20.

(3) The ARL for the mean chart of the proposed MAD* is small compare to the other methods for n=5, 10 and 20. $\bar{X}$/S and $\bar{X}$/MAD have the same ARL for n=3, 5 and 10.

(4) The ARL for the dispersion chart of the proposed MAD* is small compare to the other methods for n=3, 5, 10 and 20. $\bar{X}$/S and $\bar{X}$/MAD charts have the same ARL for n=3. The ARL of $\bar{X}$/S chart is less than that of $\bar{X}$/MAD chart for n=5 while the ARL of $\bar{X}$/MAD chart is less than that of $\bar{X}$/S chart for n= 10 and 20.

(5) The charts behave the same way with varying parameters.

(6) As the number of sample is increasing, both the CLI and the ARL are decreasing.

(7) The CLI of $\bar{X}$/MAD chart is less than that of other methods. The ARL of the proposed mean chart is less than that of other methods. Figure 1 shows the placement of the control limits of the three methods for both mean and dispersion charts. Hence, the results of the real data application show that the proposed $\bar{X}$/MAD* charts performed better than the other methods.

6. Conclusion

This research work has been able to evaluate Robust Scale Estimator Based Control Charts for MOILLD for monitoring process data when the standard value of sigma is not given and Median Absolute Deviation for MOILLD (MAD*) is used as the estimate. The ARL values in Table 2 and 3 clearly showed that control limits of MAD for MOILLD are more accurate compared to the earlier control limits for all the distributional data sets under consideration to detect out-of-control. The CLI values indicate $\bar{X}$/S and $\bar{X}$/MAD charts have a tight control limits compare to the proposed MAD* chart which could raise a false alarm when there is none. The results of the real data application show that the proposed $\bar{X}$/MAD* performed better than the other methods. This indicates that the median absolute deviation for MOILLD is more accurate and more suitable to construct control limits than the standard deviation and MAD proposed by Adekeye *et al*. (2012) for highly skewed data. Therefore, it is recommended that the proposed control limits for mean and dispersion charts be used when monitoring past/skewed data when there is no specified standard value for sigma. Thus, control chart users are advised to first investigate the distribution of the process data (real-life data) before the use of a control chart.

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Appendix

R code

```
Nsim=30; R=3
a1=4.5; b1=5.8;
set.seed(??)
x=replicate(R,(((1/runif(Nsim))-1)/b1)^(-1/a1))
x=as.data.frame(x,row.names = NULL)
####### for S
SD=apply(x,1,sd) #median absolute deviation
Xbar1=apply(x,1,mean)
Xbarbar=mean(Xbar1)
bn=1.495; B3=0; B4=2.568; A3=1.95; A6=2.5893
#X-bar chart
UCL=Xbarbar+A3*mean(SD); CL=Xbarbar; LCL=Xbarbar-A3*mean(SD)
CLI1=UCL-LCL
```

```
#S chart
UCL_s=B4*mean(SD); CL_s=mean(SD); LCL_s=B3*mean(SD)
CLI_s1=UCL_s-LCL_s
#ARL
cval <-(LCL < Xbar1 & Xbar1< UCL)
lnside<- sum(cval)/30
ARL1=1/(1-lnside)
cvals <-(LCL_s < SD & SD< UCL_s)
lnside<- sum(cvals)/30
ARLs1=1/(1-lnside)
##### for MAD
MAD=apply(x,1,mad) #median absolute deviation
Xbar1=apply(x,1,mean)
Xbarbar=mean(Xbar1)
#X-bar chart
UCL=Xbarbar+A6*mean(MAD); CL=Xbarbar; LCL=Xbarbar-A6*mean(MAD)
CLI2=UCL-LCL
#MAD chart
UCL_s=B4*bn*mean(MAD); CL_s=bn*mean(MAD); LCL_s=B3*bn*mean(MAD)
CLI_s2=UCL_s-LCL_s
cval <-(LCL < Xbar1 & Xbar1< UCL)
lnside<- sum(cval)/30
ARL2=1/(1-lnside)
cvals <-(LCL_s < MAD & MAD< UCL_s)
lnside<- sum(cvals)/30
ARLs2=1/(1-lnside)
##### MAD for MOILLD
lik<-function(par){
a<-par[1]; g<-par[2]
n*log(a)+n*log(g)- (g+1)*sum(log(x))-2*sum(log(1+a*x∧(-g)))
}
d<- maxLik::maxLik(lik,start=c(1,1))
a=d$estimate[[1]]; g=d$estimate[[2]]
Xbar=(a∧(1/g)*(3.142/g))/sin(pi*(g-1)/g)
MD=a∧(1/g)
y=abs(x-MD)
MAD=apply(y,1,median)
#X-bar chart
UCL=Xbar+A6*mean(MAD); CL=Xbar; LCL=Xbar-A6*mean(MAD)
CLI3=UCL-LCL
#MAD* chart
UCL_s=B4*bn*mean(MAD); CL_s=bn*mean(MAD); LCL_s=B3*bn*mean(MAD)
CLI_s3=UCL_s-LCL_s
G<- function(x,a,g) {
1/(1+ a*x∧(-g) ) }
b=G(UCL_s,a,g)-G(LCL_s,a,g)
ARLs3=1/(1-b)
b=G(UCL,a,g)-G(max(0,LCL),a,g)
ARL3=1/(1-b)
c1=rbind(CLI1,CLI2,CLI3)
c2=rbind(CLI_s1,CLI_s2,CLI_s3)
A1=cbind(c1,c2)
a1=rbind(ARL1,ARL2,ARL3)
a2=rbind(ARLs1,ARLs2,ARLs3)
A=cbind(a1,a2)
```